CENTRO DE INVESTIGACIÓN Y DOCENCIA ECONÓMICAS, A.C.

QUANTAL RESPONSE EQUILIBRIUM IN A CITIZEN-CANDIDATE EXPERIMENT

TESINA

QUE PARA OBTENER EL GRADO DE

MAESTRO EN ECONOMÍA

PRESENTA

DARIO TRUJANO OCHOA

DIRECTOR DE LA TESINA:

DOCTOR, ALEXANDER ELBITTAR

CIUDAD DE MÉXICO JUNIO, 2017

Este trabajo va para las noches de desvelo,
por todos los cigarros fumados y el cafe bebido,
para mi ventana en "el para" y
para esa habitación donde el tiempo siempre corrió demasiado rápido.
Dedico esto a las estrellas.

Agradecimientos

Quiero agradecer al CIDE y a los profesores que me dieron clase por permitir que un psicólogo se metiera a estudiar Economía. En especial a Alexander Elbittar, con quien trabajé este proyecto y me convenció de que esto era lo que estaba buscando.

Un agradecimiento a todos las personas que conocí en este lugar: compañeros y amigos que nos vimos sufrir y esforzarnos para ver si lo lográbamos: ¡Al final lo logramos! Recordaré a Paquiñe, Victor, Roger Danilo, Darwin, Christian, los demás cuzcos y al kinder team. Hay tantas anécdotas entre nosotros, que están más allá de este pequeño espacio, y no sé si sería apropiado mencionarlas. Pero recordaremos los cumpleaños de Leonel, el centro, el para y demás lugares. Estoy seguro que a todos nos irá bien después de esto. Al final, aprendimos a ser resistentes.

Igualmente quisiera agradecer a Gustavo, a Inti y a los demás que vivimos enfrente; vimos como nos desvelábamos, a veces desesperábamos y muchas veces festejábamos. Fuera de la ME, recordaré también a todas las personas por quienes incluso decían que ya era mappero. Aprendí de todos pequeñas y grandes cosas, y nunca lo olvidaré.

Un agradecimiento enorme a toda mi familia, quienes ya no deben reconocerme, no sólo porque estos dos años me han cambiado para bien, sino porque se les debe estar olvidando como es mi cara. Gracias a mis padres, que siempre estuvieron apoyándome y me dieron la seguridad de que, pasara lo que pasara, siempre estarían ahí para mí. Han sido un gran ejemplo toda mi vida. Este trabajo el el fruto mi esfuerzo y su confianza; espero estén orgullosos como yo lo estoy de ustedes.

Contents

List of figures

List of tables

Introduction

In any democracy, politicians have to decide if they run for office before elections. We suppose they are strategic in doing so: politicians consider possible payoffs and their own chances, which depend on electoral preferences and who their adversaries are going to be. Nevertheless, few people would categorically affirm that politicians, as any other person, are completely rational: choosing always the more profitable option, as Nash equilibrium assumes. Quantal Response Equilibrium (QRE) (McKelvey and Palfrey, 1995; Goeree, Holt, and Palfrey, 2016) relax perfect maximizing assumption. A decision making theory, as this, is needed to understand and predict who campaigns and why, along with a model about the political process. In order to do that, Citizen-Candidate model (Osborne and Slivinski, 1996) was implemented and its results compared with the QRE prediction.

In this thesis I address the next question: can the QRE theory, with more realistic assumptions, explain better the behavior than Nash Equilibrium (NE) in an experiment based in Citizen-Candidate model?

The contribution of this thesis is to answer this question with the analysis of data from the election experiment assuming that agents are not rational, but they are strategic. In this sense, this work is located at the intersection of two perspectives: first, Behavioral Game Theory (Camerer, 2003; Gächter, 2004), where QRE can be considered; and second, Political Economy (Besley, 2007), where the citizen-candidate model is an important instance. I conclude that, considering stochastic and non-perfect maximizing decision rule, it is possible to better describe candidates' decisions in the experiment.

In political economics, it is more common to study voters' behavior (Besley, 2007), and candidates' behavior is frequently studied trying to prove median voter theorem under different assumptions (Palfrey, 2009). In contrast with the Hotelling-Downs model, candidate's ideal policy matters in the Citizen-Candidate model because, once in office, they freely chose the policy they prefer. This model has been empirically implemented before (Cadigan, 2005; Elbittar and Gomberg, 2009), but results has been compared with the Nash equilibrium. Considering more realistic assumptions about behavior lead to more precise description and understanding of equilibrium outcomes seen in other experiments (Goeree, Holt, and Palfrey, 2003), and these results can be applied to real settings with more reliance.

Elections are interactions that can be modeled with the help of game theory: rules should be explicit for participants in any democracy and realistic assumptions about their possible actions can be considered. The objective of political competition models is to determine who will campaign, predict the winner, and the public policies implemented. Nevertheless, in order to evaluate the effect of relevant variables, and consider assumptions of the political process, the experimental approach is the most advisable; observational data is noisy and/or do not show variance in the parameters of the model -this approach is now part of the basic tools in political economics investigation (Palfrey, 2009). In the experiment, costs of entry and different electoral systems were modified to evaluate accurately the predictions of the model. The experimental implementation was exactly the same as that used by Elbittar and Gomberg (2009).

The Citizen-Candidate model assumes voters' preferences across an unidimensional line in $\mathbb{R}$, as the Hotelling-Downs model, although in this case, candidates can not choose locations. Each citizen decides whether to compete depending on the campaign cost, winning benefits, and the voting system. The general model allows any citizen to participate in the competition. However, in the present experiment, only a finite subset of citizens have this possibility. [1]

This model makes fairly clear prediction about what to expect from the data: a behavior in line with NE. However, there is great variability within the responses of each individual and a

[1]This can be due to different barriers to entry: highly enough costs for the others citizens.

tendency to over-participate was reported by Elbittar and Gomberg (2009). In the same article, Quantal Response Equilibrium was proposed as a tool to explain this phenomenon.

Frequently, we observe others behaving as if randomly: people make decisions against their own benefit. This concern is larger in interactions, when others' decisions directly impact our own welfare. In consequence, we must take into account the possibility of others' mistakes and our own behavior. However, decisions are not as erratic as they seem: we expect others -and ourselves- to commit less mistakes when payoffs are bigger. These characteristics are captured by QRE in what is called a regular response function (or stochastic best response) which states a mixed strategy profile: probabilities of every possible action to be chosen given their expected payoffs. A regular response function generalizes best response correspondence in classical game theory, and the concept of equilibrium is completely analogous: a point where beliefs and strategies coincide for every player.

The QRE theory take into account that agents have a certain level of rationality: they are not perfect maximizers, but they are strategic: they think about others' actions and irrationality. In this theory, NE is a especial case with perfectly rational agents. Evaluation of QRE in the context of elections presents an opportunity to check for robustness of the theory and its assumptions, whereas providing a more reliable prediction of elections.

The rest of the thesis is organized as follows: in chapter one, I summarize citizen-candidate model and quantal response equilibrium; in chapter two, I present the methodological settings; in chapter three, I compare data with NE predictions, analyze the effect of costs and electoral system over entry and adjust the model by maximum likelihood; in the last chapter, I summarize the main conclusions and possible future paths.

Chapter 1

Model

The implemented model of political competition is the Citizen-Candidate (Osborne and Slivinski, 1996), the empirical approach come from Elbittar and Gomberg (2009), and Quantal Response Theory (QRE) is used to explain deviations from NE. In this chapter I present the theoretical model of elections, the Nash Equilibria (NE) in the experimental conditions, and the theory of QRE.

The Citizen-Candidate model assumes that preferences can be represented in the real line following the Hotelling-Downs's location model. It is common to normalize preferences in the $[0, 1]$ interval but, given the empirical setup of the model, the $A = [0, 100]$ interval is considered. In the original version of the model, Osborne and Slivinski (1996) mention that the main results hold even if only a subset of the citizens can be elected whereas the rest just vote. The empirical model was implemented in this fashion: the discrete subset $Q = q_1, ..., q_n, q_i \in A$ represents ideal policies, and each citizen can be referred to by their ideal which is indicated exogenously.

Possible candidates consider the cost of participation (c), the possible benefits of being elected or ego rent (b), and their preferences over the possible final policies implemented. In order to model these considerations, equation 1.1 represent the preferences of citizens:

$$u_i(x, q_i) = -\alpha||x - q_i|| - cs_i + bw_i(s) \tag{1.1}$$

The parameter α indicates the importance of the final policy chosen: proportional to the distance from their ideal point (q_i). Thus, citizens prefer policies closer to their locations. The variables $s_i \in \{0, 1\}$ and $w_i \in \{0, 1\}$ stand for the entry decision ($entry = 1$), and the final result ($win = 1$), respectively. Notice that winning depends on the voting system and the profile of decision made by citizens ($s \in \Pi s_i$), which defines the candidates set.

There are important implications from the decisions' timing and other assumptions that will be now described.

1.1 Stages of the Political Process

Following Besley and Coate (1997) the three stages of the elections are presented in inverse order. This clarify the subsequent Nash Equilibria calculus as a consequence of backward induction reasoning.

1.1.1 Policy Choice

Once having won and holding office, the winner candidate (q_i^*) can implement the policy she prefers. At this moment, it is clear that any winner will choose her ideal policy ($x = q_i^*$) maximizing equation 1.1. Therefore, we can define $x : w \in Q \to [0, 100]$, where w stands for the winner. It will be clear that x is a random variable while w is drawn from a set of candidates.

An important assumption is perfect information: all players know the preferences of others which means that none candidate can cheat about the policy she is going to implement once in office. This assumption, and the freedom winner have once in office, contrast with the classical Hotelling-Downs model where candidates are enforced to keep their campaign promises.

1.1.2 Voting

Given the subset of citizens who have decided to participate in the election ($C \subseteq Q$), there is a voting system that designates the winners set ($W : C \to C$). Plurality Rule (PR) or Run-off

(RO) determine function W. In both systems, when two or more candidates get the same number of votes, and more than others, they belong to W from were w is randomly sampled. These electoral systems are described in section 1.2. In both cases, it is assumed that the distribution of voters over A is uniform. Citizens vote sincerely and choose the closer candidate to their own location. The assumption of sincere vote does not allow for coordination between voters.

1.1.3 Entry

Each citizen decides simultaneously whether to campaign. They do so thinking strategically: considering what other citizens would do evaluating expected payoffs and also thinking strategically. Therefore, the resulting set C is a NE.

In the case where no citizen presents a candidacy, everyone receives a payoff of $-D$ which is high enough to deter this case in the equilibrium set.

Note that the cardinality of $W(C)$ could be more than 1 ($\#((W(C)) \geq 1$). In such a case, the winner is chosen with equal probability due to sincere voting. Then, the citizen i's probability of being elected is defined: $P_i(C) = 1/\#((W(C))$ if $q_i \in W(C)$ and 0 otherwise. Therefore, the winner is a random variable.

In consequence, given the vector of parameters $\theta = (\alpha, c, b, D)$, the expected utility for each citizen can be defined.

$$U_i(C;\theta) = \Sigma P_j(C)(u_i(x = q_j, q_i; \theta)) \tag{1.2}$$

Because the candidates set is defined from the preceding entry decision ($C = C(S)$), it is convenient to write the expected value as a function of the profiles:

$$U_i(s), s \in \Pi_{i \in N} S_i$$

From this equation, the existence of a NE can be shown in a standard way using the fixed point theorem.

1.2 Electoral Systems

1.2.1 Plurality Rule

In this System, the candidate who get more votes, calculated as in the Hotelling-Downs's location model, is elected. In the case of a tie the winner is chosen randomly as stated before.

1.2.2 Runoff

This system take the two candidates with highest votes from a first round, and then a second round of voting is held only with this two options. In the case that one candidate got more that a half of the votes in the first round, she wins without a second round. Winner is chosen randomly if there is a tie in the second round.

1.3 Quantal Response Equilibrium

The Quantal Response Equilibrium (QRE) proposed by McKelvey and Palfrey (1995) is constructed on the base of a stochastic best response function. The most used implementation is the logistic function over the difference between the expected payoff of the options:

$$\sigma_{ij} = \frac{e^{(\pi_j)\lambda}}{e^{(\pi_j)\lambda} + e^{(\pi_{i \neq j})\lambda}} = \frac{1}{1 + e^{(\pi_{i \neq j} - \pi_j)\lambda}} \tag{1.3}$$

In this model the parameter λ determines the degree of stochasticity of the election. When $\lambda \to 0$ the election is a fair coin tossing for each option independent of the expected payoffs (minimum level of rationality), and when $\lambda \to \infty$ the election is deterministic with the more profitable action being chosen with certainty (complete rationality). The expected payoff of action j is represented by π_j.

An easy way to visualize the effect of λ over decisions is to remember the logistic regression: the dependent variable is the probability to choose option j, and the independent variable is the difference between expected payoffs of the two options. Intercept is fixed in zero -which imply equal probability when options' expected payoffs are the same-, and slope is precisely λ, what determines how step is the logistic function. For this reason, λ is also interpreted as the level of rationality: as it increases, decision will be better in terms of expected payoffs and less uncertain.

With equation 1.3 a stochastic best response is defined:

$$\sigma_{ij}^{*}(\lambda, \beta, \pi_j(\sigma_{-i})) \tag{1.4}$$

, which is the player i's probability of choose the actions j (i.e. Entry or Not Entry in the Citizen-Candidate model). Vector β commonly refers to economic parameters that can measure risk aversion or altruism. In the present analysis, I do not consider these possibilities, but because β refers to changes in the original payoff matrix, I will use β to refer to different games.

With equation 1.3, a distribution over actions for each player is defined:

$$\sigma_i^{*}(\lambda, \beta, \sigma_{-i})$$

This stochastic best response have the proprieties of a regular quantal response function stated by Goeree et al. (2016):

- Interiority: $\sigma_{ij} > 0$
- Continuity: σ_{ij} is continuous and differentiable.
- Responsiveness: $\partial\sigma_{ij}/\partial\pi_{ij} > 0$
- Monotonicity: $\pi_{ij} > \pi_{ik} \Rightarrow \sigma_{ij} > \sigma_{ik}$

Notice that stochastic best response is a function of others' mixed strategies (σ_{-i}). This is the case because, as seen in equation 1.3, probability depends on π_j which is a function of others' strategies: $\pi_j(\sigma_{-i})$. In the case of the Citizen-Candidate model, π_j is the expected payoff of equation 1.1.3.

Define then the function:

$$\sigma = (\sigma_1^*, ..., \sigma_N^*) \tag{1.5}$$

Then, quantal response equilibrium (σ^*) is a fixed point of equation 1.5, that now is a function only of λ and β. Due to the proprieties of the stochastic best response, and using the fixed point theorem, the equilibrium existence is assured. Figure 1.1 shows σ^* as a function of λ for the four games used in the experiment.

Note that a different equilibrium is predicted depending of the value of λ and that players tend towards pure strategies as λ goes larger. This equilibrium is called logit equilibrium (Goeree et al., 2016). The software Gambit (McKelvey, McLennan, and Turocy, 2014) allows to see those equilibria as a function of the rationality parameter λ. I realized the calculus in *R* (R Core Team, 2017) with the package *nleqslv* (Hasselman, 2017), and compare the result with gambit's; they are the same.

The parameter λ can be adjusted by maximum likelihood to empirical data as is shown in chapter 3.

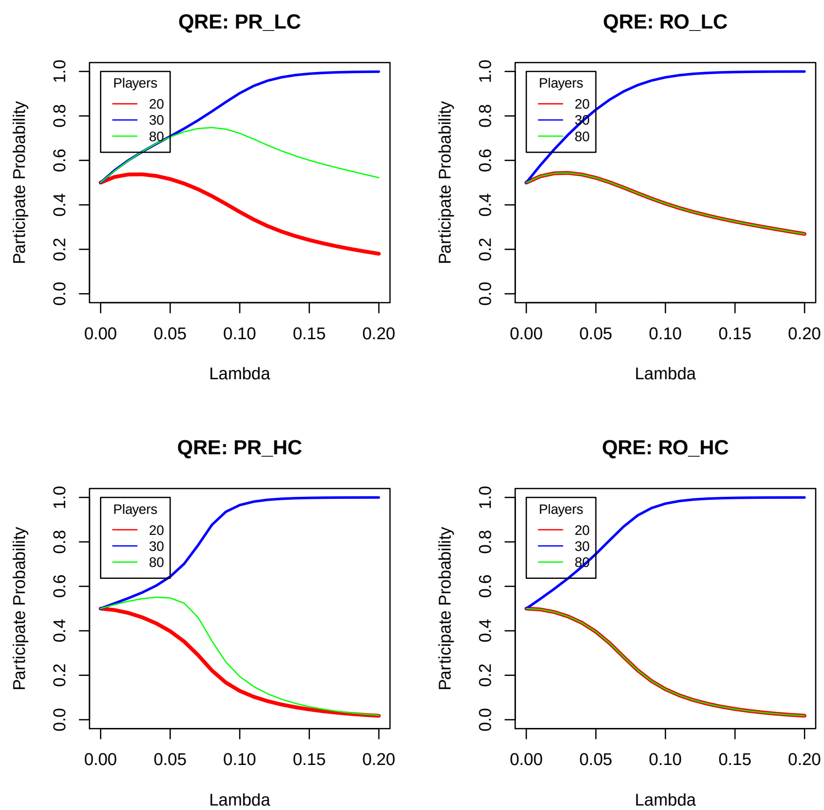

Figure 1.1: Quantal Response Equilibria as function of lambda

Chapter 2

Methodology

2.1 Experimental Design

In this chapter I present the experimental methodology, the NE (pure and mixed) in each condition, and a general descriptive analysis of data. The design and data of the current experiment come from Elbittar and Gomberg (2009), and the instruction shown to participants are reported in the appendix.

There are four different games from the combination of two voting systems: plurality rule (PR) and run-off (RO), and two levels of costs: high ($20) and low ($5) (i.e. PR_LC, PR_HC, RO_LC and RO_HC). For each game there are three sessions with different participants who were students of various undergraduate programs at ITAM in Mexico City. They played three practice trials at the beginning of each session and at most during 30 effective trials. Participants were randomly matched and assigned an ideal point ($Q = 20, 30, 80$) each trial. Less trials happen if participants lost all the money given or because they were not a multiple of three and then one or two of them waited until the next trial-match. Each one initiate with $\$140$, this amount increased or decreased through the session according with the payoffs that depend on behavior of others and game's structure. Participants were allowed to continue until they finished a trial with negative balance.

Table 2.1 summary the sessions' characteristics. As expected, there are more bankruptcy in high cost games (HC). Conditional to this, plurality rule (PR) game have a larger number of bankruptcy participants.

Voting System	Costs	No. Participants	No. Bankrupcy	Session
PR	LC	19	0	1
		18	0	2
		20	0	3
	HC	15	9	1
		20	13	2
		23	16	3
RO	LC	26	0	1
		16	0	2
		27	2	3
	HC	15	4	1
		15	5	2
		15	6	3
Total:		229	55	12

Table 2.1: There were three different sessions by game. Last column shows the number of the participants that lost all the money given.

2.2 Expected Nash Equilibria

Particular values of the parameters in the utility function and the ideal policies of the citizens are needed in order to calculate an specific NE. Table 2.2 displays parameter values used in the experiment.

The matrices with the payoffs for the four games are in figures 2.1 to 2.4. They represent the possible action profiles and payoffs for each citizen in order: s_{20}, s_{30}, s_{80}. For example, in the figure 2.1, if we look at the first matrix, second row and first column, there are the payoffs: $(20, -1, -6)$, it must be read that player 1 (q_{20}) and 2 (q_{30}) were not candidates and citizen 3 (q_{80}) was alone in the campaign ($s = (0, 0, 1)$).

It is important to mention that, in this case -with only three participants-, payoffs are equal

Parameter	Value(s)
α	0.1
c	$\{5, 20\}$
b	25
D	40
Q	$\{20, 30, 80\}$

Table 2.2: Parameters used in the current experiment.

q_{80}

Not Entry ——————————- Entry

q_{30}		Not Entry	Entry
q_{20}	N	$-40, -40, -40$	$-1, 5, -5$
	E	$5, -1, -6$	$-21, 5, -5$

q_{30}		Not Entry	Entry
q_{20}	N	$-6, -5, 5$	$-1, 5, -25$
	E	$-10.5, -3, -10.5$	$-26, -25, 5$

Figure 2.1: PR_HC normal form representation.

q_{80}

Not Entry ——————————- Entry

q_{30}		Not Entry	Entry
q_{20}	N	$-40, -40, -40$	$-1, 5, -5$
	E	$5, -1, -6$	$-21, 5, -5$

q_{30}		Not Entry	Entry
q_{20}	N	$-6, -5, 5$	$-1, 5, -25$
	E	$-10.5, -3, -10.5$	$-21, 5, -25$

Figure 2.2: RO_HC normal form representation.

q_{80}

Not Entry ——————————- Entry

q_{30}		Not Entry	Entry
q_{20}	N	$-40, -40, -40$	$-1, 20, -5$
	E	$20, -1, -6$	$-6, 20, -5$

q_{30}		Not Entry	Entry
q_{20}	N	$-6, -5, 20$	$-1, 20, -10$
	E	$4.5, -3, 4.5$	$-11, -10, 20$

Figure 2.3: PR_LC normal form representation.

q_{80}: Not Entry ——— Entry

q_{80} = Not Entry

q_{20} \ q_{30}	Not Entry	Entry
N	$-40, -40, -40$	$-1, 20, -5$
E	$20, -1, -6$	$-6, 20, -5$

q_{80} = Entry

q_{20} \ q_{30}	Not Entry	Entry
N	$-6, -5, 20$	$-1, 20, -10$
E	$4.5, -3, 4.5$	$-6, 20, -10$

Figure 2.4: RO_LC normal form representation.

Sets of SNE	Plurality Rule	Run-off
High Cost (20)	$\{(0,1,0)\}$	$\{(0,1,0)\}$
Low Cost (5)	$\{(0,1,0),(1,0,1)\}$	$\{(0,1,0)\}$

Table 2.3: Strict Nash Equilibria profiles for each game ((s_{20}, s_{30}, s_{80}) with 0=Not Entry, 1=Entry).

between voting rules, except in the case when all citizen decide to participate. In such a case, PR makes q_{80} the winner, and RO system award q_{30}.

In order to calculate NE, I used the Gambit program (McKelvey et al., 2014). I report these Pure and Mixed equilibria in the next sub sections. This software search for the solutions of a non linear equations system when calculating mixed strategy equilibria.

2.2.1 Pure Strategy Equilibria

In the literature, it is common just to consider de Pure Nash Equilibria and I will refer here to them as strict NE (SNE) because it is the case that NE definition apply with strict inequality. Table 2.3 reports those equilibria in each game.

Note that q_{30} campaigning alone is always an equilibrium, and that the only game with more than one SNE is plurality rule with low cost. It is the case because, as expected-utility-maximizers, q_{20} and q_{80} are dispose to tie while expected benefits from wining overcome the cost of entry.

MSE in Plurality Rule	Ideal Points		
Costs	20	30	80
High Cost (20)	1	0.3103	0.2143
	0.9091	0	0.9091
Low Cost (5)	0.7812	0.5122	1

Table 2.4: Mixed Strategies Equilibria. Each cell shows the probability to enter.

2.2.2 Mixed Strategy Equilibria

In the original model, only SNE are considered. It seems a reasonable approach if we consider to implement error in the agents' decisions. Young (1998) demonstrated that stable equilibria along a dynamic setting are pure SNE in the one-shot setting. Nevertheless it is possible that there were not SNE.

With the current parameter values, there are no MSE in the Run-off games. Plurality Rule system has one and two MSE for low and high cost respectively. Table 2.4 shows these MSE.

Chapter 3

Results

In this section I present the main results from the analysis. The effects of the relevant experimental variables are evaluated with a logistic regression, and the adjusted λ is discussed at different levels of aggregation.

3.1 Descriptive Statistics

There are at most two possible equilibria in pure strategies in each game. However, many other profiles occurred. Table 3.1 shows the proportion each configuration was played by game. First columns display the eight possible profiles. For example, profile 3 ($[0, 1, 0]$) means that only citizen q_{30} enter the competition. This profile is a SNE in all games, and is the most frequent in RO games. Notice that the more frequent profiles are 3 and 4 in PR_HC game and, 4 and 8 in the PR_LC. This phenomena can be explained first because citizen q_{80} wins in most scenarios where others deviate from equilibrium; and second because, with low costs, citizen q_{20} face the same logic and then profile 8 (every citizen campaign) increases its probability. This kind of considerations are captured by the QRE theory.

It is important to review the aggregated proportion of entrance by ideal point precisely because these proportions are the direct prediction of QRE theory. Table 3.2 displays these proportions by game. This table makes clear that once a participant was assigned the ideal point 30,

	Profiles			Games			
	20	30	80	PR_LC	PR_HC	RO_LC	RO_HC
1	0	0	0	0.013	0.029	0.006	0.020
2	0	0	1	0.048	0.066	0.003	0.006
3	0	1	0	0.067	**0.305**	**0.347**	**0.589**
4	0	1	1	**0.402**	**0.366**	**0.252**	**0.151**
5	1	0	0	0.009	0.013	0.008	0.011
6	1	0	1	0.063	0.018	0.011	0.000
7	1	1	0	0.044	0.079	0.210	0.146
8	1	1	1	**0.354**	0.124	0.164	0.077
	Total trials:			540	380	648	350

Table 3.1: For each game, the proportions of the eight possible combinations of strategies played are shown ($1 : entry$, $0 : abstain$). The total number of actual trials are set at the bottom.

it is quite probable for her to enter. The probability of enter being 80 increases until reach 30's in the PR_LC condition precisely for de reason explained above.

	PR_LC	PR_HC	RO_LC	RO_HC
20	0.470	0.234	0.392	0.234
30	0.867	0.874	0.972	0.963
80	0.867	0.574	0.429	0.234

Table 3.2: Proportion of entry by type of player.

Figure 3.1 displays the boxplots of the entrance proportion. It can be seen a clear tendency to enter when participants were chosen to be q_{30}. The entrance is less for any other ideal point and, in general, with less variability. The most interesting observation is in condition PR_LC where entrance is equal between q_{30} and q_{80}, as shown in table 3.2, and the median is even higher for q_{80}.

3.1.1 Logistic Regression

In order to evaluate the effect of costs, voting rule, and if there are some difference in proportions through time, I run a logistic regression with random effects using the *lme4* package (Bates et

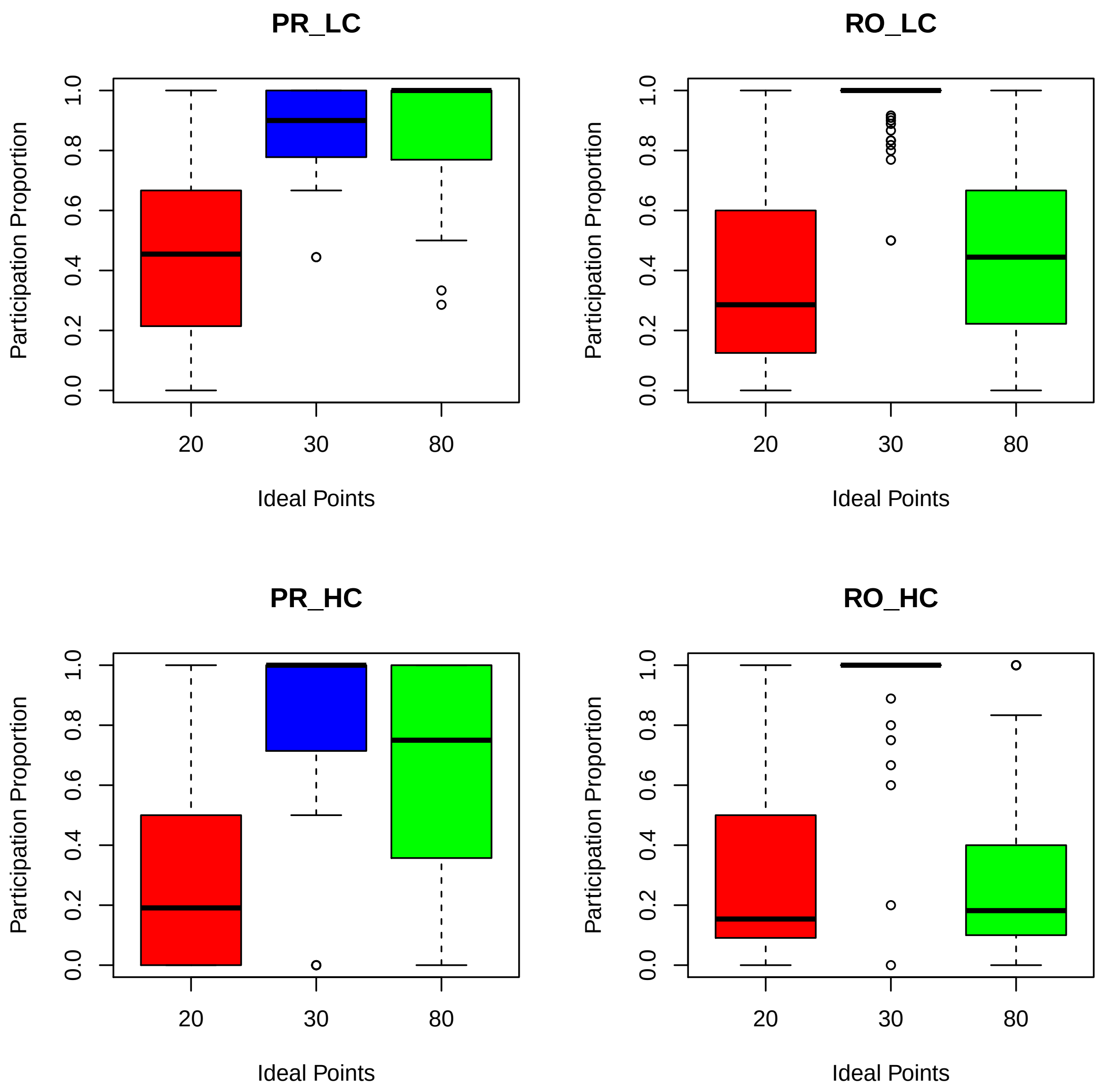

Figure 3.1: Proportion of entrance by game and by ideal points.

al., 2015). Table 3.3 shows[1] the results from three regressions for each ideal point.

Period is a dummy variable -as the other ones- constructed from trials, it split them in two: first 14 trials and the rest. Final number of trials was different for each participant because it depends on the random matchings and bankruptcy, the more equitable distribution is reached with 14 trials for period zero. The effect of this variable is robust across ideal points, which shows that, through time, players tend towards NE: q_{30} campaigning alone.

The results confirm statistically the visual analysis made with figure 3.1: there is an effect of high cost and RO system towards NE. Additionally, there is no effect of costs over q_{30} possibly due to a ceiling effect -they almost always participate-, for q_{20} there is not effect by election system. Finally, it is worth noting that q_{80}'s entrance rate is very sensible to the relevant variables.

	Dependent variable:		
	Entry		
	20	30	80
	(1)	(2)	(3)
HighCost	-1.221^{***}	0.072	-1.875^{***}
	(0.277)	(0.321)	(0.284)
RunOff	-0.199	1.896^{***}	-2.862^{***}
	(0.269)	(0.348)	(0.293)
Period	-0.984^{***}	1.118^{***}	-1.922^{***}
	(0.133)	(0.221)	(0.154)
Constant	0.237	2.043^{***}	3.695^{***}
	(0.236)	(0.277)	(0.294)
Observations	1,918	1,918	1,918
Akaike Inf. Crit.	2,028.730	908.160	1,795.807
Bayesian Inf. Crit.	2,056.525	935.955	1,823.602
Note:			*p<0.1; **p<0.05; ***p<0.01

Table 3.3: Logistic Regression with Random Effects

[1]This table was made with the help of *stargazer* package (Hlavac, 2015).

3.2 Maximum Likelihood Estimation

Data analysis at different levels was calculated by maximum likelihood, following directions from Goeree et al. (2016). The equilibrium correspondence approach was used: the QRE is calculated for a given λ and other parameters (β): $\sigma *_{ij} (\lambda, \beta)$. In order to find this function, a non linear equations system with no analytical solution has to be solved. This was done using the R package *nleqslv* (Hasselman, 2017).

Once this function is constructed, it is possible to find the general logLikelihood objective function, and the then optimize it.

$$logL(\lambda) = \Sigma_{i=1}^{n} \Sigma_{j=1}^{J_i} f_{i,j} log(\sigma_{ij}^{*}(\lambda, \beta)) \tag{3.1}$$

In the objective function 3.2: $n = 3$, $J_i = 2$ and $f_{i,j}$ are the times each i-th citizen choose their j-th option.

3.2.1 Global Analysis

In this level, it is considered that every player has the same λ. Then, the resulting objective function considers the likelihood in each game according with equation 3.2.

$$logL(\lambda) = \Sigma_{g=1}^{4} \Sigma_{i=1}^{n} \Sigma_{j=1}^{J_i} f_{g,i,j} log(\sigma_{ij}^{*}(\lambda, \beta_g)) \tag{3.2}$$

Parameter β_g refers to the payoffs in the g-th game, and it is clear that the estimated equilibrium depends on the payoffs. The λ estimated this way is 0.0782. This result and those for the analysis by game are shown in the table 3.4.

	Global	PR_LC	PR_HC	RO_LC	RO_HC
λ	0.0782	0.0836	0.0723	0.0984	0.0832

Table 3.4: MLE of λ for the global analysis and for each game.

3.2.2 Game Analysis

It could be possible to observe an effect by game over λ which would imply that it is a function of payoffs. For this reason rationality parameter is calculated by game and compared with global result.

In the figure 3.2, it is possible to compare the aggregated adjustment in each game. This graph shows soft lines which state the predicted rate of entrance for each citizen type as a function of λ. The vertical lines are located at the adjusted λ; solid and dashed lines are in global and by game calculation respectively. Horizontal lines are located at the proportions observed in data, they used the same color pattern for each ideal point than in soft lines.

It is clear from figure 3.2 that there are no large differences between the global and by game estimates and that predictions from both (vertical and smooth lines intersections) are close to the observed proportions.

There is always heterogeneity in the participants that can be interesting to observe and measure. For this reason the analysis by individual is included.

3.2.3 Individual Analysis

In figure 3.3 it is possible to observe the heterogeneity across and inside the games. Red lines represent entrance rates of those participants who end in bankruptcy before the programed trials ended. First, it is clear that the logit equilibrium is the more frequent strategy played across participants. Also, there is observed heterogeneity across games in this respect that could be explained by the payoffs, because a mistake in high cost games is more important. This heterogeneity cannot be explained by a more random behavior in high cost games considering that λ estimations are not too much different across games -as can be seen in figure 3.2. There is the possibility of a biased measure of the strategies played in this games due to the fact that more random participants, who are more prone to bankruptcy, played less trials than more rational participants. For this reason it is important to analyze individuals in a more detailed way.

Figure 3.4 shows the histograms for MLE estimators in each game. They were calculated in

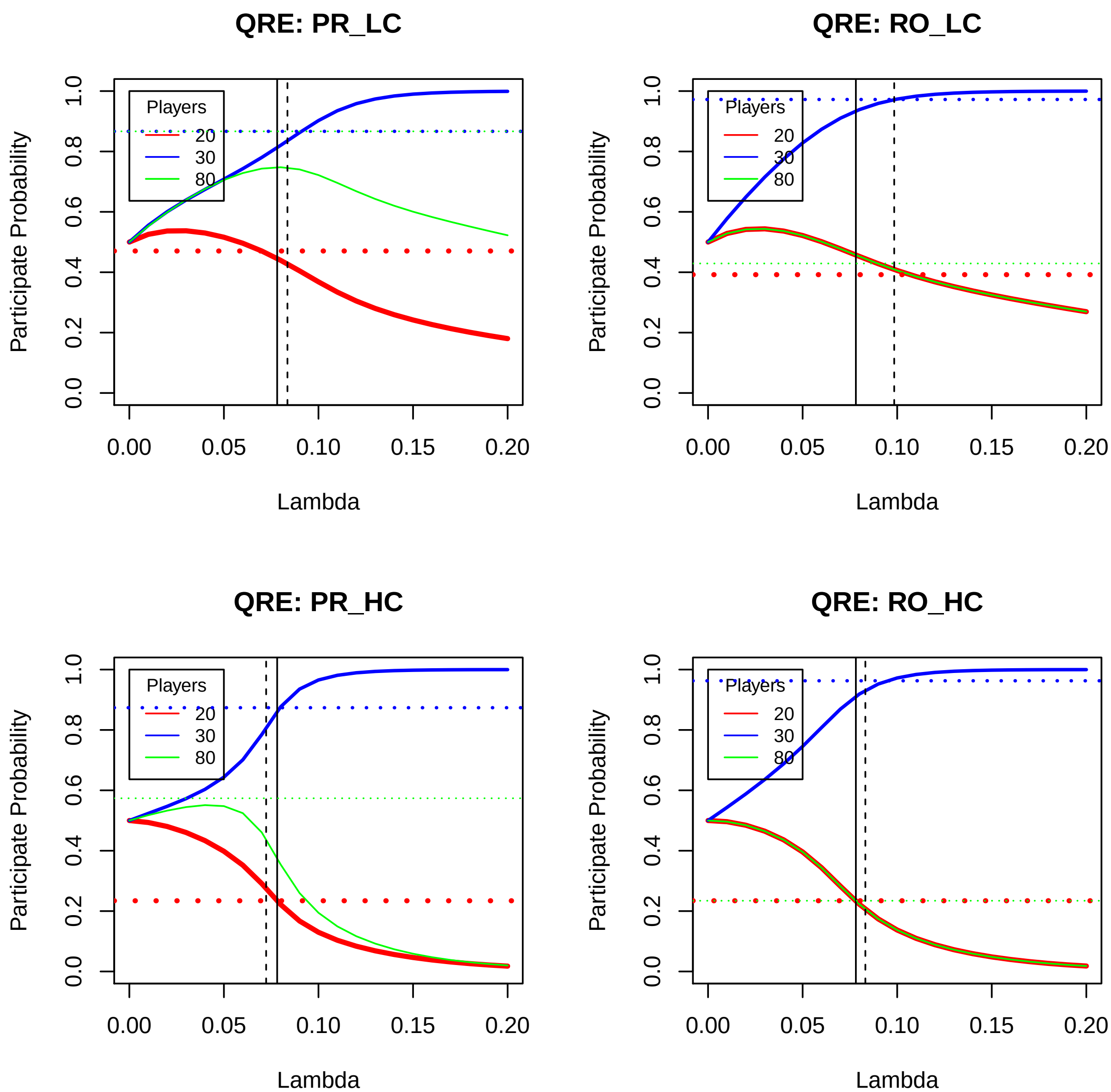

Figure 3.2: Equilibria for different Lambda

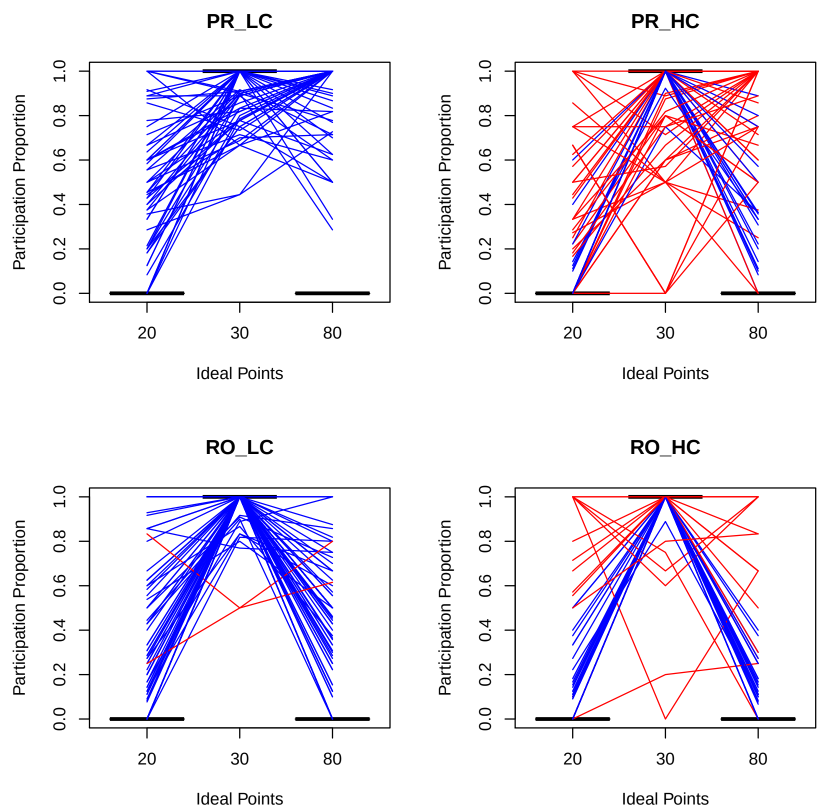

Figure 3.3: Each line connects the proportion of entry when a participant played for different ideal points.

a slight different way than Goeree et al. (2016) . In this case, for each player the proportions of their own elections in different ideal points were used to calculate λ, it is like consider that each participant play against herself.

It can be noticed that the distributions are not very different, and could be described by an exponential distribution. For high cost games it is clear that there are a few players that are apart from the main population to the right: maybe those who played the logit equilibrium almost perfectly.

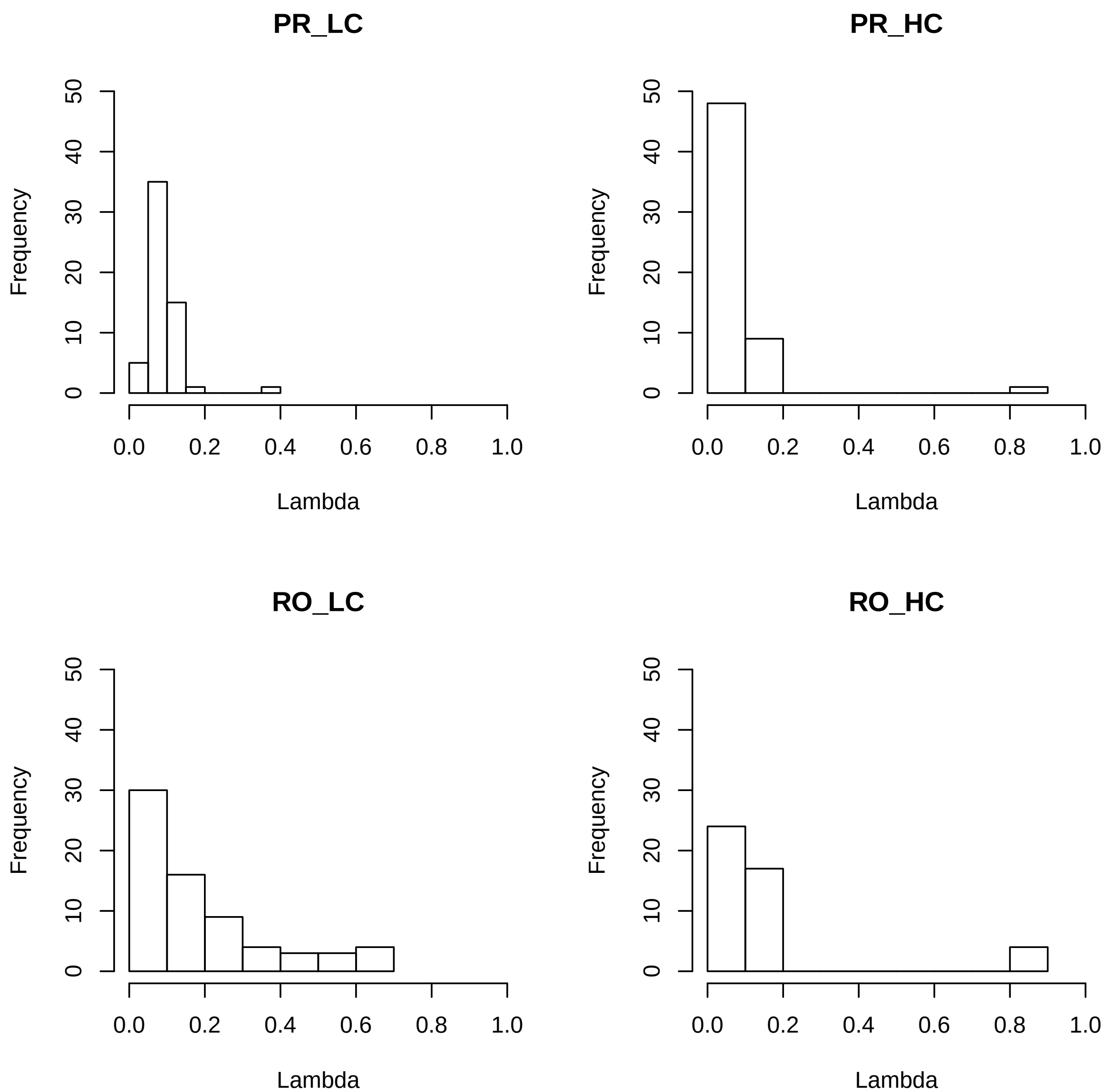

Figure 3.4: Each line connects the proportion of entry when a participant played for different ideal points.

Chapter 4

Conclusion

The quantal response Equilibrium (QRE) theory, which consider stochastic behavior, can describe candidates' decisions in the experiment. This allows more reliance when applying the citizen-candidate model to real settings.

There is variability in the data that can not be explained just by random error: there are systematic deviations from the standard prediction of NE. These phenomena can be explained by QRE theory, considering participants' decisions depends stochasticity on the expected payoffs they face in the game, which in turn depend on other's decisions that are also random, and at the same time they consider that others behave in the same way.

Considering that citizen-candidate assumption describe well enough the electoral process, the main prediction of QRE theory is that there are more candidates than expected by standard game theory. This is the result of a direct and indirect effect of the stochastic best response. First, there is a direct effect of the stochasticity that made the Nash winner (q_{30}) less probable to enter and increase the probability of others to enter. Second, others candidates increase their expected payoffs to participate because the probability of being defeated decreases relative to NE. The magnitude of this indirect effect depends on the citizen's position relative to Nash winner; distant candidates to her are more prone to participate. This implication goes according with data observed.

In the experiment, citizen q_{80} was more prone to enter than q_{20}. The effect of over participation is expected to be bigger in isolated candidates because they are already losing: the Nash winner is far from them. Meanwhile, candidates closer to the Nash winner have less to win if they are elected. This is an implication from the sincere voting assumption because closer candidates compete for the same voters.

The same logic goes for the analysis over the effects of other variables. Costs and election systems affect the expected payoffs of enter the competition. High cost and Run-off system decrease entrance of Nash losers. The effect of the last variable is due to all participants campaign, q_{30} wins in Run-off system, and q_{80} in Plurality Rule, q_{20} always lose.

Additionally, an analogous result to median voter theorem was found: the closest candidate to the median (i.e. q_{30}) entering alone was always an equilibrium.

There are no important differences in λ between global and by game analysis; it seems improbable -and also inconvenient- that the level of rationality (λ) varies across game, which would mean that it is a function of payoffs. A better adjust could be reached if non linear utility function were considered, as previously done by Goeree et al. (2003) and Trujano Ochoa (2013). But in this case, it is more difficult to implement due to random choice when ties, and goes further the objective of this thesis. Although, it seems plausible that differences between global and by game analysis would shorten.

There is individual heterogeneity, but in general all participants can be described by a single distribution. From a parsimony argument it is better to consider that all participants share the same level of rationality measured with the parameter λ. Nevertheless, this heterogeneity can be considered by multilevel statistical models. These models are complex to calculate but include the assumption that participants come from the same distribution. Bayesian Statistics analysis could be an adequate method: the inferences would not be on a specific parameter, but over the distribution of λ in the population.

Finally, it most be noted that NE is not a bad forecasting. Nevertheless, QRE not just offer a more precise description, but also predict other interesting phenomena in data.

References

Bates, D., Maechler, M., Bolker, B., Walker, S., Christensen, R. H. B., Singmann, H., … Eigen, C. (2015). "Fitting Linear Mixed-Effects Models Using {lme4}." *Journal of Statistical Software*, *67*(1), 1 – 48. doi: 10.18637/jss.v067.i01

Besley, T. (2007). "The New Political Economy." *The Economic Journal*, *117*(524), F570–F587. Retrieved from `http://www.jstor.org/stable/4625573`

Besley, T., and Coate, S. (1997). "An Economic Model of Representative Democracy." *The quarterly journal of economics*, *112*(1), 85–114.

Cadigan, J. (2005). "The Citizen Candidate Model : An Experimental Analysis." *Public Choice*(123), 197–216. doi: 10.1007/s11127-005-0262-4

Camerer, C. F. (2003). *Behavioral Game Theory: Experiments in Strategic Interaction*. Princeton University Press.

Elbittar, A., and Gomberg, A. (2009). "An Experimental Study of the Citizen-Candidate Model." In E. Aragones, C. Bevia, H. Llavador, and N. Schofield (Eds.), *The political economy of democracy* (pp. 31–46). Fundación BBVA.

Gächter. (2004). "Behavioral Game Theory." In *Hanbook of judment and desition making* (pp. 485–503). doi: 10.1002/9780470752937.ch24

Goeree, J. K., Holt, C. A., and Palfrey, T. R. (2003). "Risk averse behavior in generalized matching pennies games." *Games and Economic Behavior*, *45*(1), 97–113. Retrieved from `http://linkinghub.elsevier.com/retrieve/pii/S0899825603000526` doi: 10.1016/S0899-8256(03)00052-6

Goeree, J. K., Holt, C. A., and Palfrey, T. R. (2016). *Quantal Response Equilibrium: A Stochastic Theory of Games*.

Hasselman, B. (2017). *nleqslv: Solve Systems of Nonlinear Equations.* Retrieved from `https://cran.r-project.org/package=nleqslv`

Hlavac, M. (2015). *Package ‘ stargazer ’.* Cambridge, USA. Retrieved from `http://cran.r-project.org/package=stargazer`

McKelvey, R. D., McLennan, A. M., and Turocy, T. L. (2014). *Gambit: Software Tools for Game Theory.* Retrieved from `http://www.gambit-project.org`

McKelvey, R. D., and Palfrey, T. R. (1995). “Quantal Response Equilibria for Normal Form Games.” *Games and Economic Behavior*, *10*(1), 6–38. Retrieved from `http://dx.doi.org/10.1006/game.1995.1023` doi: 10.1006/game.1995.1023

Osborne, M. J., and Slivinski, A. L. (1996). “A model of political competition with citizen-candidates.” *Quarterly Journal of Economics*, *111*(February), 65–96.

Palfrey, T. (2009). “Laboratory experiments in political economy.” *Annual Review of Political Science*, *12*(111), 379–388. Retrieved from `http://www.annualreviews.org/doi/abs/10.1146/annurev.polisci.12.091007.122139`

R Core Team. (2017). *R: A Language and Environment for Statistical Computing.* Vienna, Austria. Retrieved from `http://www.r-project.org/`

Trujano Ochoa, D. (2013). *La Teoría del Prospecto en la Predicción de Estrategias Mixtas en un Juego 2x2.* Retrieved from `http://132.248.9.195/ptd2013/noviembre/0706151/Index.html`

Young, H. P. (1998). *Individual Strategy and Social Structure: An Evolutionary Theory of Institutions* (Vol. 110) (No. 461). Princeton University Press. Retrieved from `http://books.google.com/books?hl=en&lr=&id=iI7XCi-lnSQC&pgis=1`

Appendix

Instructions[1]

Instructions (read by participants)

This is an experiment about decision making in elections. CONACYT has provided funding for this research. The instructions are simple and if you follow them carefully and make good decisions, you can win a CONSIDERABLE MONEY AMOUNT, which will be CASH PAYED to you privately at the end of the session.

After we read the instructions, you will have the opportunity to make your decisions. - General Procedure

In this experiment you will have to decide whether or not to compete as a candidate in each of the 30 elections that we will carry out at the end of the instructions.

In each of the elections, one of 3 possible alternatives will be chosen winner by a population of voters (simulated by the computer), according to the procedure of voting that we will see later. The 3 alternatives are represented by positions 20, 30 and 80 located on the next line from 0 to 100 (figure 4.1):

- Group Formation

In each election, you will be part of a group of 3 participants. The composition of each group of participants will change randomly, so that the same group will be composed of different participants in each election. You will never know the identity of who you are participating with.

[1] Here I present the translation from Spanish of the directions given to the participants.

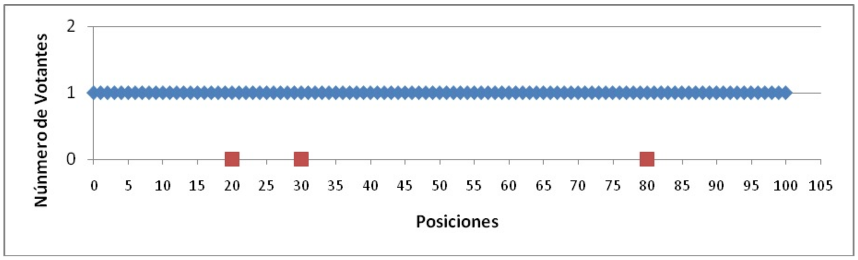

Figure 4.1: Ideal Points description

- Allocation of Alternatives

In each election, one of the alternatives mentioned will be assigned to you as your ideal position. Each participant in your group will be assigned a different position. Thus, a participant will be assigned the 20 position; To another the position 30; And to another, position 80. The allocation of alternatives for each election will be determined in a random manner.

- Candidate Application Procedure

To be considered as a candidate eligible by the voters, you must decide whether or not to post your ideal position in each of the elections. That is, you must decide whether to compete or not to be elected by the voters in each of the elections.

You can only postulate your ideal position and you will not be able to postulate any other position.

Once all the participants have made their decisions to postulate their positions, the winning candidate in each election will be determined according to the voting process described below.

- Procedure for Electing the Winning Candidate

For each election, we have a population of 101 voters. Voters are distributed along the line from 0 to 100 as follows: One voter in each integer represented on the line (figure 4.2).

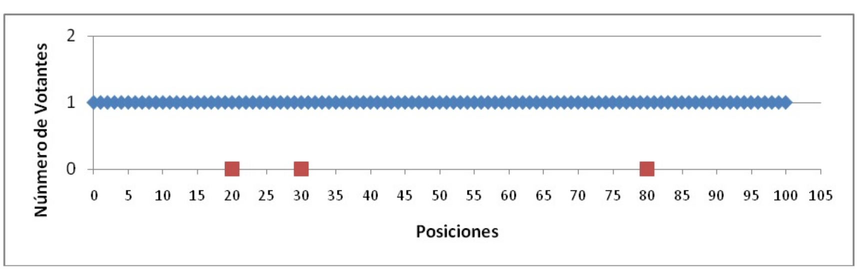

Figure 4.2: Ideal Points description

The 101 voting citizens (simulated by the team) will choose the winning candidate, according to the following voting procedure:

- **Plurality Rule:**

  1. Each citizen will vote for the candidate closest to his / her position. When there is more than one candidate with the same closeness, the citizen's vote will be randomly assigned to the closest candidates.

  2. The winning candidate will be the one who accumulates the highest number of votes. In case of a tie, the winner will be randomly selected among the candidates tied in the first place. Therefore, there will always be only one winner in case there are postulants.

- **Run-Off:**

  1. In a first round of voting, each citizen will vote for the candidate closest to his or her position. When there is more than one candidate with the same closeness, the citizen's vote will be randomly assigned to the closest candidates.

  2. After the first round of voting, the two candidates with the highest number of votes will be selected to participate in a second round of voting.

  3. In this second round of voting, each citizen will vote for the candidate closest to his or her position. In case of a tie, the winner will be determined randomly among the tied candidates. Therefore, there will always be only one winner in case there are postulants.

  4. If less than three candidates have been nominated, only the first round of voting will be held, with the candidate having the highest number of votes selected.

- Initial Balance, Profits and Payments

Each participant will start with an initial balance of 140 pesos. At each election, the opening balance will be updated as follows:

In the event that at least one alternative has been postulated:

1. Each participant will be subtracted from the amount in pesos equal to the parameter Alpha (= 0.1) multiplied by the absolute distance between his ideal position and the position of the winning candidate. That is, it will be subtracted from the amount of:

0.1 x | Your Ideal Position - Candidate Position Winner |

2. It will be subtracted to each participant who has decided to postulate his ideal position the amount of C (5 or 20) pesos.

3. The winning candidate will be added the amount of 25 pesos.

In the event that no alternative has been postulated, each participant will be subtracted from the only amount of 40 pesos.

- Accumulated Balance and Payment Procedure

The accumulated balance at the end of each election will be the sum of your initial balance plus the payments and winnings you have earned in each previous election. The balance accumulated at the end of the 30 periods will be paid in closed envelope. If you get a negative balance, you will not get any payment.

- Summary of Instructions

In each election,

1. You will be part of a new group of 3 participants.

2. Each member of the group will be assigned one of the following 3 positions on the line from 0 to 100: 20, 30 and 80.

3. Each participant must decide whether or not to run for election.

4. The 101 voting citizens (simulated by the computer) will determine the winning candidate, voting for the closest to their location.

- **Run-Off**

  A. In the first round, the two candidates with the highest number of votes are elected.

  B. In a second round, the winning candidate is chosen from the two candidates with the highest number of votes.

C. Only a first round of voting will be held if the number of nominated candidates is less than three.

5. The balances will be updated as follows after each election:

A. In the event that at least one alternative has been postulated:

I. It will be subtracted to each participant the amount in pesos equal to 0.1 x | Its Ideal Position - Position of the Winning Candidate |

Ii. It will be subtracted to each participant who has decided to postulate his ideal position the amount of C (5 or 20) pesos.

Iii. The winner will be added the amount of 25 pesos.

B. In the case that no alternative has been postulated, each participant will be subtracted from the only amount of 40 pesos.

6. The balance accumulated at the end of each election will be the sum of your initial balance plus the payments and winnings you obtained in each previous election.

- Factors that influence your earnings in each election

As you can see, your earnings are influenced by three factors:

1. The distance between the chosen winning position and your ideal position.
2. Your decision and the other participants to apply.
3. Be elected winner by voters.

Next Steps (Read by the investigator after reading the instructions)

Next we will show you the software that we have designed for you to make your decisions. Therefore, leave the instructions on the side of the computer and take the ID LIST sheet that are next to your computer.

Connection with the Server

Each participant must initiate their connection with the server using the following procedure: Enter in the User and Password box the numbers written at the top of their IDENTIFICATION RECORDS formats. Then press the send box.

• Completed records, press end of record.

Screen Reading

Then we will review the information that is now on your screen. In the upper left you will find a column where your USER NUMBER, GROUP SIZE, ROUND NUMBER, TYPE OF ROUND (which in our case we are in the test rounds), and the ACCUMULATED BALANCE (which in Our case is the initial balance of 140 pesos). In the second column on the right side, the value of the ALPHA parameter is indicated, the COST TO BE POSTED, the PAYMENT THAT IS GRANTED FOR WINNING, COST IN THE EVENT OF NO CANDIDATE, and finally NUMBER OF VOTERS.

Practice Rounds

We will now conduct 3 rounds of practice. The primary purpose of these practice rounds is to familiarize you with the software we have designed for you to make your decisions, and therefore will not count toward your payments. If you have any questions during the practice, please raise your hand and I will try to answer them.

Once all have made their decisions, they must wait until all the participants have made their decisions and the computer throw the results of the election.

In case your computer has not activated the box for decision making, it is because we do not have a number divisible by three, so you must wait until the next rounds to activate your screen.

After the practice periods, your initial balance will return to the initial amount of 140 pesos.

Generate Period 1

We now begin the 1st period of practice. (Press: Start Period)

Before they make their decisions see that next is presented a graph where the number of voters in each integer point in the line of 0 to the 100 is indicated, as well as the positions of the different alternatives, including their position. At this level on the right side are some circles where you must dial with the MOUSE to make your decision to run for or not to run.

(After the computer generates the results)

In the Results Table, a chart is indicated on the right side indicating the number of votes obtained by each of the candidates who ran. In case of no graphic display, it means that none of the participants in the group ran. On the left side, the amount of the initial balance, the payment to win, which will be greater than zero if you have won the election, the cost of having been nominated, which will be greater than zero if you decided to apply, the cost per Its distance from the winner, the cost for lack of candidates, which will be greater than zero in case no member of the group has decided to run, and finally the accumulated balance, which is the sum of their initial balance plus payments minus Costs.

Do you have questions?

Results Sheet: If you wish to keep a record of your accumulated balance, please use the back of the information record sheet. In any case, the computer will be recording its decisions and accumulated payments.

Generate Period 2

Let us proceed now to the 2nd period of practice. (Press: Start Period) Proceed to make your decisions.

Do you have questions?

Generate Period 3

Let's proceed to 3rd. Period of practice. (Press: Start Period) Proceed to make your decisions.

Do you have questions?

Periods Actual or Played by Money

Now we will proceed to carry out the 30 periods to be played for money. Your initial balance will return to the initial amount of 140 pesos.

• Check all screens.

After the experiment begins, you are not allowed to speak or communicate with other participants. Otherwise, we will be forced to exclude it from the experiment. Please concentrate on your computer screen. If you have any questions, please raise your hand and one of us will approach you and we will try to answer it.

Generate Period 1-30 (Press: Start of Period) Proceed to make their decisions.

Final payment

Your payment is the amount that appears on the balance of your screen.

Please stay in your posts. One of us will give you a final questionnaire and your payment receipt to be filled out by you. Please add up the balances obtained in both parts of the session.

Then they will be called out to receive their payment. Please go to leave all the material that was given to them.

Thank you very much for your participation!