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Endogenous Search Concentration in OTC Markets

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Abstract

To explain the core-periphery structure observed in many OTC markets we augment a random matching model in three ways: we introduce preference heterogeneity in types and allow agents to choose how intense they meet specific types. Agents with the least-concave preferences are more likely to join the intermediating core and, as quasi market-makers, have more contacts and larger transfers per meet similar to empirical observations. Further, we observe persistence in the intermediating role and, as a novel feature, can make prediction about who contacts whom: high-cost agents who are away from target contact low-cost agents more intensely than the other way around.

Keywords: OTC markets, Matching Model, Intermediating Role

Resumen

Para explicar la estructura núcleo-periferia observada en muchos mercados descentralizados, aumentamos un *matching model* aleatorio de tres maneras: introducimos heterogeneidad en las preferencias de los tipos y permitimos a los agentes elegir la intensidad de encontrar ciertos tipos. Los agentes con las preferencias menos cóncavas tienen más probabilidades de unirse al núcleo de intermediación y, como cuasi creadores de mercado, tienen más contactos y transferencias más grandes por *match*, de forma similar a las observaciones empíricas. Además, observamos persistencia en el papel de intermediario y, como característica novedosa, podemos hacer predicciones sobre quién contacta a quién: los agentes de alto costo que están lejos del centro contactan a los agentes de bajo costo más intensamente que viceversa.

Palabras claves: mercados descentralizados, *matching model,* papel de intermediario

Endogenous Search Concentration In OTC Markets

Alexander Dentler*

Revised: November 14, 2015

Abstract

To explain the core-periphery structure observed in many OTC markets we augment a random matching model in three ways: we introduce preference heterogeneity in types and allow agents to choose how intense they meet specific types. Agents with the least-concave preferences are more likely to join the intermediating core and, as quasi market-makers, have more contacts and larger transfers per meet similar to empirical observations. Further, we observe persistence in the intermediating role and, as a novel feature, can make prediction about who contacts whom: high-cost agents who are away from target contact low-cost agents more intensely than the other way around.

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1 Introduction

1.1 Motivation

Intermediation in Over-The-Counter (OTC) markets is concentrated. E.g., about 40 out of every 100 Dollars traded on the fed funds market are borrowed to be re-lend while the average per bank is only 5 Dollars.[1] This means a considerable small number of banks intermediates a large volume to generate the market-wide average. This phenomenon is also referred to as a core-periphery structure where agents in the core allow for indirect trade between agents in the periphery and themselves.[2] The importance of this stylized fact has recently surfaced in the public debate: the financial crisis starting in 2007 led several high-profile policy makers besides Janet Yellen to go on record that the "too-big-to-fail" problem require administrative intervention.[3]

One question arises naturally: how can we explain a core-periphery structure endogenously? This is crucial for policy makers: understanding an endogenous buildup of the financial architecture allows us to better understand contagion, predict the recovery after a crisis where some actors exited or how agents respond to regulations. But it is also a difficult question to answer as it involves two stages: first, an agent needs to make up her mind who she wants to trade with, and, second, the payoff of a match is determined.

We explain a core-periphery structure endogenously where agents in the core allow agents in the periphery to trade indirectly when otherwise they could not. A standard random matching model is augmented in three ways: we introduce preference heterogeneity in types and allow agents to discriminate who and how intense they meet specific types. Agents act as portfolio optimizers that try to attain a certain asset level such as the minimum reserve requirements in the fed funds market. They can only achieve this goal through trade in bilateral meetings. Higher types have a more linear flow payoff than lower types and are therefore more indifferent about changing their holding level. Therefore it is less costly for higher types to deviate from their target to let a lower type achieve hers. This preference heterogeneity can also be interpreted as variation in the cost to deviate from a target or as different degrees of risk aversion. In particular, in the

[1] Compare Afonso and Lagos [4] for the years 2005-2007. The fed funds market is essentially an interbank overnight loan market in which banks trying to meet their legal minimum reserve requirements connect to banks who have excess funds.

[2] The group of actors connecting others indirectly is generally referred to as the core of the market while the remaining agents belong to the periphery. The core-periphery nomenclature stems from the network literature and has no fixed definition. For the scope of this paper I refer to the core as the group of agents that intermediate while agents in the periphery are not agents in the core. There is similar evidence for other markets. E.g., overnight interbank lending market has been studied by Iori et al. [33] in Italy, Craig and Von Peter [20] in Germany and Boss et al. [15] in Austria. The municipal bond market shows similar properties as documented by Li and Schürhoff [40]. The CDS market has recently been studied by Peltonen et al. [43].

[3] Among them are in alphabetical order former Chairman of the Federal Reserve Bernanke [13], Bullard [16] of the Federal Reserve Bank of St Louis, Fisher [24] of the Federal Reserve Bank of Dallas, Hoenig [29] of the Federal Reserve Bank of Kansas, King [37] of the Bank of England, and former Chairman of the Federal Reserve Volcker [50].

fed funds market we empirically associate it with different degrees of net cost ratios as described below. We also find empirically that the bank-specific average of the net cost ratio is associated with variables that indicate intermediation. While the heterogeneity is necessary to pin down who becomes an intermediary it is not sufficient to model the degree of concentration we desire. We require a purposeful direction of contacting and therefore propose a technology that a) resembles contacting choices in an OTC-like environment and b) is easy to implement. The technology can be best described by a trader knowing other traders phone numbers and general aspects about them but not their position. She therefore places phone calls based on her observables. We reduce the general aspects to one dimension, namely the rank position of the heterogeneity, and the result of her choice become matching intensities: who she calls is a mixed strategy where more weight is placed on agents who are more attractive in expectations.

Knowing each type but not each others holding determines our information asymmetry. It leads an agent far away from her target level to contact a more indifferent type with high intensity while an agent close to her target searches a low-type to trade against the stronger curvature. A more indifferent type can therefore expect more meetings with more volume per trade and therefore she becomes even more indifferent about her current holding level. This comparative advantage leads higher types to join the core with a higher probability. Further, they experience more contacts, trade higher volume per meetings and as a result trade a higher total volume which is all in line with the empirical literature. Thus, small exogenously-assumed differences in bank types manifest themselves in large differences in banks' observable gross trading positions. With the agents' types fixed we also observe persistence in core membership which is a key feature in OTC-markets. Further, we can make predictions about who initiates the contact which is a novel contribution of this paper.

We proceed as follows: the remainder of this section is dedicated to connect our work with the existing literature. We shortly motivate the fed funds market and the heterogeneity in the banking sector. The next section describes the model and discusses our assumptions. Next, we describe the equilibrium and motivate equilibrium behaviour. Section 4 shows some equilibrium predictions. A short conclusion summarizes and gives an outlook for future work.

1.2 Literature

Our model matches the empirical evidence that OTC-markets concentrate trade flows through a small group of agents. E.g. Bech and Atalay [10] observe bilateral flows of fed funds throughout the day and find that about 5% of all banks populate the core.[4] Afonso and Lagos [4] find statistics pointing to similar results. The overnight interbank

[4] They define the core as a group of agents whose members form a *giant strongly connected component* in a network. In short, on a given day there is a set of banks who can reach each other following a set of directed links set out by transferred fed funds. Further, they state that 10% of all banks active on a given day populate the core. We want to distinguish that nonparticipation is also a result of choice and since about half of all banks are inactive on any given day we quote the 5%.

lending market has also been studied in Italy by Iori et al. [33], in Germany by Craig and Von Peter [20] and in Austria by Boss et al. [15]. They all point out that intermediation is concentrated among a few banks. Li and Schürhoff [40] study the municipal bond market and find that 10 to 30 dealers act as hubs and connect several hundred peripheral dealers. Peltonen et al. [43] find similar results for the CDS market where only 10 of 946 parties account for 73% of gross purchases and sales.

There are other empirical facts worth pointing out that our model can explain at least qualitatively. Members of the core have more trading partners, trade higher volume per trade and, ultimately, have a higher absolute trading volume.[5] There is also a high degree of persistence in the core-membership. Bech and Atalay [10] document that the chance a bank becomes a member of the core is 76% conditional on her being in the core the preceding day. Afonso and Lagos [4] document that agents far away from their target level will actually trade to move further away.

The large number of participants and the short time intervals between trades point to the main problem why intermediation takes place: agents might know each other because they repeatedly trade with each other, but they do not know who has what. This is the essential reason to use search models and there is a range of work done in OTC-markets using random search models. Gehrig [27] shows that price-posting market makers with access to a Walrasian market can extract surplus from agents in a market riddled with search frictions because they provide immediacy. Our endogenous core also extracts surplus to a degree that makes them disproportionally better off. Duffie et al. [22] explore the environment for market makers that suffer from contacting friction with investors. In both papers the intermediating group is exogenously defined while it arises endogenously in our model. Afonso and Lagos [3] use a random-search model without exogenous middlemen. Intermediation is speculative but by chance: agents average their pre-trade holdings to determine their post-trade holdings due to their homogenous payoff. This deserts intermediation of any meaning besides being a chance happening. While they can replicate some moments they claim "it would be a fruitful task to extend the quantitative work to allow for heterogeneity among banks". Our model specifically addresses the issue of a more purposeful intermediation.

Rubinstein and Wolinsky [46] provide a first analysis of how intermediation arises in search environments. Middlemen are only active if their search friction is comparatively lower than that between buyers and sellers. We find that being contacted more frequently increases the chances of becoming an intermediary in our model. Recently, Neklyudov [42] uses exogenous variation in trade execution efficiency to establish a core with dealers that depend less on private valuations. We find that agents whose valuation is more linear, and therefore depend less on their current holding level, get more endogenous contacts

[5] While the average bank has 6.6 trading partners in 2006 this number shots to 28.4 for banks in the core according to Bech and Atalay [10]. For the same year and source the average loan volume between an actively trading pair is $ 219 million US Dollar per day while it increases to 390 million in the core. The average bank has a total loan volume of $ 720 million US Dollar per day given she traded that day, while it skyrockets to 11074 million in the core.

and appear more efficient in their trade execution. Further, to reconcile a decreasing bid-ask spread for more efficient dealers Neklyudov [42] alludes to the idea that agents shop around. We find that the bid-ask spread is indeed larger in the core when the high types receive a call. Our model differs in that peripheral low-type agents are more likely to be responsible for the contact as they are desperate to trade.

OTC markets are spontaneous and no single agent can either exclude another agent from becoming a middleman nor is the status a tradable right. Therefore we choose not to use entry costs but exploit ex-ante heterogeneity to determine ”good candidates” that are more likely to intermediate while allowing all agents to become a middleman. Using storage costs Kiyotaki and Wright [38] determine what good will become a medium of exchange and, implicitly, who becomes an intermediator in the economy. Agents in our model have different cost to deviate from an optimal portfolio. This deviation cost might be determined by access to other markets. E.g., Corbae et al. [18] develop a model where banks have different degrees of spatial access and diversification. Kashyap and Stein [36] show that smaller banks are slower to adapt to changes in monetary policy pointing to heterogeneous adaption capacities.

Corbae et al. [19] determine who meets whom under full information in equilibrium. Their equilibrium concept requires stability against a two-agent deviation.[6] We circumvent this problem by exploiting the continuous time environment and assume that negotiation and trade are instantaneous and costfree: the alternative to a proposed match is no match and trade at all. The participation constraint ensures that no individual agent deviates and rejects a proposed match.

Further, the contacting technology is unilaterally driven which facilitates drawing from a matching distribution. E.g., Neklyudov [42] essentially needs to reweight an equilibrium distribution by the contacting frequencies, and Atkenson et al. [8] use an entry cost that skews a holding distribution to a distribution of holdings among actively trading agents. The latter also find that larger banks are in the core as entry costs are lower on average and their interdealer market is characterized by a close to common price which is in line with our results but in contrast to empirical findings (compare Li and Schürhoff [40]).

One common shortcoming of random matching models is that states and amounts are usually binary or discrete. Hugonnier et al. [32] introduce valuation heterogeneity by drawing from a continuous distribution. Agents far away from target buy/sell quickly while agents with moderate valuations hold on to the asset longer, and both sell and buy making them de facto intermediaries. While the former resembles the same search effort pattern our agents exert with respect to holding levels we find that the meeting frequency among intermediaries is larger which is in line with empirical facts (compare Li and Schürhoff [40]).

Our model is also closely related to a Chang and Zhang [17]. Highly volatile types match with low volatile types through contingency trade contracts before shocks are realized. A key distinction is that we form links after agents know their realizations

[6] This concept is identical to ”pairwise stability” used in the network formation literature (compare Jackson et al. [34]).

which are continuous. This allows for any type to meet any other type in probability and motivates why high types match with other high types in equilibrium.

There is a vast body of literature analyzing the benefits of repeated relationships both, empirically and theoretically.[7] The idea to use invested relationships to establish a core periphery structure has brought forward two recent proposals: Babus [9] lets agents face the trade-off between pledging costly collateral or going through an intermediary with whom they have a long-standing relationship in which they invested. This mechanism results in star networks as the equilibrium outcome. The model proposed here explicitly ignores the effects of repeated interactions which would surely bolster the concentration of intermediation. Farboodi [23] motivates concentration in network formation by forcing banks to establish credit lines before they realize whether a distinct subgroup of agents obtains an investment project. Because banks try to reduce the distance to the profitable object investment banks become hubs as they are "more likely" to be net demanders of funds. We eventually keep the model symmetric so that agents establish a core-periphery structure without knowing net demand.

1.3 The fed funds market

The federal funds market serves as a narrative example. Therefore let us briefly summarize the institutional framework. Fed funds are held at the Federal Reserve by commercial banks to satisfy legally mandated reserve requirements. These reserves safeguard banks from being short of liquidity due to unexpected shocks such as interbank clearing, loan delinquencies etc. Insufficient account levels require a loan from the discount window at a punitive rate. Monitoring by the monetary authority takes place at the end of each business day. This in turn creates opportunities for overnight lending between banks that have excessive funds and banks that have insufficient funds. The fed funds market provides and absorbs these needs for liquid funds through unsecured overnight loans.

With 7387 commercial banks in the US in 2006 Bech and Atalay [10] find only around 986 ever were active on the fed funds during that year. Most smaller banks are not active themselves but use a bankers bank that act on their behalf. This large bank-small bank dichotomy goes further. Furfine [26] documents that larger banks are on average net buyers. Larger banks are more diversified and therefore display less a risk as a potential borrower. Another reason brought forth by Allen and Saunders [7] is that larger banks are publicly known while the creditworthiness of small, rural banks is harder to judge, and therefore the latter is priced out. We will not focus on the link between size and net demand for the heterogeneity used in this paper.

The reserve requirements are supposed to be held over an average of a two week maintenance period. This means that banks can be below their respective target and not use the discount window as long as they have a) excess reserves on other days during the same two week period and b) no negative balances at each end of the business day. Hamilton [28] provides evidence of periodicity. Banks build up a good average early in

[7] Compare Affinito [2], or Afonso et al. [5].

the maintenance period so that the interest rate decreases during the first week of the maintainance period.

1.4 Heterogeneity

The form of heterogeneity used here implies that a) agents are essentially portfolio optimizer in an environment with two goods and b) that there is variation of costs when agents hold a suboptimal portfolio. The optimal portfolio for banks in the fed funds market is determined by the reserve requirements: banks do not want to hold an excess amount as they hold interest-free or low-interest cash[8] but holding too little results in a punitive loan from the discount window. But note that banks can effectively build up credit early or make up insufficient funds later so that they only need to hold the level of the reserve requirements on average. Therefore, excess reserves can be converted into an interest bearing asset by issuing loans, and banks can issue large denominated certificate of deposits that do not affect reserve requirements to acquire funds. Our source of heterogeneity stems from the fact that they can do so with different ability, and we approximate this variation by a bank-specific average net cost ratio. This ratio reflects the competitiveness of banks outside the fed funds market. A bank with a ratio of 1 can issue a new loan but she will not gather a net return in doing so. Therefore if she had excess returns she is not able to convert this into a gain. Similarly, she can issue certificates of deposits to gain funds but she has costs associated with it that are equivalent to her investment opportunities. In other words, she can compensate for her lack of funds on the deposit market but will not offset the cost of the discount window. On the other hand a bank with an average net cost ratio below 1 can issue extra loans at a market rate with smaller costs associated. Similarly, she can raise funds to make a profit. We therefore adopt the idea that whenever a bank is off her target she can adapt better over the two week maintenance period the smaller her average net cost ratio is.[9]

We use the quarterly Reports of Condition and Income by the Chicago federal reserve from 2003 to 2006 to display this heterogeneity among banks. We obtain 24859 total observations in an unbalanced panel data set with 2399 banks over 16 quarters. The majority of dropped observations from the original 7387 stems from the fact that smaller banks do not have a fed funds account. We annualize not fed-funds related interest and non-interest expenses and income for commercial banks and denote the net cost ratio as the ratio of relative expenses over relative income. Let us denote this variable by $\upsilon_{b,q}$ for bank b in quarter q. We obtain the bank-specific average net cost ratio, drop observations below 0 and above 1 and denote it $\bar{\upsilon}_b$. This leaves us with 2173 observations.

Table 1 provides an overview about some summary statistics. First, note that mean and median are in the expected region between 0 and 1. Second, the large within-quarter

[8] The federal reserve now pays (some) interest on excess balances.

[9] Furfine [25] uses adjustment costs and quotes costs associated with excessive shrinking due to severed relationships as described by Diamond [21] and the ending of economies of scale such as in Berger et al. [12].

variation and the small between-quarter variation tell us that banks are heterogeneous with respect to their net cost margin at any point of time. Third, the within-bank variation is much smaller than the between-bank variation. The heterogeneity with respect to cost forwarding persists over time.

	net cost ratio	average net cost ratio
Mean	0.54	0.72
Median	0.72	0.73
Min	-8326.44	0.02
Max	820.27	1.00
Total variation	53.47	0.13
Within-bank-variation	1.92	
Between-bank-variation	28.14	
Within-quarter-variation	15.69	
Between-quarter-variation	1.19	
No observations	24652	2173
No banks	2374	2173
No quarters	16	

Table 1: Summary statistics for net cost margin and average net cost margin. The within-bank variation is given by $\frac{1}{B}\sum_b \sqrt{\frac{1}{Q}\sum_q (\upsilon_{b,q} - \bar{\upsilon}_b)^2}$ and the (complimentary) between-bank variation is given by $\sqrt{\frac{1}{B}\sum_b (\bar{\upsilon}_b - \bar{\upsilon})^2}$. The within-quarter variation is given by $\frac{1}{Q}\sum_q \sqrt{\frac{1}{B}\sum_b (\upsilon_{b,q} - \bar{\upsilon}_q)^2}$ and the between-quarter variation is given by $\sqrt{\frac{1}{Q}\sum_q (\bar{\upsilon}_q - \bar{\upsilon})^2}$.

Next we take a look how the average net cost ratio is associated with variables that indicate intermediating activity at the fed funds market. Average volume traded is the bank-specific average of the quarterly average of fed funds bought and sold. Excess funds reallocation is the bank-specific average of the difference between the sum of fed funds bought and sold and the net position change for the last business day. The variable gross-over-net divides the total of fed funds traded by the net demand determined by a banks fed funds position and reserve requirements. A size is given by her average quarterly asset size and because of the skewed distribution we also report the logarithm thereof.

A higher average net cost ratio is associated with less volume traded, less excess funds reallocation, less gross-over-net and smaller banks. In other words, the smaller the average net cost ratio the more likely a bank will intermediate on the fed funds market.

	Pearson	Kendall	Spearman
average volume traded	-0.02	-0.26	-0.37
	(0.3)	(0)	(0)
excess funds reallocation	-0.03	-0.13	-0.17
	(0.2)	(0)	(0)
gross-over-net	-0.04	-0.11	-0.16
	(0.07)	(0)	(0)
size	-0.05	-0.27	-0.39
	(0.03)	(0)	(0)
log(size)	-0.33	-0.27	-0.39
	(0)	(0)	(0)

Table 2: Correlation statistics for the average net cost ratio with different variables that indicate intermediation activity on the fed funds market. The p-values are in parenthesis. All values are rounded to two digits.

2 Model

2.1 Environment

Time is continuous[10] and any future payoff is uniformly discounted with r. There is a unit mass of non-atomistic agents. Each agent is associated with a time-invariant type denoted $i \in [0,1]$ and i is uniformly distributed among the population. Further, each agents type is public knowledge. An agent can hold and store an indisposable asset $x \in \mathbb{R}$ whose level is private information unless two agents are matched to trade. Holding x results in a flow payoff $u_i(x) : [0,1] \times \mathbb{R} \to \mathbb{R}$ which is bounded, continuous and strictly concave in x. We further impose that $\frac{\partial^2 u_i(x)}{\partial x^2}$ is monotonically nondecreasing in i. At any given point of time an agent can experience an innovation to her holding level: her new level x is drawn from distribution G which is non-degenerate and has a continuous and bounded density g(x). Note that her new holding is independent of her old level. These events arrive at the homogenous Poisson arrival rate Λ^x. Further, an agent can produce a good $y \in \mathbb{R}$ at constant marginal cost and, similarly, consume the same good y giving the agent constant marginal payoff equal to the marginal cost.[11]

All trade is bilateral and meeting events only take place at random times that are idealized as event times of a Poisson process with intensity Λ^c. There are two ways an agent of type i holding x is matched to trade with another party: first, to meet a particular type i' she can actively exert search effort $\lambda_i^{i'}(x) \geq 0$ against cost given by $c(\lambda) = \frac{\alpha}{\kappa}\lambda^\kappa$ with $1 < \kappa \leq \infty$ and $0 < \alpha < \infty$. This results in an agent of type i holding x meeting

[10] We will focus on a stationary environment eventually, and therefore avoid notation that implies time-dependency from the start.

[11] We make the simplifying yet reasonable assumption that agents do not produce to consume by themselves. The good y is used as numeraire and a medium of exchange. For the fed funds market x can be thought of as currently held fed funds while y is the net present value of the certain repayment.

an agent of type i' as a draw from a distribution with density $f_i(x)\lambda_i^{i'}(x)/\Lambda^c$ at a meeting event. Note that she has no control over the level of x' as this is private knowledge. The holding level of an eventual match is modeled as a random draw from the type-conditional equilibrium distribution which has a density function $f_{i'}(x')$. Also note that she can search for all types simultaneously paying individual cost for each type. Second, she can be contacted by an agent of type i' holding x' in a converse fashion. In the first event we will refer to the agent (i, x) as the caller while she is the receiver of the call in the second version. The economywide calling intensity is defined by $\Lambda^c = \iiint f_i(x)\lambda_i^{i'}(x)didxdi'$ which is equal to the economywide receiving intensity.[12]

Trade is effortless, instantaneous and governed as follows: after two agents are matched their holding levels x and x' are revealed to each other. They bargain over a possible exchange of the two type of goods. The terms of trade, denoted $(X_{i,i'}(x, x'), Y_{i,i'}(x, x'))$, are reached using a technology whose solution is given by Nash bargaining where the receiver has bargaining parameter $\theta \in [0, 1]$. After the agents conclude on their terms of trade they exchange goods with certainty, and depart. Further, no agent has a technology to remember the other agents holding after they split.

We will focus on the stationary version of this economy. Therefore, an agents state is solely defined by his type i and his holding x.

2.2 Discussion

The environment described above fits basic facts about typical OTC-markets such as the fed funds market, the market for municipal or corporate bonds, the market for credit default swaps or other exotic financial products. First and foremost, trade is bilateral and executed among a (potentially) large set of agents.

Random search models assume that a match is created by pure randomness. Neither the frequency of meetings nor who is met are considered a result of choice. But agents generally have different needs to trade which is captured in our model by the level-choice of search intensity. While it is true that the OTC-environment is not designed to reveal exact information about potential trading partners agents know basic characteristics exactly or in distribution either by repeated interaction, by reputation or some other inference. We therefore introduce a search technology that allows agents to direct their relative efforts towards publicly known types. Further, meetings are assumed to be initialized by one party and accepted by the other resembling contacting choices made by a phone call. This also allows us to learn more about who initiates the contacts in these markets when the model is eventually brought to the data, a feature not addressed by any model that we are aware of.

We assume trade to be instantaneous because trade in OTC-markets is fast-paced. This also allows us to avoid queuing solutions where some agents are rationed away which

[12] An alternative explanation where each possible meeting event follows a Poisson process would be more intuitive. But the second Borel -Cantelli lemma prohibits this as the sum of probabilities diverges to infinity.

resembles the fact that OTC-markets are not known to have queues nor rationing.

Further, the information structure resembles that of a market where agents know each others "type", but not each others holdings before bargaining with two caveats: first, we switch from asymmetric information to symmetric knowledge when agents trade. It serves as a simple first approximation. Using Nash-style bargaining retains the strategic component of the threat to leave the bargaining table. This outside option in turn is largely defined by the environment which is the primary focus of any equilibrium analysis including this one. Second, not allowing for any record keeping forbids agents to "go back with a purpose" to an earlier bargaining partner after meeting a third party. This possibly reduces the buildup of a "core" of an economy where agents are connected with transfers pointing in both direction. The alternative of Bayesian-updating of believes of each others holdings after a meet approaches computational impossibility.

The concavity in the flow payoff lends itself to an interpretation where agents face a portfolio problem with two goods. E.g., in case of the federal funds market banks hold fed funds to satisfy reserve requirements where holding too little results in a discount window loan at a punitive high interest rate while holding too much results in excess fed funds that yield no or little interest. One can think of good x as federal funds and good y as the promise of a discounted and certain repayment. In other markets y can be thought of the good that is easiest transformed into direct utility, i.e. the most liquid good. A possible "target level" is defined not only by an agents flow payoff but also by the distribution of holdings across the economy. E.g., a solution without search friction requires that at any point of time agents equalize their marginal flow payoff, $\frac{\partial u_i(x)}{\partial x} = \frac{\partial u_{i'}(x')}{\partial x}$ in which case the allocation only depends on the total quantity of x available.

The time-invariant heterogeneity in the flow payoff implies that some agents are more eager to reach a target level, and its public information characterization suggests that this exogenous variation is known by other agents. E.g., larger banks that are active on the fed funds market have lower average net cost ratio. They can therefore buy assets with less cost associated to drop excess fed funds than smaller banks. Similarly, an inter-dealer market for bonds or other financial products will have some parties whose customer base is larger so that an excess position of x can be unloaded more easily. E.g., Li and Schürhoff [40] document that some dealers have access to smaller markets. Alternatively, one can interpret the heterogeneity as difference in diligence managers require from their trading desks when ask to reach a certain target level.

Nash-style bargaining retains social optimality, is easy to compute and the solution is equivalent of a strategic bargaining protocol a la Rubinstein [45]. With one quasilinear good it is also equivalent to Kalai [35], another axiomatic bargaining solution. It turns out that any socially optimal bargaining protocol will yield an equilibrium solution similar to the one as described below. The notable difference might be the exchange of the quasilinear good y.

Pairwise stability in the sense of Jackson et al. [34] is ensured by the participation constraint of the bargaining protocol and by the instantaneity of trade in a search friction environment: every agent is as least as good off if he agrees to a match and no agent can deviate and form another coalition.

A shock to an agents holding level is not only a technical necessity to avoid a stationary solution with a degenerate distribution but has some micro foundations. E.g., banks clear checks that are issued between customers which alters their fed funds level throughout the day. Alternatively, these innovations can be thought of changes in agents relative preferences over the two goods or as changes in their respective information sets that alter their optimal portfolio allocation. Dealer can learn about their customers hedging needs and change their optimal holding level. Hugonnier et al. [32] also uses renewals that are independent of past values.

2.3 Calling technology

At any point of time an agent faces the question how much effort she should exert to increase her chance to meet an agent of type $i' \in [0,1]$. Let $S^c_{i,i'}(x,x')$ denote the net surplus from bargaining a caller of type i holding x obtains after being matched with an agent of type i' holding x'. The surplus of a meeting depends not only on her type, her holding and the receivers type but also on the unknown holding of the receiver. Therefore she needs to take expectations over the receivers holding x' to obtain her expected net surplus from calling type i'.

$$ES_i^{i'}(x) = \int_{\mathbb{R}} S^c_{i,i'}(x,x') f_{i'}(x') dx' \tag{1}$$

where we assume that S^c and f is equilibrium knowledge. She now trades off the linearly increasing chance to meet with an increasing and convex cost for each type i'.

$$W_i^{i'}(x) = \max_{\lambda \geq 0} \left\{ \lambda ES_i^{i'}(x) - \frac{\alpha}{\kappa} \lambda^\kappa \right\} \tag{2}$$

where $1 < \kappa \leq \infty$ and $0 < \alpha < \infty$. α levels the cost while κ shifts the marginal increase of costs. With κ close to 1 the costs are almost linear so that a small increase in $ES_i^{i'}(x)$ results in a steep change in any optimal solution of $\lambda_i^{i'}(x)$. Similarly, letting κ larger makes a solution less discriminant. In fact, $\kappa \to \infty$ yields a random search model.

2.4 Bargaining

Say an agent of type i holding x, the caller, is matched with agent of type i' holding x', the receiver. Then the Nash bargaining problem is described as follows:

$$\begin{aligned} S_{i,i'}(x,x') = & \max_{\{X \times Y \in \mathbb{R}^2\}} \Big\{ \left(V_i(x-X) + Y - V_i(x)\right)^{1-\theta} \\ & \times \left(V_{i'}(x'+X) - Y - V_{i'}(x')\right)^{\theta} \Big\} \qquad (3) \\ \text{s.t.} \quad & V_i(x-X) + Y - V_i(x) \geq 0 \qquad \text{PC Caller} \\ & V_{i'}(x'+X) - Y - V_{i'}(x') \geq 0 \qquad \text{PC Receiver} \end{aligned}$$

where V denotes the maximal attainable payoff and (X,Y) are the terms of trade. The two constraints are the participation constraints of the caller and the receiver, respectively.

2.5 Payoff

A state is pinned down by the type i and holding x. An agent remains in this state and experiences the same flow payoff and cost to search until one of three events occur: a shock to her holding level, she successfully initiates a meeting as a caller or she is contacted as a receiver. Let us denote the time until a shock to her holding level occurs by τ^x. Similarly, τ^c denotes the random variable until she is a caller and τ^r denotes the random time until she is a receiver. While the first is exogenously given the latter ones are subject to choice. All three can be described as homogeneous Poisson processes in a stationary environment. As rivaling processes we denote the minimal arrival time of any event by $\tau = \min(\tau_x, \tau_c, \tau_r)$. Her potential future state is continuously discounted by r. Let $\mathbb{I}_\bullet$ be the indicator function that is equal to one when the statement $\bullet$ is true and zero otherwise. Taking expectations over τ we can denote an agents maximal attainable payoff by the following renewal equation:

$$
\begin{aligned}
V_i(x) =\mathbb{E}_\tau \Bigg\{ & \underbrace{\int_0^\tau e^{-rz}\left[u_i(x) - \int_0^1 c(\lambda_i^{i'}(x))di'\right] dz}_{\text{flow payoff and search cost}} + \underbrace{e^{-r\tau}}_{\text{discount}} \Bigg[\underbrace{\mathbb{I}_{\{\tau=\tau_x\}}}_{\textbf{H}\text{olding}} \underbrace{\int_{\mathbb{R}} V_i(\hat{x})g(\hat{x})d\hat{x}}_{\text{renewal}} \\
& + \underbrace{\mathbb{I}_{\{\tau=\tau_c\}}}_{\textbf{C}\text{alling}} \int_0^1 \int_{\mathbb{R}} \underbrace{\frac{\lambda_i^{i'}(x) f_{i'}(x')}{\Lambda_i^c(x)}}_{\text{call of } (i,x) \text{ to } i'} \underbrace{\{V_i(x - X_{i,i'}(x,x')) + Y_{i,i'}(x,x')\}}_{\text{gross posttrade payoff}} dx'di' \\
& + \underbrace{\mathbb{I}_{\{\tau=\tau_r\}}}_{\textbf{R}\text{eceiving}} \int_0^1 \int_{\mathbb{R}} \underbrace{\frac{\lambda_{i'}^{i}(x') f_{i'}(x')}{\Lambda_i^r}}_{\text{call of } (i',x') \text{ to i}} \underbrace{\{V_i(x + X_{i',i}(x',x)) - Y_{i',i}(x',x)\}}_{\text{gross posttrade payoff}} dx'di' \Bigg] \Bigg\}
\end{aligned} \tag{4}
$$

where $\Lambda_i^c(x) = \int_0^1 \lambda_i^{i'}(x)di'$ denotes the total intensity of making a call while $\Lambda_i^r = \int_0^1 \int_{\mathbb{R}} \lambda_{i'}^i(x')f_{i'}(x')dx'di'$ denotes the intensity at which an agent of type i is contacted. Both terms normalize each event to 1.

2.6 Law of motion

For any type-holding combination (i,x) the law of motion can be described by the following equation.

$$
\begin{aligned}
\dot{f}_i(x) = &-\int_0^1\int_{\mathbb{R}} \mathbb{I}_{\left\{x+X_{i',i}(x',x)\neq x\right\}} f_i(x) f_{i'}(x')\lambda_{i'}^{i}(x')dx'di' && \text{(outflow by receiving)}\\
&-\int_0^1\int_{\mathbb{R}} \mathbb{I}_{\left\{x-X_{i,i'}(x,x')\neq x\right\}} f_i(x) f_{i'}(x')\lambda_{i}^{i'}(x)dx'di' && \text{(outflow by calling)}\\
&+\int_0^1\int_{\mathbb{R}}\int_{\mathbb{R}} \mathbb{I}_{\left\{\hat{x}+X_{i',i}(x',\hat{x})= x\right\}} f_i(\hat{x}) f_{i'}(x')\lambda_{i'}^{i}(x')dx'd\hat{x}di' && \text{(inflow by receiving)}\\
&+\int_0^1\int_{\mathbb{R}}\int_{\mathbb{R}} \mathbb{I}_{\left\{\hat{x}-X_{i,i'}(\hat{x},x')= x\right\}} f_i(\hat{x}) f_{i'}(x')\lambda_{i}^{i'}(\hat{x})dx'd\hat{x}di' && \text{(inflow by calling)}\\
&-\Lambda^x f_i(x) + \Lambda^x g(x) && \text{(innovations)}
\end{aligned}
\tag{5}
$$

The first line captures the events where an agent (i,x) is called by an agent of type (i', x') and leaves her state. The second line similarly leads to an outflow but this time (i,x) initiated the contact. The third line (5) yields to an inflow of an agent of type $(i, \hat{x})$ to (i,x) because she got called while the fourth line captures the mass of agents flowing to (i,x) because they call (i', x'). Finally, the fifth line capture out and inflow because a mass of agents receives an innovation shock.

3 Equilibrium

This section will summarize the solutions to the agents individual choice problems, the value function and the distribution function. We will further motivate how a core is established. But let us first define the equilibrium. An equilibrium in the environment described above is characterized as follows:

Definition 1. *A stationary and symmetric equilibrium in the environment described above is defined by a value function V, an equilibrium density f, and policies $X, Y,$ and λ s.t.*

- *f satisfies the law of motion described below and $\dot{f}_i(x) = 0$ everywhere.*
- *V is maximal attainable payoff.*
- *X and Y solve Nash.*
- *λ solves search problem described below.*

3.1 Bargaining

The following proposition pins down some important characteristics about the bargaining problem.

Proposition 1. *Let V be a real-valued and bounded function that is continuous, differentiable and strictly concave in x. The solution to (3) is uniquely given by*

$$\begin{aligned} X_{i,i'}(x,x') =& \underset{\{X\in\mathbb{R}\}}{\arg\max}\big\{V_i(x-X) - V_i(x) + V_{i'}(x'+X) - V_{i'}(x')\big\} && (6)\\ Y_{i,i'}(x,x') =& (1-\theta)\left(V_{i'}(x'+X_{i,i'}(x,x')) - V_{i'}(x')\right) \\ & - \theta\left(V_i(x - X_{i,i'}(x,x')) - V_i(x)\right) && (7) \end{aligned}$$

and the bounded, nonnegative social net surplus is given by

$$S_{i,i'}(x,x') = \max_{\{X\in\Gamma(x,x')\}}\big\{V_i(x-X) - V_i(x) + V_{i'}(x'+X) - V_{i'}(x')\big\} \quad (8)$$

We require some common properties on V to have a solution for the terms of trade. In fact, we need V to be strictly concave for a unique solution but do not require V to be monotone. Further, note that the social net surplus is the sum of the callers and the receivers net surplus where the terms for $Y_{i,i'}(x,x')$ drops because of linearity. Corollary 2 essentially shows that the net surplus is split between caller and receiver by θ.

Corollary 2. *The net surplus from bargaining for the caller and the receiver are, respectively:*

$$\begin{aligned} V_i(x - X_{i,i'}(x,x')) + Y_{i,i'}(x,x') - V_i(x) = (1-\theta)S_{i,i'}(x,x') \\ V_{i'}(x' + X_{i,i'}(x,x')) - Y_{i,i'}(x,x') - V_{i'}(x') = \theta S_{i,i'}(x,x') \end{aligned}$$

It is easy to see from (16) that a set of matched agents $((i,x),(i',x'))$ will not trade if their marginal payoff of holding the good x is identical. Naturally, $S_{i,i'}(x,x') = 0$ follows. This is the case in particular when $i = i'$ and $x = x'$, or when an agent meets an identical copy. A social net surplus is only positive if $\frac{\partial V_i(x)}{\partial x} \neq \frac{\partial V_{i'}(x')}{\partial x}$. The following lemma summarizes this fact.

Lemma 3. *$S_{i,i'}(x,x')$ is strictly decreasing (increasing) in $x < \bar{x}$ ($x > \bar{x}$) and has a unique minimum for $x = \bar{x}$ at $\frac{\partial V_i(\bar{x})}{\partial x} = \frac{\partial V_{i'}(x')}{\partial x'}$ with $S_{i,i'}(\bar{x},x') = 0$. Similarly, $S_{i,i'}(x,x')$ is strictly decreasing (increasing) in $x' < \bar{x}'$ ($x' > \bar{x}'$) and has a unique minimum for $x' = \bar{x}'$ at $\frac{\partial V_i(x)}{\partial x} = \frac{\partial V_{i'}(\bar{x}')}{\partial x'}$ with $S_{i,i'}(x,\bar{x}') = 0$.*

It is easy to see that the larger the difference between the marginal payoff of holding x the larger the surplus from trading. It should also be obvious that given strict concavity the mass of events where no trade takes place is essentially zero.

3.2 Calling technology

The following proposition pins down some important characteristics about the search problem.

Proposition 4. *Let V be a real-valued and bounded function that is continuous, differentiable and strictly concave in x. Let f be a well-defined, continuous density function from a nondegenerate distribution. Further, let $c(\lambda) = \frac{\alpha}{\kappa}\lambda^{\kappa}$ with $1 < \kappa < \infty$ and $0 < \alpha < \infty$ define the parameterized cost function.*

A solution to (2) exists and is uniquely defined by

$$\lambda_i^{i'}(x) = \left(\frac{ES_i^{i'}(x)}{\alpha}\right)^{\frac{1}{\kappa-1}} \tag{9}$$

and the net surplus from calling an agent of type i' for an agent of type i holding x is

$$W_i^{i'}(x) = \frac{\kappa-1}{\kappa}\left(\frac{ES_i^{i'}(x)^{\kappa}}{\alpha}\right)^{\frac{1}{\kappa-1}} \tag{10}$$

Because of the participation constraints and how surplus is split we know $ES_i^{i'}(x)$ is nonnegative. The return is linear in λ and the cost is increasing and convex so that we are guaranteed a solution (17). Solution (18) pins down the expected surplus from calling a type i' for an agent of type i holding x. The following corollary states that all agents search for all types regardless of their type and holding.

Corollary 5. $\lambda_i^{i'}(x) > 0$ *and* $W_i^{i'}(x) > 0$ *everywhere if* $\theta < 1$.

The reason is straightforward: an agent of type i holding x will only not trade with another agent of type i' if they have the same marginal payoff. But with V strictly concave and f nondegenerate there is always some positive mass of agents where trade is beneficial. Therefore $ES_i^{i'}(x) > 0$ and with $\frac{\partial c(0)}{\partial \lambda} = 0$ we have positive search intensity everywhere. With the same line of arguments we find $W_i^{i'}(x) > 0$. Of course this only holds if the caller gets a fraction of the surplus, or if $\theta < 1$.

3.3 Payoff

The following lemma gives us a more convenient form of (4) as the result of some rearrangements.

Lemma 6. *Let V be a real-valued and bounded function that is continuous, differentiable and strictly concave in x. Let f be a well-defined, continuous density function. Further, let $c(\lambda) = \frac{\alpha}{\kappa}\lambda^{\kappa}$ with $1 < \kappa < \infty$ and $0 < \alpha < \infty$ define the parameterized cost function.*

Then (4) has a linear transformations of the form

$$\begin{aligned}\bar{V}_i(x) =& \frac{1}{r+\Lambda^x}\Bigg[u_i(x) - c_i(x) \\ &+ \int_0^1 \int_{\mathbb{R}} \Big((1-\theta)\lambda_i^{i'}(x) + \theta\lambda_{i'}^{i}(x')\Big) f_{i'}(x') S_{i,i'}(x,x') dx' di'\Bigg] \end{aligned} \tag{11}$$

where $c_i(x) = \int_0^1 c(\lambda_i^{i'}(x))di'$.

The agent strictly prefers participating over leaving the economy when $\theta < 1$. This follows from Corollary 5: agents expect to be able to trade at least with some subfraction of agents of every type to a Pareto improvement. Therefore the expected net surplus $ES_i^{i'}(x)$ is positive, and they will contact others with a positive frequency, and they will be contacted by others at a positive frequency.

The following proposition shows that (20) is actually a contraction mapping.

Proposition 7. *Let B be the space of real-valued, bounded functions defined on $[0,1] \times \mathbb{R}$ which are continuous in x, and denote their elements by V and $\hat{V}$. Let $D : B \times B \to \mathbb{R}^+$ be the supnorm metric in the sense of $D(V,\hat{V}) = \sup_{i \times x \in [0,1]\times\mathbb{R}} \left\{\left|V_i(x) - \hat{V}_i(x)\right|\right\}$. Further, let T be defined on B. In particular,*

$$\begin{aligned}T(\bar{V}_i(x)) =& \frac{1}{r+\Lambda^x}\Bigg[u_i(x) - c_i(x) \\ &+ \int_0^1 \int_{\mathbb{R}} \Big((1-\theta)\lambda_i^{i'}(x) + \theta\lambda_{i'}^{i}(x')\Big) f_{i'}(x') S_{i,i'}(x,x') dx' di'\Bigg] \end{aligned} \tag{12}$$

where λ and S are functions of V. Then T is a global contraction. Further, V is strictly concave and differentiable.

The proof is given in the appendix and it is a fairly general result. We do not require cost heterogeneity in type. We also conjecture that the model can be further relaxed. E.g., the proof for Proposition 7 does not hinge on homogeneity in the liquidity shocks: neither the frequency nor the distribution must be the same across type. The cost of the contacting technology can also depend on who is calling and who is being reached. The discounting factor (very similar to the flow payoff) as well as bargaining power can also be non-uniform, and the proof should go through for either cases. Any socially optimal trading technology can be used. Note that Kalai [35] is equivalent in the solution to Nash [41] with one good of the exchange being linear.

A distinction of heterogeneous-agent economies that are based on bilateral exchange is the lack of a central clearing mechanism. E.g., Aiyagari [6], Bewley [14], or Huggett

[31] have markets that match supply and demand using a price which is in turn is used to establish existence of (at least) stationary equilibria.[13] Here, meeting a trading partner establishes the equivalent of a consumption or investment technology, and strict concavity ensures boundedness of any trading solution. This in turn guarantees existence and uniqueness in these individual meetings, but cannot be used directly to establish existence or uniqueness of the economy itself.

The following statement enables us to confine our attention to symmetric environments where the amount of agents with excess assets equals the amount of agents of the same type with assets that are equally far below their optimal target level. Further, a deviation from an optimal holding $x = 0$ in one way is equally costly as a deviation in the other way.[14]

Corollary 8. *TV is even given u and f are even and the assumptions in Proposition 7.*

3.4 Law of motion

The following lemma gives a more convenient form for (5) in when f is in steady state.

Lemma 9. *Let $\lambda_i^{i'}(x)$ and $X_{i,i'}(x, x')$ be scalar-valued policies. Further, let $X_{i,i'}(x, x')$ be strictly monotone in x'. f is a well-defined continuous density function. For $\dot{f}_i(x) = 0\ \forall i \in [0, 1], x \in \mathbb{R}$ (5) becomes*

$$f_i(x) = \frac{1}{\Lambda_i^r + \Lambda_i^c(x) + \Lambda^x} \Bigg(\Lambda^x g(x) + \int_0^1 \int_{\mathbb{R}} f_{i'}(h_{i,i'}(\bar{x}, x)) f_i(\bar{x}) \times \\ \left(\lambda_{i'}^i(h_{i,i'}(\bar{x}, x)) + \lambda_i^{i'}(x) \right) \sqrt{1 + H_{i,i'}(\bar{x}, x)^2} d\bar{x} di' \Bigg) \tag{13}$$

where $h_{i,i'}(\bar{x}, x) : \bar{x} - X_{i,i'}(\bar{x}, h_{i,i'}(\bar{x}, x)) = x,\ H_{i,i'}(\bar{x}, x) = \frac{\partial_- h_{i,i'}(\bar{x},x)}{\partial \bar{x}}$.

The terms h and H stem from converting the indicator functions to a line integral. $H_{i,i'}(\bar{x}, x)$ is the left-sided derivative of $h_{i,i'}(\bar{x}, x)$ with respect to $\bar{x}$. The following proposition establishes that Lemma 9 is actually a contraction mapping.

Proposition 10. *Let $\mathbb{F}$ be the space of real-valued, bounded functions defined on $[0, 1] \times \mathbb{R}$ which are continuous in x with elements $f, \hat{f}$, and $D : \mathbb{F} \times \mathbb{F} \to \mathbb{R}^+$ be the supnorm metric in the sense of $D(f, \hat{f}) = \sup_{i \times x \in [0,1] \times \mathbb{R}} \left\{ \left| f_i(x) - \hat{f}_i(x) \right| \right\}$. Further, define T' by equation (21).*

[13] Compare Achdou et al. [1].

[14] The most accessible way to think about this symmetry is to visualize f to be a standard normal density so that it is symmetric around zero and $u_i(x) = -\delta_i x^2$ where $\frac{\partial \delta_i}{\partial i} < 0$. Also note that symmetry around 0 and evenness refer to the same property.

In particular,

$$T'(f_i(x)) = \frac{1}{\Lambda_i^r + \Lambda_i^c(x) + \Lambda^x}\left(\Lambda^x g(x) + \int_0^1 \int_{\mathbb{R}} f_{i'}(h_{i,i'}(\bar{x},x)) f_i(\bar{x}) \times \right.$$
$$\left. \left(\lambda_{i'}^{i}(h_{i,i'}(\bar{x},x)) + \lambda_i^{i'}(x)\right)\sqrt{1 + H_{i,i'}(\bar{x},x)^2}\, d\bar{x}\, di'\right)$$

Then $T'f$ is a global contraction. Further, f is a well-defined, nondegenerate and continuous density function.

The following statement compliments Corollary 8 which allows us to focus on symmetric environments.

Corollary 11. *f is even given V and g are even in x, and the assumptions of Proposition 10.*

Two essential features in a symmetric economy are a) $F_i(0) = 0.5\ \forall i \in [0,1]$, or the mass of asset holdings less than 0 is exactly half, and b) $\frac{\partial V_i(0)}{\partial x} = \frac{\partial V_{i'}(0)}{\partial x}\ \forall i, i' \in [0,1]$, or the marginal payoff for holding x is identical for all agents holding 0.

3.5 Calling

The following proposition pins down that agents who are moving away from their optimal holding will increase their search effort.

Proposition 12. *$\lambda_i^{i'}(x)$ is strictly decreasing (increasing) for $x < 0$ ($x > 0$) and has a unique minimum at $x = 0$, or agents further away from their target search more given V and f are even and the assumptions of Proposition 4.*

The proof is given in the appendix but a rough intuition is straightforward: An agent who has an asset surplus x and obtains more ($\hat{x}$) will increase the surplus from meeting agents whose marginal payoff for that asset is equal or higher than her own. The latter are essentially agents whose asset position is less than the type-specific $\bar{x}_i^{i'}(x)$ that equates the marginal payoff of an i'-type with our potential caller (i,x). Figure (1) visualizes this fact when an agent meets his own type. This is at least half of all agents in an even economy as the agent who is indifferent to trade with her is also above her target level for all types. On the other hand the mass of agents whose surplus from trading diminishes is below half.

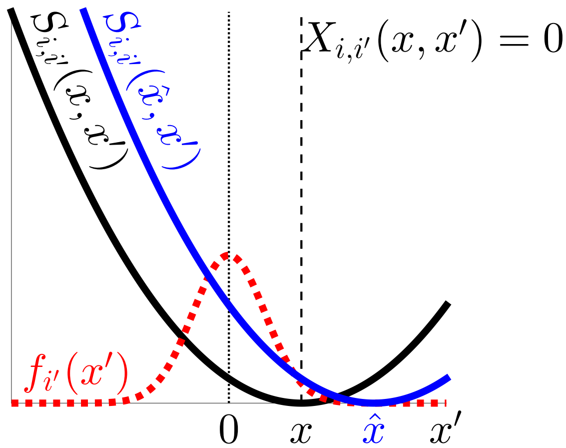

Figure 1: Surplus from bargaining when meeting an agent of the same type holding x and $\hat{x}$.

It would be desirable to verify whether $\frac{\partial^2 V_i(x)}{\partial x^2}$ is also monotonically nondecreasing in i. Unfortunately we can make little predictions about the form of V and f in equilibrium in this regard. But we can also make the following conjectures and motivate under the assumption that $\frac{\partial^2 V_i(x)}{\partial x^2}$ is indeed monotonically nondecreasing in i.

Conjecture 1. $\frac{\partial^2 \lambda_i^{i'}(x)}{\partial i' \partial x} = -\frac{\partial^2 \lambda_i^{i'}(-x)}{\partial i' \partial x} > 0$, *or agents further away from their target prefer higher types increasingly.*

The intuition is straightforward. Fix an agent of type i with holding x and assume she calls another agent of the same type $i' = i$. Assume without loss of generality that $x > 0$. Because of strict concavity two agents of the same type will leave the meeting with the same holding level. Basically, the two will average their holdings in an almost mechanical fashion so that both will hold $\bar{x} = \frac{x+x'}{2}$. Then there exists a region Q where the receiver will move away from an optimal holding level in absolute terms. This region's upper bound is x and the lower bound is $\hat{x} = -\frac{\hat{x}+x'}{2}$, so that $Q = \{x' || x'| \leq |x' + X_{i,i'}(x,x')|\}$. The significance of this becomes more apparent when we remember the social surplus from bargaining, $S_{i,i'}(x,x') = V_i(x - X_{i,i'}(x,x')) - V_i(x) + V_{i'}(x' + X_{i,i'}(x,x')) - V_{i'}(x')$. The receivers contribution is $V_{i'}(x' + X_{i,i'}(x,x')) - V_{i'}(x')$ which is strictly negative for the region Q when V is symmetric around 0. Therefore agents inside the region Q appear like an investment technology where shallow curvature in the payoff function is desirable while agents outside this region are similar to a consumption technology where a steeper curvature promises higher returns. The further an agent move away the larger the region Q becomes. This region Q can be constructed for all types but figure (2) visualizes it for meeting the same type.

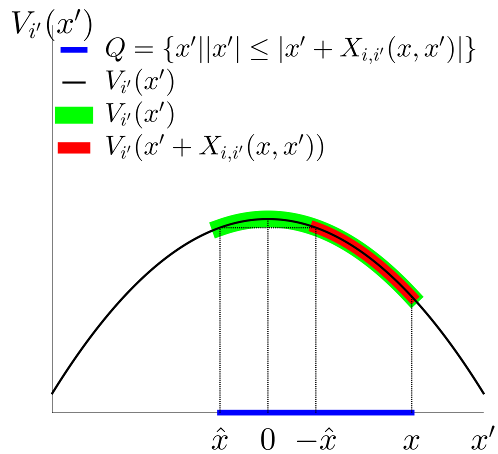

Figure 2: Contribution to the net surplus from bargaining when meeting an agent of the same type holding x and $\hat{x}$.

Conjecture 2. $\frac{\partial \lambda_i^{i'}(0)}{\partial i'} < 0$, *or agents exactly on target prefer smaller types.*

The motivation for this conjecture is similar to the one above. One must only realize that the region Q disappears completely with $x = 0$.

Conjecture 3. $\frac{\partial \lambda_i^{i'}(x)}{\partial i} = \frac{\partial \lambda_i^{i'}(-x)}{\partial i} \leq 0$, *or agents of higher type search less actively.*

Note that higher types have less curvature to worry about in the first place. Therefore they profit the least from calling other agents.

4 Simulation study

4.1 Choices in simulation study

We use the negative of the inverse tangents multiplied by $\frac{2\xi}{\pi}$ for the derivative of the flow payoff. The corresponding flow payoff function has the nice property that the marginal payoff is decreasing from ξ to $-\xi$. So an agent with $x \to -\infty$ has marginal payoff of ξ and an agent with $x \to \infty$ has marginal payoff of $-\xi$. This in turn can be interpreted as upper and lower bounds of the value of the asset, and in particular with the fed funds we can interpret ξ as a policy choice, namely the difference between having excess funds and having to pay a discount window loan. In particular, we have $\frac{\partial u_i(x)}{\partial x} = -\frac{2\xi}{\pi} tan^{-1}(\delta(i)x)$ where $\delta(i) = \delta_1 - \delta_2 i^{\delta_3}$ with $\xi = 0.005, \delta_1 = 3, \delta_2 = 2.7, \delta_3 = 3, \alpha = 0.00006, \kappa = 1.5, \Lambda^x = 1, r = 1.01^{\frac{1}{365}} - 1, \theta = 0.5$ for a currently uncalibrated version. Further, we use a standard normal distribution for the innovation distribution.

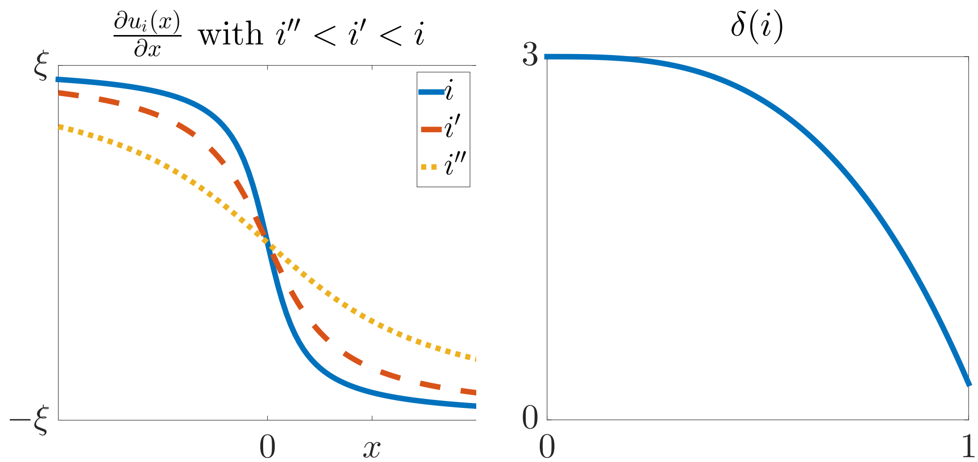

Figure 3: Marginal payoff of holding the asset x on the left, and the heterogeneity function on the right.

4.2 The core result

An agent is intermediating (and belongs to the core) if she buys and sells between two innovations. An agent is a customer if he either only sells or only buys and she is considered nonparticipating when she does neither. Figure (4) shows the conditional distribution that an agent takes on a particular role between two innovations.

The chance that an agent is intermediating is highest for the high type. This stems from the fact that she will receive a mass of contacts from agents that are far away from target. A second reason is that the highest type is most indifferent therefore going either direction when the counterparty is sufficiently far away from target. The majority of agents become one-sided customers while the chance of nonparticipating decreases in type. In total 2.7% of all agents are intermediating with 49.2% nonparticipating and the remaining 48.1% are customers.

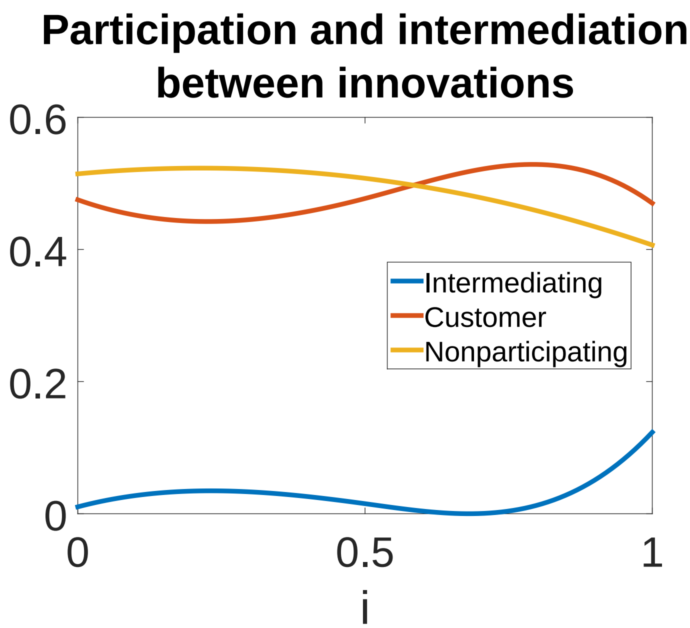

Figure 4: Probability that an agent of type i intermediates, becomes a customer or remains nonparticipating between two innovations.

Another feature of the given heterogeneity is that it creates persistence, another stylized fact of OTC-markts.[15] With 2.7% of all agents intermediating we can predict that conditional on having intermediated in a prior interval the chances of repeating the service is 4.7%.

4.3 Trade observables

We can make predictions about average trade behaviour between two innovations that are in line with empirical observations. Figure (5) plots these patterns. The top left panel shows the expected number of trades a type-agent can expect in a trading interval. The highest type will receive the most contacts and therefore trade the most. This is in line that core-agents not only interconnect but also has more contacts. E.g., Bech and Atalay [10] document that that agents in the core have on average 19.1 lenders and 9.3 borrowers. The top right panel shows the average transfer size which increases in type. This is also found by Bech and Atalay [10]. But since we can distinguish whether trade is initiated by the agent as a caller or whether he just receives a phone call. As one would expect lower types have higher transactions when they call while transfers are lower when they receive a call. This changes as we increase the type. In fact, the highest type will make the most transfers when contacted by someone else, and their average transfer is decreasing when they search for someone else compared to lower types. This is because

[15]Compare Bech and Atalay [10] for the fed funds market

they are more indifferent when searching and therefore will end up meeting lower types who are more stringent about going to target them self. The bottom left panel shows the accumulation of more and larger trades: higher types trade more in total. The bottom right panel corresponds to another phenomenon observed in many OTC studies: agents in the core have a high gross to net ratio. We calculate the expected value traded for each type and holding and divide it by its holding which represents an agent net regulatory demand, or what she really wants to trade when she would have access to a centralized exchange.

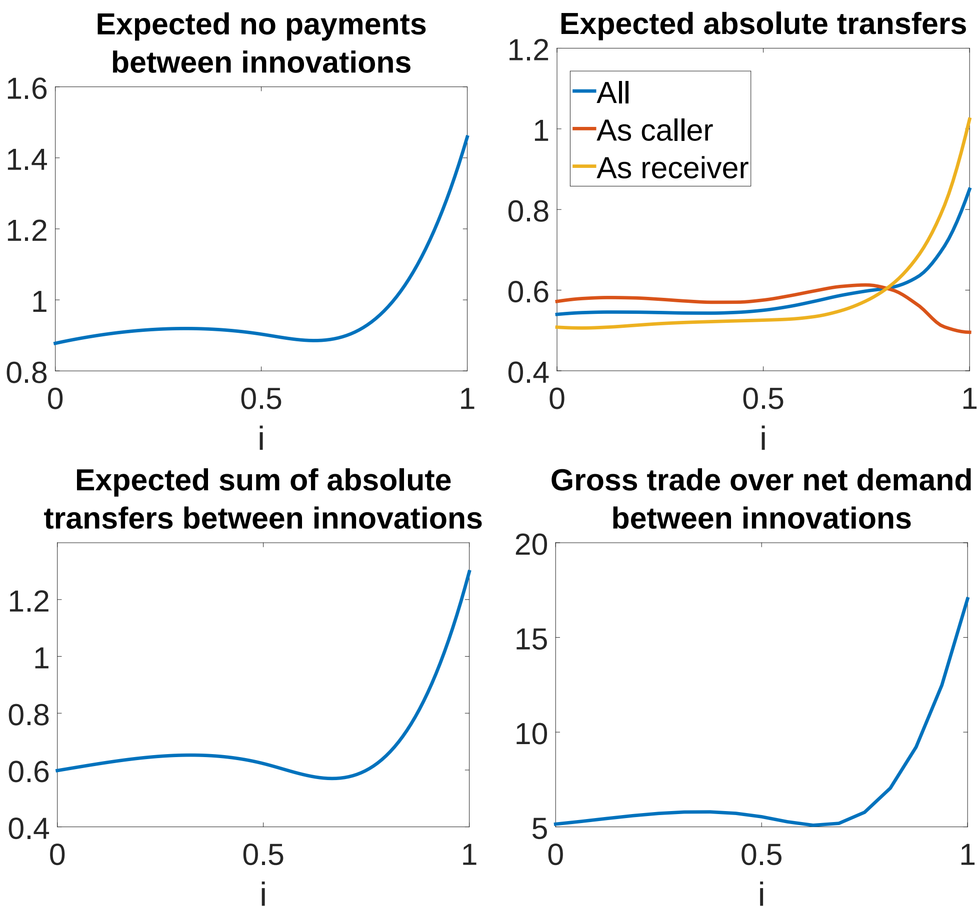

Figure 5: Trade data predictions. The top left panel shows how many contacts a type-agent expects between two innovations. The top right panel shows the average transfer size for all types of contact and when a type is either a caller or a receiver. The bottom left panel shows the accumulated value traded between two innovations and the bottom right shows the average net volume traded over the gross demand an agent has between two innovations.

4.4 Welfare

Agents prefer to participate in an OTC market over autarky as we discussed above. But this form of trade is not ideal. It is straightforward to see that agents also prefer to access a centralized exchange over autarky. This has two reasons. First, they would not pay the search costs. Second, access to a centralized exchange satisfies the participation constraint. The question remains how OTC markets fare against a centralized exchange. Our result on welfare takes a nuanced stand, namely that some agents not only prefer to participate in an OTC market but we find that the highest type can strictly prefer an OTC structure. Of course, a central planner could use side payments to make all agents prefer a centralized exchange. Figure (6) displays this.

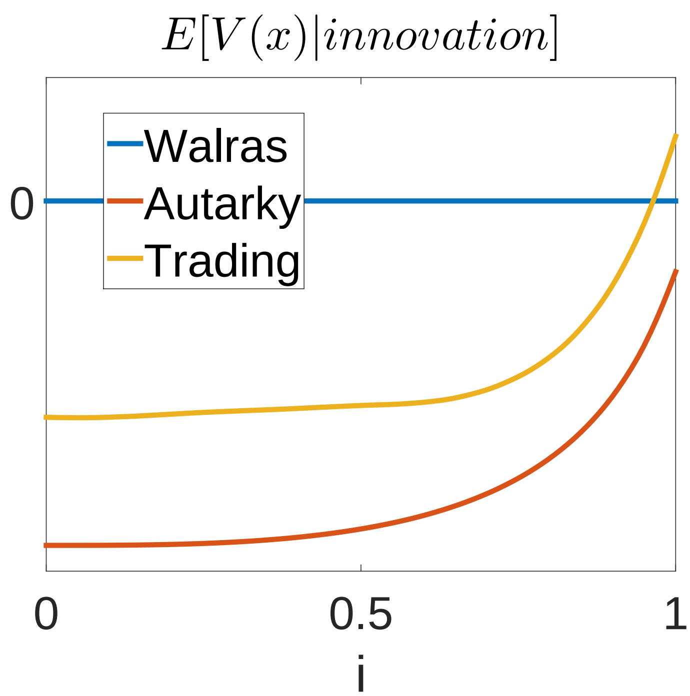

Figure 6: Expected welfare for type-agents.

4.5 Push-Pull Ratio

We borrow a notion used in logistic and marketing: on markets the consumer "pulls" the good or information she demand for her needs, while the supplier "pushes" the good toward the consumers.[16] A typical example is the issue of the new iPhone. Consumers standing in line for hours are evidence for the good being pulled while an expensive marketing campaign would be considered a push by the seller. We can observe who is contacting whom and therefore can distinguish whether intermediation services are pulled

[16] Compare Klaas-Wissing [39] for a more technical definition of these terms.

or pushed. The left panel of figure (7) shows that trade is concentrated between low types and the highest type in the yellow area. The right panel indicates that contacts with the highest types are usually initiated by the lower types. In other words, high types do not provide their services door-to-door in anticipation of a customer in need. It is rather that agents that are far off their target are the source of most trades.[17] This results in fewer high value transactions.

This emphasizes the importance of using a model such as ours when one wants to address the concentration of trade and intermediation among a small group of agents. A model using random search would observe much more 'coincidental trade' upon a calibration using the value of all transactions or something closely related: agents exchange goods because they happen to have different marginal payoffs. But this is the case nearly all the time, and these (smaller) transactions add up resulting in many trades. A model which discriminates with search effort, i.e. agents pay for establishing contacts, will observe less meetings to achieve the same transaction value. This is due to the fact that a form of self-selection takes place.[18] Another step towards more volume concentration is induced by the heterogeneity: Higher types trade a higher volume per transaction because of their more linear curvature. Eventually, allowing for agents to type-discriminate in their search focuses the attention of agents far away from target towards higher types. This means there are fewer trades but with higher value per transaction compared to a random matching model.

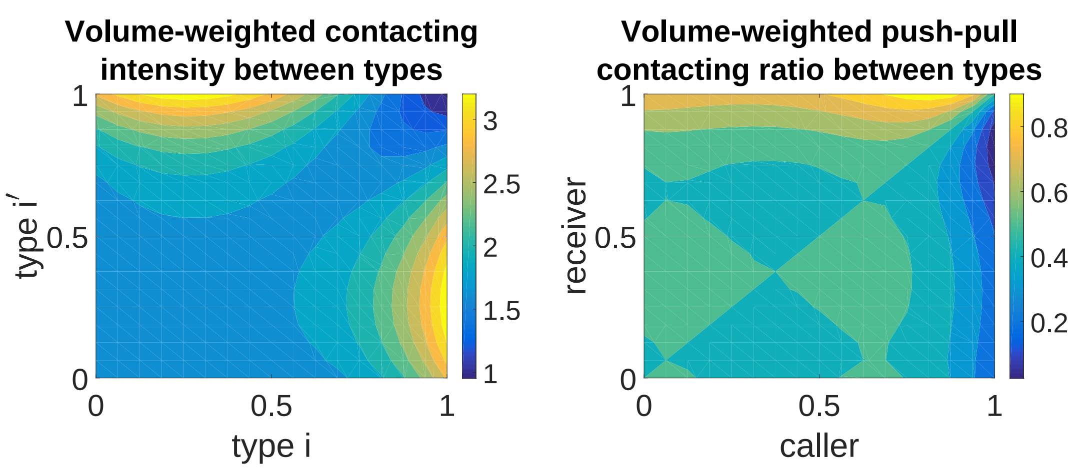

Figure 7: Contacting between types. The left panel shows the trade volume weighted average contacting intensities between types and the right panel shows the ratio when the volume weighted contacts are initiated by the caller.

[17] This relies on the fact that shocks are independent over time and that agents do not remember each others holdings. If both of these assumptions are be dropped we would observe repeated relationships where agents with negatively correlated shocks build pairs.

[18] It is irrelevant whether it is a conventional bilateral search model where both parties need to pay or a unilateral one like the one described here.

5 Conclusion

We introduced a model that forms an explanation for our desired prediction: Agents form a core-periphery structure endogenously. They do so to satisfy their own need to reach a desired target level. Agents with the least cost to deviate become middlemen and connect agents with opposing trading needs that would otherwise not find each other. We find persistence in the role given their exogenously given heterogeneity, and we can further match some stylized facts in the literature. Our model further predicts that it is high cost agents far away from their desired target level that are initiating high volume trade contacts which contribute to a concentration that concerns public debate.

The next step is to use data from the Reports of Condition and Income to calibrate the model, and employ a calibrated version to consider policy variations. One variation already discussed informally is that the discount window rate was lowered after the financial crisis. This lead to a decrease in potential profits as the maximal bid-ask spread shrank. It is unclear whether this increased or decreased a trade concentration on the federal funds market. Another important policy consideration discussed elsewhere is to impose a limit on banks exposure to individual counter parties. By putting a cap on the intensity at which agents meet we can force them to diversify their exposure to other agents. In particular, high-cost agents far away from target would then meet any other agents with the same probability. This would lead away from a concentration of intermediation and trade among the low-cost agents but it would also exacerbate the coordination problem inherent in this market.

Appendices

A Proofs

A.1 Bargaining solution

Proposition 1. *Let V be a real-valued and bounded function that is continuous, differentiable and strictly concave in x. The solution to (3) is uniquely given by*

$$X_{i,i'}(x,x') = \underset{\{X\in\mathbb{R}\}}{\arg\max}\left\{V_i(x-X) - V_i(x) + V_{i'}(x'+X) - V_{i'}(x')\right\} \tag{14}$$

$$\begin{aligned} Y_{i,i'}(x,x') =& (1-\theta)\left(V_{i'}(x'+X_{i,i'}(x,x')) - V_{i'}(x')\right) \\ & - \theta\left(V_i(x - X_{i,i'}(x,x')) - V_i(x)\right) \end{aligned} \tag{15}$$

and the bounded, nonnegative social net surplus is given by

$$S_{i,i'}(x,x') = \max_{\{X\in\Gamma(x,x')\}}\left\{V_i(x-X) - V_i(x) + V_{i'}(x'+X) - V_{i'}(x')\right\} \tag{16}$$

Proof: Assume $Y_{i,i'}(x,x')$ is an interior solution. Then the first order condition with respect to Y imply:

$$\begin{aligned} &\frac{\partial\left[\left(V_i(x-X)+Y-V_i(x)\right)^{1-\theta}\left(V_{i'}(x'+X)-Y-V_{i'}(x')\right)^{\theta}\right]}{\partial Y} = 0 \Leftrightarrow \\ &0 = (1-\theta)\left(V_i(x-X)+Y-V_i(x)\right)^{-\theta}\left(V_{i'}(x'+X)-Y-V_{i'}(x')\right)^{\theta} \\ &-\theta\left(V_i(x-X)+Y-V_i(x)\right)^{1-\theta}\left(V_{i'}(x'+X)-Y-V_{i'}(x')\right)^{\theta-1} \\ &\leftrightarrow (1-\theta)\left(V_{i'}(x'+X)-Y-V_{i'}(x')\right) = \theta\left(V_i(x-X)+Y-V_i(x)\right) \\ &Y = (1-\theta)\left(V_{i'}(x'+X)-V_{i'}(x')\right) - \theta\left(V_i(x-X)-V_i(x)\right) \end{aligned}$$

which is bounded as V is bounded. But this implies that the original optimization problem is equivalent to

$$\begin{aligned} &\underset{\{X\times Y\in\mathbb{R}^2\}}{max}\left(V_i(x-X)+Y-V_i(x)\right)^{1-\theta}\left(V_{i'}(x'+X)-Y-V_{i'}(x')\right)^{\theta} \\ &\equiv \underset{\{X\times Y\in\mathbb{R}\times G\}}{max}(1-\theta)^{1-\theta}\theta^{\theta}\left(V_i(x-X)-V_i(x)+V_{i'}(x'+X)-V_{i'}(x')\right) \\ &\equiv \underset{\{X\times Y\in\mathbb{R}\times G\}}{max}\left(V_i(x-X)-V_i(x)+V_{i'}(x'+X)-V_{i'}(x')\right) \\ &\rightarrow \underset{\{X\in\mathbb{R}\}}{max}\left(V_i(x-X)-V_i(x)+V_{i'}(x'+X)-V_{i'}(x')\right) \end{aligned}$$

where $G = \left\{Y\in\mathbb{R} : Y = (1-\theta)(V_{i'}(x'+X_{i,i'}(x,x')) - V_{i'}(x')) - \theta(V_i(x-X_{i,i'}(x,x')) - V_i(x))\right\}$. Now, V is bounded so that the last expression is bounded. Therefore, a maximum exists. Further, a solution is in the interior of the real line and it is unique. To see this take the derivative and strict concavity increases (decreases) $\frac{\partial V_i(x-X)}{\partial x}$ $(\frac{\partial V_{i'}(x'+X)}{\partial x})$ as X increases. An optimal $X_{i,i'}(x,x')$ and the first optimality condition yields (15) as optimal $Y_{i,i'}(x,x')$ which is indeed in the interior of $\mathbb{R}$. It is easy to see that the participation constraints are never violated by contradiction. The social net surplus of optimal trading is identical to the optimal value of the maximization for the reduced optimization problem, and is indeed (16). Boundedness follows from the boundedness of V and nonnegativity follows from the participation constraints. $\square$

Let us state the following corollaries:

Corollary 2. *The net surplus from bargaining for the caller and the receiver are, respectively:*

$$V_i(x - X_{i,i'}(x,x')) + Y_{i,i'}(x,x') - V_i(x) = (1-\theta)S_{i,i'}(x,x')$$
$$V_{i'}(x' + X_{i,i'}(x,x')) - Y_{i,i'}(x,x') - V_{i'}(x') = \theta S_{i,i'}(x,x')$$

Proof: The split of surplus can be verified by simply plugging the solution for $Y_{i,i'}(x,x')$ in the surplus the caller and the receiver get, respectively. $\square$

Lemma 13. *The solution for the problem (3) is symmetric for caller and receiver. In particular, $X_{i,i'}(x,x') = -X_{i',i}(x',x)$ and $S_{i,i'}(x,x') = S_{i',i}(x',x)$.*

Proof: Fix two agents, (i,x) and (i',x'), and change their roles. Note that this does not hold for $Y_{i',i}(x',x)$. $\square$

Lemma 14. *The maximal attainable social surplus from trading in equation (16) is continuous in x and x'.*

Proof: Note that $\mathbb{R}$ is not compact-valued. But we can always create a maximization problem with an identical objective function as in (3) with a choice set

$$\hat{\Gamma}(x,x') = \left\{ \hat{X} \in \mathbb{R} | X_{i,i'}(x,x') - \epsilon \leq \hat{X} \leq X_{i,i'}(x,x') + \epsilon \right\}$$

for an arbitrary $0 < \epsilon < \infty$. Note that $\hat{\Gamma}(x,x')$ is now a compact-valued and continuous correspondence if $\epsilon < \infty$. $V_i(x-X) - V_i(x) + V_{i'}(x'+X) - V_{i'}(x')$ is continuous in x, and so is the identical function $\hat{V}_i(x-\hat{X}) - \hat{V}_i(x) + \hat{V}_{i'}(x'+\hat{X}) - \hat{V}_{i'}(x')$ we try to maximize over $\hat{X}$ subject to $\hat{\Gamma}(x,x')$. We can then use the theorem of the maximum as stated by Stokey and Lucas [48] (theorem 3.6 on page 62). $\hat{S}_{i,i'}(x,x')$ is continuous for the neighborhood $N_v(x)$, and by equivalence, so is $S_{i,i'}(x,x')$. The only thing left to show is that $\epsilon < \infty$ always holds. But by Proposition 1 we know $X_{i,i'}(x,x')$ is bounded, and so is $\hat{X}_{i,i'}(x,x')$ for any ϵ. Therefore, $\epsilon < \infty$ suffices. The result for x' results from a similar line of arguments. $\square$

Lemma 15. *X is strictly increasing (decreasing) in x (x').*

Proof: Note that $(V_i(x-X) - V_i(x) + V_{i'}(x'+X) - V_{i'}(x'))$ shows strictly increasing (decreasing) differences in (X,x) $((X, x'))$ because of the strict concavity of $V_i(x)$ in x. We establish the result by invoking theorem 10.6 on page 258 of Sundaram [49]. □

Lemma 16. *X is strictly increasing in i (i') for $x - X_{i,i'}(x, x') < 0$ ($x' + X_{i,i'}(x, x') > 0$) if V is even and has decreasing strict concavity in type.*

Proof: Note that $(V_i(x-X) - V_i(x) + V_{i'}(x'+X) - V_{i'}(x'))$ shows strictly increasing differences in (X,i) $((X, i'))$ because of the decreasing strict concavity of $V_i(x)$ in x for i. We establish the result by invoking theorem 10.6 on page 258 of Sundaram [49]. □

Lemma 17. *The maximal attainable gross social surplus from trading $S^g_{i,i'}(x, x') = \max_{\{X \in \Gamma(x,x')\}} \big\{V_i(x - X) + V_{i'}(x'+X) - V_{i'}(x')\big\}$ is continuous and concave in x. Further, $X_{i,i'}(x, x')$ is continuous in x and x'.*

Proof: We can use the same line of arguments as in Lemma 14 to create an identical maximization problem, only that we invoke the theorem of the maximum under convexity as stated by Sundaram [49] (theorem 9.17 on page 237). The result for $X_{i,i'}(x, x')$ follows from the fact that the gross optimization problem is just constant transformations of the net optimization problem. □

Lemma 3. *$S_{i,i'}(x, x')$ is strictly decreasing (increasing) in $x < \bar{x}$ ($x > \bar{x}$) and has a unique minimum for $x = \bar{x}$ at $\frac{\partial V_i(\bar{x})}{\partial x} = \frac{\partial V_{i'}(x')}{\partial x'}$ with $S_{i,i'}(\bar{x}, x') = 0$. Similarly, $S_{i,i'}(x, x')$ is strictly decreasing (increasing) in $x' < \bar{x}'$ ($x' > \bar{x}'$) and has a unique minimum for $x' = \bar{x}'$ at $\frac{\partial V_i(x)}{\partial x} = \frac{\partial V_{i'}(\bar{x}')}{\partial x'}$ with $S_{i,i'}(x, \bar{x}') = 0$.*

Proof: $X_{i,i'}(x, x')$ is continuous in x by Lemma 17. Then the envelope theorem yields

$$\frac{\partial S_{i,i'}(x, x')}{\partial x} = \frac{\partial V_i(x - X_{i,i'}(x, x'))}{\partial x} - \frac{\partial V_i(x)}{\partial x} \begin{cases} > 0 & \text{if } X_{i,i'}(x, x') > 0 \\ = 0 & \text{if } X_{i,i'}(x, x') = 0 \\ < 0 & \text{if } X_{i,i'}(x, x') < 0 \end{cases}$$

where the split in cases follows because of strict concavity of V. The proof for convexity in x' follows a similar argument. □

Lemma 18. *$S_{i,i'}(x, x')$ is even in (x, x') if V is even.*

Proof: $S_{i,i'}(x, x') = S_{i,i'}(-x, -x')$ can be verified by plug in. □

A.2 Calling technology

Proposition 4. *Let V be a real-valued and bounded function that is continuous, differentiable and strictly concave in x. Let f be a well-defined, continuous density function from* *a nondegenerate distribution. Further, let $c(\lambda) = \frac{\alpha}{\kappa}\lambda^{\kappa}$ with $1 < \kappa < \infty$ and $0 < \alpha < \infty$ define the parameterized cost function.*

A solution to (2) exists and is uniquely defined by

$$\lambda_i^{i'}(x) = \left(\frac{ES_i^{i'}(x)}{\alpha}\right)^{\frac{1}{\kappa-1}} \tag{17}$$

and the net surplus from calling an agent of type i' for an agent of type i holding x is

$$W_i^{i'}(x) = \frac{\kappa-1}{\kappa}\left(\frac{ES_i^{i'}(x)^{\kappa}}{\alpha}\right)^{\frac{1}{\kappa-1}} \tag{18}$$

Proof: First, note that $ES_i^{i'}(x) = (1-\theta)\int\limits_{\mathbb{R}} S_{i,i'}(x,x')f_{i'}(x')dx'$ exists under the assumptions of Proposition 4: Proposition 1 defines $S_{i,i'}(x,x')$ and shows it is bounded. Since $f_{i'}(x')$ is a well-defined density function we know $(1-\theta)\int\limits_{\mathbb{R}} S_{i,i'}(x,x')f_{i'}(x')dx' \leq (1-\theta)\sup\limits_{x'\in\mathbb{R}}\{S_{i,i'}(x,x')\}$. Further, $ES_i^{i'}(x)$ is nonnegative because $S_{i,i'}(x,x')$ is nonnegative.

If $ES_i^{i'}(x) = 0$ we set $\lambda_i^{i'}(x) = 0$, while if the expected surplus is positive we have an interior solution. By the first order condition

$$\lambda_i^{i'}(x) = \left(\frac{ES_i^{i'}(x)}{\alpha}\right)^{\frac{1}{\kappa-1}}$$

so that this equation fully pins down optimal $\lambda_i^{i'}(x)$. And the result for $W_i^{i'}(x)$ follows from plugging in the solution. □

Corollary 5. *$\lambda_i^{i'}(x) > 0$ and $W_i^{i'}(x) > 0$ everywhere if $\theta < 1$.*

Proof: The result follows from Lemma 3 and the fact that each type has a density with a nondegenerate distribution. □

Lemma 19. *$ES_i^{i'}(x)$ is even in x if $V_i(x)$ and $f_i(x)$ are even in x $\forall i \in [0,1]$ under the assumptions of Proposition 4.*

Proof: The claim is $ES_i^{i'}(x) = ES_i^{i'}(-x) \leftrightarrow (1-\theta)\int\limits_{\mathbb{R}} S_{i,i'}(x,x')f(x')dx' = (1-\theta)\int\limits_{\mathbb{R}} S_{i,i'}(-x,x')f_{i'}(x')dx'$. Symmetry of f and Lemma 18 yields $ES_i^{i'}(x) = (1-\theta)\int\limits_{-\infty}^{0} S_{i,i'}(x,-x')f(-x')dx' + (1-\theta)\int\limits_{0}^{\infty} S_{i,i'}(x,-x')f_{i'}(-x')dx'$. The result follows. □

Lemma 20. *$ES_i^{i'}(x)$ is strictly monotone in x and has a unique minimum at $x = 0$ if $V_i(x)$ and $f_i(x)$ are even in x $\forall i \in [0,1]$ and the assumptions of Proposition 4 hold.*

Proof: From Lemma 3 we know that S is strictly monotone in x. But then $ES_i^{i'}(x)$ is convex in x as a (weighted) sum of S.

By Leibniz we know

$$\frac{\partial ES_i^{i'}(x)}{\partial x} = (1-\theta)\int_{-\infty}^{\infty} \frac{\partial S_{i,i'}(x,x')}{\partial x} f_{i'}(x')dx'$$

With V even in x we know S is even in (x, x'). Fix x=0 which makes S even in x'. Then the derivative $\frac{\partial S_{i,i'}(0,x')}{\partial x}$ is odd in x'. By symmetry of f we conclude that $\frac{\partial ES_i^{i'}(0)}{\partial x} = 0$. □

Proposition 21. *$\lambda_i^{i'}(x)$ is even in x given V and f are even and the assumptions of Proposition 4.*

A.3 Payoff function

Lemma 6. *Let V be a real-valued and bounded function that is continuous, differentiable and strictly concave in x. Let f be a well-defined, continuous density function. Further, let $c(\lambda) = \frac{\alpha}{\kappa}\lambda^\kappa$ with $1 < \kappa < \infty$ and $0 < \alpha < \infty$ define the parameterized cost function. Then (4) has a linear transformations of the form*

$$\begin{aligned}\bar{V}_i(x) =& \frac{1}{r+\Lambda^x}\Bigg[u_i(x) - c_i(x) \\ &+ \int_0^1\int_{\mathbb{R}} \Big((1-\theta)\lambda_i^{i'}(x) + \theta\lambda_{i'}^{i}(x')\Big) f_{i'}(x')S_{i,i'}(x,x')dx'di'\Bigg]\end{aligned} \tag{19}$$

where $c_i(x) = \int_0^1 c(\lambda_i^{i'}(x))di'$.

Proof: The minimal arrival time of independent homogenous Poisson processes is itself a Poisson process where the intensity is equal to the sum of the underlying processes, or here $\Lambda_i(x) = \Lambda^x + \Lambda_i^c(x) + \Lambda_i^r$. The probability of each event is simply $\frac{\Lambda^x}{\Lambda_i(x)}$, $\frac{\Lambda_i^c(x)}{\Lambda_i(x)}$ and $\frac{\Lambda_i^r}{\Lambda_i(x)}$, respectively. The algebra fairy reminds us that $\int_0^\tau e^{-rx}dx = \frac{1}{r}(1-e^{-r\tau})$. Then (4) becomes

$$\begin{aligned}
V_i(x) =& \int_0^\infty \Lambda_i(x) e^{-\Lambda_i(x)\tau} \Bigg\{ \int_0^\tau e^{-rz} dz \left[u_i(x) - c_i(x)\right] + e^{-r\tau} \Bigg[\frac{\Lambda^x}{\Lambda_i(x)} \int_{\mathbb{R}} V_i(\hat{x}) g(\hat{x}) d\hat{x} \\
&+ \frac{\Lambda_i^c(x)}{\Lambda_i(x)} \int_0^1 \int_{\mathbb{R}} \frac{\lambda_i^{i'}(x)}{\Lambda_i^P(x)} f_{i'}(x') \left\{ V_i(x - X_{i,i'}(x,x')) + Y_{i,i'}(x,x') \right\} dx' di' \\
&+ \frac{\Lambda_i^r}{\Lambda_i(x)} \int_0^1 \int_{\mathbb{R}} \frac{\lambda_{i'}^i(x') f_{i'}(x')}{\Lambda_i^R} \left\{ V_i(x + X_{i',i}(x',x)) - Y_{i',i}(x',x) \right\} dx' di' \Bigg] \Bigg\} d\tau \\
=& \int_0^\infty e^{-\Lambda_i(x)\tau} \Bigg\{ \frac{\Lambda_i(x)}{r} (1 - e^{-r\tau}) \left[u_i(x) - c_i(x)\right] + e^{-r\tau} \Bigg[\Lambda^x \int_{\mathbb{R}} V_i(\hat{x}) g(\hat{x}) d\hat{x} \\
&+ \int_0^1 \int_{\mathbb{R}} \lambda_i^{i'}(x) f_{i'}(x') \left\{ V_i(x - X_{i,i'}(x,x')) + Y_{i,i'}(x,x') \right\} dx' di' \\
&+ \int_0^1 \int_{\mathbb{R}} \lambda_{i'}^i(x') f_{i'}(x') \left\{ V_i(x - X_{i,i'}(x,x')) + Y_{i,i'}(x,x') \right\} dx' di' \Bigg] \Bigg\} d\tau
\end{aligned}$$

where we applied symmetry properties of the bargaining solution stated in Lemma 13.

Remember that $\int_0^\infty e^{-k\tau}d\tau = \frac{1}{k}$. Then using Corollary 2

$$
\begin{aligned}
V_i(x) = & \frac{u_i(x)-c_i(x)}{r} + \frac{1}{r+\Lambda_i(x)}\Bigg[\Lambda^x \int_{\mathbb{R}} V_i(\hat{x})g(\hat{x})d\hat{x} - \frac{\Lambda_i(x)}{r}(u_i(x)-c_i(x)) \\
& + \int_0^1\int_{\mathbb{R}} \lambda_i^{i'}(x) f_{i'}(x')\left\{(1-\theta)S_{i,i'}(x,x') + V_i(x)\right\} dx'di' \\
& + \int_0^1\int_{\mathbb{R}} \lambda_{i'}^{i}(x') f_{i'}(x')\left\{\theta S_{i,i'}(x,x') + V_i(x)\right\} dx'di'\Bigg] \\
= & \frac{1}{r}\left(1-\frac{\Lambda_i(x)}{r+\Lambda_i(x)}\right)(u_i(x)-c_i(x)) + \frac{1}{r+\Lambda_i(x)}\Bigg[\Lambda^x\int_{\mathbb{R}} V_i(\hat{x})g(\hat{x})d\hat{x} \\
& + (\Lambda_i^P(x)+\Lambda_i^R)V_i(x) \\
& + \int_0^1\int_{\mathbb{R}} f_{i'}(x')\left\{(1-\theta)\lambda_i^{i'}(x) + \theta\lambda_{i'}^{i}(x')\right\} S_{i,i'}(x,x')dx'di'\Bigg] \\
\leftrightarrow & (r+\Lambda^x)V_i(x) = u_i(x)-c_i(x) + \Bigg[\Lambda^x\int_{\mathbb{R}} V_i(\hat{x})g(\hat{x})d\hat{x} \\
& + \int_0^1\int_{\mathbb{R}} f_{i'}(x')\left\{(1-\theta)\lambda_i^{i'}(x) + \theta\lambda_{i'}^{i}(x')\right\} S_{i,i'}(x,x')dx'di'\Bigg]
\end{aligned}
$$

Then we can get a linear transformation of V so that the result follows. □

The following results are used in the proof for Proposition 7.

Lemma 22. *T is a local radial contraction under the assumptions of Proposition 7.*

Proof: Note that (B,D) is a complete metric space and $T:(B,D)\to(B,D)$. We fix (i,x). Proposition 4 shows that we can rewrite T(V):

$$
\begin{aligned}
T(\bar{V}_i(x)) = & \frac{1}{r+\Lambda^x}\Bigg[u_i(x) + \frac{\kappa-1}{\kappa}\alpha^{\frac{1}{1-\kappa}}(1-\theta)^{\frac{\kappa}{\kappa-1}}\left(\int_{\mathbb{R}} S_{i,i'}(x,\bar{x})f_{i'}(\bar{x})d\bar{x}\right)^{\frac{\kappa}{\kappa-1}} \\
& + \theta\left(\frac{(1-\theta)}{\alpha}\right)^{\frac{1}{\kappa-1}}\int_0^1\int_{\mathbb{R}}\left(\int_{\mathbb{R}} S_{i,i'}(\hat{x},x')f_i(\hat{x})d\hat{x}\right)^{\frac{1}{\kappa-1}} f_{i'}(x')S_{i,i'}(x,x')dx'di'\Bigg]
\end{aligned}
$$

Then by the triangle inequality we find $\left|T(V_i(x)) - T(\hat{V}_i(x))\right|$

$$
\begin{aligned}
&\leq \frac{1}{r+\Lambda^x}\Bigg[\Bigg|\frac{\kappa-1}{\kappa}\alpha^{\frac{1}{1-\kappa}}(1-\theta)^{\frac{\kappa}{\kappa-1}}\left(\int_{\mathbb{R}} S_{i,i'}(x,\bar{x})f_{i'}(\bar{x})d\bar{x}\right)^{\frac{\kappa}{\kappa-1}}\Bigg| \\
&+\left|\theta\left(\frac{(1-\theta)}{\alpha}\right)^{\frac{1}{\kappa-1}}\int_0^1\int_{\mathbb{R}}\left(\int_{\mathbb{R}} S_{i,i'}(\hat{x},x')f_i(\hat{x})d\hat{x}\right)^{\frac{1}{\kappa-1}} f_{i'}(x')S_{i,i'}(x,x')dx'di'\right| \\
&+\left|\frac{\kappa-1}{\kappa}\alpha^{\frac{1}{1-\kappa}}(1-\theta)^{\frac{\kappa}{\kappa-1}}\left(\int_{\mathbb{R}} \hat{S}_{i,i'}(x,\bar{x})f_{i'}(\bar{x})d\bar{x}\right)^{\frac{\kappa}{\kappa-1}}\right| \\
&+\left|\theta\left(\frac{(1-\theta)}{\alpha}\right)^{\frac{1}{\kappa-1}}\int_0^1\int_{\mathbb{R}}\left(\int_{\mathbb{R}} \hat{S}_{i,i'}(\hat{x},x')f_i(\hat{x})d\hat{x}\right)^{\frac{1}{\kappa-1}} f_{i'}(x')\hat{S}_{i,i'}(x,x')dx'di'\right|\Bigg]
\end{aligned}
$$

For any $S_{i,i'}(x,x') = \max_{\{X\in\mathbb{R}\}}\{V_i(x-X) - V_i(x) + V_{i'}(x'+X) - V_{i'}(x')\}$ we can construct a sequence $\{X_j\}_{j=0}^C$ so that $X_0 = 0$, $X_C = X_{i,i'}(x,x')$, $X_j - X_{j-1} = c\ \forall j \in [1,C]$ and $\max(\sup_{j\in[1,C]} D(V_i(x-X_j), V_i(x-X_{j-1})), \sup_{j\in[1,C]} D(V_{i'}(x'+X_j), V_{i'}(x'+X_{j-1}))) \leq D(V,\hat{V})$ where the latter comes from the continuity of V. Then we use the triangle inequality:

$$
\begin{aligned}
|S(x,x')| \leq\ & |V_i(x-X(x,x')) - V_i(x-X_C)| + \sum_{j=1}^{C}|V_i(x-X_j) - V_i(x-X_{j-1})| \\
&+ |V_i(x-X_0) - V_i(x)| + |V_{i'}(x'+X(x,x')) - V_{i'}(x'+X_C)| \\
&+ \sum_{j=1}^{C}|V_{i'}(x'+X_j) - V_{i'}(x'+X_{j-1})| + |V_{i'}(x'+X_0) - V_{i'}(x')| \\
\leq\ & 2CD(V,\hat{V})
\end{aligned}
$$

and we can do the same for any $\hat{S}(x,x')$. This yields

$$
\left|T(V(x)) - T(\hat{V}(x))\right| \leq KD(V,\hat{V})^{\frac{\kappa}{\kappa-1}}
$$

where $K = \frac{1}{r+\Lambda^x}\left(\frac{1-\theta}{\alpha}\right)^{\frac{1}{\kappa-1}} 2^{\frac{2\kappa-1}{\kappa-1}} C^{\frac{\kappa}{\kappa-1}}\left[\frac{\kappa-1}{\kappa}(1-\theta)^{\kappa} + \theta\right]$, and it holds for any (i,x) and arbitrary (V,$\hat{V}$). Now fix V and let $\hat{C}$ denote the integer associated with the distance between (V,$\hat{V}$). Then $\forall \hat{V} \in N_\epsilon(V)$ we have a collection of finite $\hat{C}$. Then $\exists\ \epsilon > 0$ so that

$$
\epsilon(V) \leq \sup_{\hat{V}\in N_\epsilon(V)}\left\{D(V,\hat{V}) : KD(V,\hat{V})^{\frac{1}{\kappa-1}} < 1\ \forall \bar{V} \in N_\epsilon(V)\right\}
$$

The result follows from the arbitrariness of V and $\kappa < \infty$. $\square$

Lemma 23. *The space of continuous functions defined on* $\mathbb{R}$ *which are concave,* B^c*, is a closed subset of B.*

Proof: Say B^c is not closed. Then $\exists v_n$ which is Cauchy but $v_n \underset{n\to\infty}{\to} \bar{v} \notin B^c$. Then for $\xi > 0$ we find $0 = \bar{v}(\tau x_1 + (1-\tau)x_2) + \xi - \tau\bar{v}(x_1) - (1-\tau)\bar{v}(x_2) \geq \bar{v}(\tau x_1 + (1-\tau)x_2) - v_n(\tau x_1 + (1-\tau)x_2) + \xi - \tau(\bar{v}(x_1) - v_n(x_1)) - (1-\tau)(\bar{v}(x_2) - v_n(x_2))$. Now we let n be large enough so that $0 \geq 3\epsilon + \xi$ for some $\epsilon > 0$ which contradicts that $\bar{v}$ is not concave. $\square$

Lemma 24. *Let V be defined on the convex set* $\mathbb{R}$*, real-valued and concave. Then* $\forall i \in [0,1]$ *and* $\forall x_0 \in int(\mathbb{R})$ *and their neighbourhood D we can create a concave and differentiable function W so that* $V_i(x_0) = W(x_0)$ *and* $V_i(x) > W(x) \forall x \in D(x_0)$*. Then* $V_i(x_0)$ *is differentiable and* $\frac{\partial V_i(x_0)}{\partial x} = \frac{\partial W(x_0)}{\partial x}$.

Proof: An arbitrary subgradient w of V must hold $w(x - x_0) \geq V_i(x) - V_i(x_0) \geq W(x) - W(x_0)\ \forall x \in D(x_0)$. w becomes unique with differentiability of W (as we take the limit $x \to x_0$), and any concave function with a unique subgradient at an interior point x_0 is differentiable at x_0 (compare Rockafellar [44], theorem 25.1 on page 242). $\square$

Let us now restate Proposition 7 and show the proof.

Proposition 7. *Let B be the space of real-valued, bounded functions defined on* $[0,1] \times \mathbb{R}$ *which are continuous in x, and denote their elements by V and* $\hat{V}$*. Let* $D : B \times B \to \mathbb{R}^+$ *be the supnorm metric in the sense of* $D(V, \hat{V}) = \sup\limits_{i \times x \in [0,1]\times\mathbb{R}} \left\{ \left| V_i(x) - \hat{V}_i(x) \right| \right\}$*. Further, let T be defined on B. In particular,*

$$\begin{aligned} T(\bar{V}_i(x)) =& \frac{1}{r + \Lambda^x} \Bigg[u_i(x) - c_i(x) \\ &+ \int_0^1 \int_{\mathbb{R}} \Big((1-\theta)\lambda_i^{i'}(x) + \theta\lambda_{i'}^i(x') \Big) f_{i'}(x') S_{i,i'}(x, x') dx' di' \Bigg] \end{aligned} \tag{20}$$

where λ *and S are functions of V. Then T is a global contraction. Further, V is strictly concave and differentiable.*

Proof: We know the supnorm is a metric and B is complete under this metric[19]. The fact that $T : B \to B$ is easily established using the assumptions given and the properties of continuous functions. We show that T is a local radial contraction and use a theorem by Hu and Kirk [30] to establish that TV is a global contraction. We establish

[19] Compare theorem 7.15 of Rudin [47].

strict concavity, and we get to differentiability in a similar fashion as Benveniste and Scheinkman [11].

Let us look into the definition of local contraction and local radial contraction from Hu and Kirk [30].

Definition: Let (B,D) be a complete metric space, $k < 1$ and T a mapping of B into B. If for each $V \in B$ there exists a neighborhood N(V) such that for each $\hat{V}, \bar{V} \in N(V)$, $D(T\hat{V}, T\bar{V}) \leq kD(\hat{V}, \bar{V})$, then T is said to be **locally contractive** with constant k. If it is assumed only that $D(T\hat{V}, TV) \leq kD(\hat{V}, V)\ \forall\ \hat{V} \in N(V)$ then T is a **local radial contraction**.

And the theorem they proof is

Theorem: Let (B,D) be a complete metric space and $T : (B, D) \to (B, D)$ is a local radial contraction. Suppose for some $V^0 \in B$ the points V^0 and TV^0 are joint by a path of finite length. Then T has a unique fixed point in B.

Lemma 22 establishes that T is indeed a local radial contraction. The graph joining any V^0 with TV^0 can be described by $g_{V^0,T}(l) = lV^0 + (1-l)TV^0$ where $l \in [0, 1]$. Note that $\frac{\partial g_{V^0,T}(l)}{\partial l} = V^0 - TV^0 < \sup\limits_{(i,x)\in[0,1]\times\mathbb{R}} \{V_i^0(x) - TV_i^0(x)\} < \infty$ is continuous in l as all V are bounded. By theorem 6.27 on page 137 of Rudin [47] we know that such a graph is of finite length.

Regarding strict concavity with respect to x: We invoke Corollary I from Stokey and Lucas [48] on page 52. Lemma 23 shows B^c is closed under B. Now we need to show that T actually maps into B^c. We start by adding $\frac{\theta\Lambda_i^r}{r+\Lambda^x}V_i(x)$ to both sides of equation (20) and explicitly stating the search problem again:

$$\bar{V}_i(x) = \max_{\{\lambda^{i'}\geq 0\}_{i'\in[0,1]}} \left\{ \frac{1}{r + \Lambda^x + \theta\Lambda_i^r} \left[u_i(x) - \int_0^1 c(\lambda^{i'})di' \right.\right.$$
$$\left.\left. + (1-\theta)\int_0^1 \lambda^{i'} \int_{\mathbb{R}} f_{i'}(x')S_{i,i'}(x,x')dx'di' + \theta \int_0^1\int_{\mathbb{R}} \lambda_{i'}^i(x')f_{i'}(x')S_{i,i'}^g(x,x')dx'di' \right]\right\}$$

The right hand side is strictly concave in $\lambda^{i'}$ as c is strictly convex and $\lambda^{i'}$ is linear otherwise. We can equivalently state that

$$\bar{V}_i(x) = \max_{\{\lambda^{i'}\geq 0\}_{i'\in[0,1]}} \left\{ \frac{1}{r + \Lambda^x + \theta\Lambda_i^r + (1-\theta)\int_0^1 \lambda^{i'}di'} \left[u_i(x) - \int_0^1 c(\lambda^{i'})di' \right.\right.$$
$$\left.\left. + (1-\theta)\int_0^1 \lambda^{i'} \int_{\mathbb{R}} f_{i'}(x')S_{i,i'}^g(x,x')dx'di' + \theta \int_0^1\int_{\mathbb{R}} \lambda_{i'}^i(x')f_{i'}(x')S_{i,i'}^g(x,x')dx'di' \right]\right\}$$

where the problem is now strictly concave in x as $S^g_{i,i'}(x,x')$ is weakly concave and $u_i(x)$ is strictly concave. From the assumption on c we know any solution is in the interior and hence we can establish strict concavity of $\bar{V}_i(x)$ using the the theorem of the maximum under convexity as stated by Sundaram [49] (theorem 9.17 on page 237).

Let us show that the function V is differentiable with respect to x. Lemma 24 shows that we require a concave and differentiable function W that touches V from below. Denote by $S^g_{i,i'}(x,x_0,x') = V_i(x - X_{i,i'}(x_0,x')) + V_{i'}(x' + X_{i,i'}(x_0,x')) - V_{i'}(x')$. Clearly, $S^g_{i,i'}(x_0,x_0,x') = S^g_{i,i'}(x_0,x')$, and $S^g_{i,i'}(x,x_0,x') \leq S^g_{i,i'}(x,x')$. In fact we can show that the inequality never binds as $X_{i,i'}(x,x')$ is strictly monotone in x. Nonetheless, $S^g_{i,i'}(x,x_0,x')$ is still weakly concave in x. Then we can use the above manipulation and find

$$W_i(x,x_0) = \max_{\left\{\lambda^{i'}\geq 0\right\}_{i'\in[0,1]}} \left\{ \frac{1}{r + \Lambda^x + \theta\Lambda^r_i + (1-\theta)\int_0^1 \lambda^{i'} di'} \left[u_i(x) - \int_0^1 c(\lambda^{i'}) di' \right.\right.$$
$$\left.\left. + (1-\theta)\int_0^1 \lambda^{i'} \int_{\mathbb{R}} f_{i'}(x') S^g_{i,i'}(x,x_0,x') dx' di' + \theta \int_0^1 \int_{\mathbb{R}} \lambda^i_{i'}(x') f_{i'}(x') S^g_{i,i'}(x,x_0,x') dx' di' \right] \right\}$$

Now, $W_i(x,x_0)$ is differentiable, strictly concave and $W_i(x,x_0) = \bar{V}_i(x)$ if $x = x_0$ and $W_i(x,x_0) < \bar{V}_i(x)$ otherwise. □

The following lemma allows us to use corollary I from Stokey and Lucas [48] again to establish that TV is a contraction in the set of even function conditional on TV being even.

Lemma 25. *The space of continuous functions defined on $\mathbb{R}$ which are even, B^e, is a closed subset of B.*

Proof: A space B is closed if every Cauchy sequence finds its limit inside B. Say B^e is not closed. Then $\exists v_n$ which is Cauchy but $v_n \underset{n\to\infty}{\to} \bar{v} \notin B^e$. Then for $\xi > 0$ and some $x \in \mathbb{R}$ we find without loss of generality $0 = \bar{v}(x) - \bar{v}(-x) + \xi = \bar{v}(x) - \bar{v}(-x) + \xi - v_n(x) + v_n(-x)$ which obviously contradicts that $\bar{v}$ is the limit as we let n go to infinity. □

The following corollary establishes evenness of V as an equilibrium property under evenness of u and f.

Corollary 8. *TV is even given u and f are even and the assumptions in Proposition 7.*

Proof: Lemma 25 establishes that B^e is complete. Given Lemma 18 and Lemma 19 and the evenness of u together with the symmetry of f we conclude that 20 is even. □

A.4 Law of motion

Lemma 9. *Let $\lambda_i^{i'}(x)$ and $X_{i,i'}(x,x')$ be scalar-valued policies. Further, let $X_{i,i'}(x,x')$ be strictly monotone in x'. f is a well-defined continuous density function. For $\dot{f}_i(x) = 0\ \forall i \in [0,1], x \in \mathbb{R}$ (5) becomes*

$$f_i(x) = \frac{1}{\Lambda_i^r + \Lambda_i^c(x) + \Lambda^x}\left(\Lambda^x g(x) + \int_0^1\int_{\mathbb{R}} f_{i'}(h_{i,i'}(\bar{x},x))f_i(\bar{x})\times \left(\lambda_{i'}^i(h_{i,i'}(\bar{x},x)) + \lambda_i^{i'}(x)\right)\sqrt{1+H_{i,i'}(\bar{x},x)^2}d\bar{x}di'\right) \tag{21}$$

where $h_{i,i'}(\bar{x},x) : \bar{x} - X_{i,i'}(\bar{x}, h_{i,i'}(\bar{x},x)) = x,\ H_{i,i'}(\bar{x},x) = \frac{\partial_- h_{i,i'}(\bar{x},x)}{\partial \bar{x}}$.

Proof: By Lemma 13 we can rewrite the statements involving the bargaining solutions and collect terms.

$$\begin{aligned}
\dot{f}_i(x) &= -f_i(x)\int_0^1\int_{\mathbb{R}} \mathbb{I}_{\{x - X_{i,i'}(x,x') \neq x\}} f_{i'}(x')\left(\lambda_{i'}^i(x') + \lambda_i^{i'}(x)\right)dx'di' \\
&+ \int_0^1\int_{\mathbb{R}}\int_{\mathbb{R}} \mathbb{I}_{\{\hat{x} - X_{i,i'}(\hat{x},x') = x\}} f_{i'}(x')f_i(\hat{x})\left(\lambda_{i'}^i(x') + \lambda_i^{i'}(x)\right)dx'd\hat{x}di' + \Lambda^x(g(x) - f_i(x)) \\
&= \Lambda^x(g(x) - f_i(x)) - f_i(x)\int_0^1\int_{\mathbb{R}} f_{i'}(x')\left(\lambda_{i'}^i(x') + \lambda_i^{i'}(x)\right)dx'di' \\
&+ f_i(x)\int_0^1\int_{\mathbb{R}} \mathbb{I}_{\{x - X_{i,i'}(x,x') = x\}} f_{i'}(x')\left(\lambda_{i'}^i(x') + \lambda_i^{i'}(x)\right)dx'di' \\
&+ \int_0^1\int_{\mathbb{R}}\int_{\mathbb{R}} \mathbb{I}_{\{\hat{x} - X_{i,i'}(\hat{x},x') = x\}} f_{i'}(x')f_i(\hat{x})\left(\lambda_{i'}^i(x') + \lambda_i^{i'}(x)\right)dx'd\hat{x}di' \\
&= A + B + C
\end{aligned}$$

where A, B and C are defined by the third to last, second to last and last line, respectively.

Lets look at A for a moment:

$$A = \Lambda^x(g(x) - f_i(x)) - f_i(x)\left(\int_0^1\int_{\mathbb{R}} f_{i'}(x')\lambda_{i'}^i(x')dx'di' + \int_0^1 \lambda_i^{i'}(x)\int_{\mathbb{R}} f_{i'}(x')dx'di'\right) = g(x)\Lambda^x - f_i(x)\left(\Lambda_i^r + \Lambda_i^c(x) + \Lambda^x\right)$$

Now, by Lemma 15 we know that $X_{i,i'}(\hat{x},x')$ is strictly monotone in x'. Then $\exists h : \hat{x} - X_{i,i'}(\hat{x}, h_{i,i'}(\hat{x},x)) = x$ and h is bounded and monotonically decreasing in $\hat{x}$. Further, we know from Proposition 1 that $X_{i,i'}(x,x')$ are in the interior of the real line and therefore a left hand sided derivative must exist. We substitute using shorthand $h_{i,i'}(\hat{x},x) = h(\hat{x})$ and $\frac{\partial_- h_{i,i'}(\hat{x},x)}{\partial \hat{x}} = H_{i,i'}(\hat{x},x) = H(\hat{x})$

$$
\begin{aligned}
B &= f_i(x)\int_0^1\int_{\mathbb{R}} \mathbb{I}_{\left\{x - X_{i,i'}(x,x') = x\right\}} f_{i'}(x')\left(\lambda_{i'}^i(x') + \lambda_i^{i'}(x)\right) dx'di' \\
&= f_i(x)\int_0^1\int_{h^{-1}(\mathbb{R})} \mathbb{I}_{\left\{x - X_{i,i'}(x,h(\bar{x})) = x\right\}} f_{i'}(h(\bar{x}))\left(\lambda_{i'}^i(h(\bar{x})) + \lambda_i^{i'}(x)\right) H(\bar{x}) d\bar{x}di' \\
&= -f_i(x)\int_0^1\int_{\mathbb{R}} \mathbb{I}_{\{x=\bar{x}\}} f_{i'}(h(\bar{x}))\left(\lambda_{i'}^i(h(\bar{x})) + \lambda_i^{i'}(x)\right) H(\bar{x}) d\bar{x}di' = 0
\end{aligned}
$$

where the second to last step comes from the fact that h^{-1} is decreasing and the last step acknowledges that we integrate over zero measure.

Similarly,

$$
\begin{aligned}
C &= \int_0^1\int_{\mathbb{R}}\int_{\mathbb{R}} \mathbb{I}_{\left\{\hat{x} - X_{i,i'}(\hat{x},x') = x\right\}} f_{i'}(x') f_i(\hat{x})\left(\lambda_{i'}^i(x') + \lambda_i^{i'}(x)\right) dx'd\hat{x}di' \\
&= \int_0^1\int_{\mathbb{R}}\int_{\mathbb{R}} \mathbb{I}_{\{h(\hat{x}) = x'\}} f_{i'}(x') f_i(\hat{x})\left(\lambda_{i'}^i(x') + \lambda_i^{i'}(x)\right) dx'd\hat{x}di'
\end{aligned}
$$

where the inner two integrations over $\mathbb{R}\times\mathbb{R}$ are just a line integral which we parameterize with $\bar{x}$, or $\hat{x} = \bar{x}$ and $x' = h(\bar{x})$

$$
C = \int_0^1\int_{\mathbb{R}} f_{i'}(h(\bar{x})) f_i(\bar{x})\left(\lambda_{i'}^i(h(\bar{x})) + \lambda_i^{i'}(x)\right)\sqrt{1 + H(\bar{x})^2} d\bar{x}di'
$$

This establishes

$$
\begin{aligned}
\dot{f}_i(x) =& g(x)\Lambda^x - f_i(x)\left(\Lambda_i^r + \Lambda_i^c(x) + \Lambda^x\right) \\
&+ \int_0^1\int_{\mathbb{R}} f_{i'}(h_{i,i'}(\bar{x},x)) f_i(\bar{x})\left(\lambda_{i'}^i(h_{i,i'}(\bar{x},x)) + \lambda_i^{i'}(x)\right)\sqrt{1 + H_{i,i'}(\bar{x},x)^2} d\bar{x}di'
\end{aligned} \tag{22}
$$

The steady state solution follows from simple rearranging. □

The following results are used in the proof for Proposition 10.

Lemma 26. $\mathbb{F}^d \subset \mathbb{F}$ *is closed.*

Proof: We start with the nonnegative part: Say it is not closed! Then $\exists f_n \in \mathbb{F}^d$ which is Cauchy but its limit f is not in $\mathbb{F}^d$. Fix (i,x). For some $\xi > 0$ we have $0 = f_i(x) + \xi = f_i(x) - \{f_i(x)\}_n + \{f_i(x)\}_n + \xi$. But for n large enough we find $\epsilon + c + \xi > 0$ for some $\epsilon > 0$ and $c \geq 0$ which contradicts that $f \notin \mathbb{F}^d$.

Now, say $\left\{\int\limits_0^1\int\limits_{\mathbb{R}} f_i(x)dxdi\right\}_n = F_n = 1$ is Cauchy but its limit point $F \neq 1$. Without loss of generality say $F < 1$. Then $0 = F - 1 + \xi = F - F_n + F_n - 1 + \xi = \epsilon + \xi$. But this means F cannot be the limit point of F_n unless $\xi = 0$. □

Lemma 27. *Under the assumptions of Proposition 10 we find $T'f$ to be a local radial contraction.*

Proof: The elements of $T'f = \frac{C_i(x)}{\Lambda_i(x)}$ are defined implicitly from the formula in Proposition 10. Fix (i,x). Then

$$\left|T'f_i(x) - T'\hat{f}_i(x)\right| = \left|\frac{C_i(x)}{\Lambda_i(x)} - \frac{\hat{C}_i(x)}{\hat{\Lambda}_i(x)}\right|$$

Without loss of generality we can state $\frac{C_i(x)}{\Lambda_i(x)} \geq \frac{\hat{C}_i(x)}{\hat{\Lambda}_i(x)}$. Then there are two distinctions. First, $\Lambda_i(x) \geq \hat{\Lambda}_i(x)$. Then

$$\begin{aligned}
&\left|T'f_i(x) - T'\hat{f}_i(x)\right| \leq \frac{1}{\Lambda^x}\left|C_i(x) - \hat{C}_i(x)\right| \\
&\leq \frac{1}{\Lambda^x}\left|\int\limits_0^1\int\limits_{\mathbb{R}}\int\limits_{\mathbb{R}}\left[f_{i'}(h(\bar{x}))f_i(\bar{x})f_i(\tilde{x}) - \hat{f}_{i'}(h(\bar{x}))\hat{f}_i(\bar{x})\hat{f}_i(\tilde{x})\right]\times\right. \\
&\left. S_{i,i'}(\tilde{x}, h(\bar{x}))\sqrt{1+H(\bar{x})^2}(1-\theta)did\tilde{x}d\bar{x}\right| \\
&+\left|\int\limits_0^1\int\limits_{\mathbb{R}}\int\limits_{\mathbb{R}}\left[f_{i'}(h(\bar{x}))f_i(\bar{x})f_{i'}(\tilde{x}) - \hat{f}_{i'}(h(\bar{x}))\hat{f}_i(\bar{x})\hat{f}_{i'}(\tilde{x})\right]\times\right. \\
&\left. S_{i,i'}(\bar{x}, h(\bar{x}))\sqrt{1+H(\bar{x})^2}(1-\theta)did\tilde{x}d\bar{x}\right|
\end{aligned}$$

Now we know that S and H(x) are bounded because V is bounded and continuous which prohibits infinite-sized jumps. So, the only critical part is in the brackets. We write he first bracket out and expand

$$\begin{aligned}
&\left|f_{i'}(h(\bar{x}))f_i(\bar{x})f_i(\tilde{x}) - \hat{f}_{i'}(h(\bar{x}))\hat{f}_i(\bar{x})\hat{f}_i(\tilde{x})\right| = \left|f_{i'}(h(\bar{x}))f_i(\bar{x})\left[f_i(\tilde{x}) - \hat{f}_i(\tilde{x})\right]\right. \\
&\left.+\left[f_{i'}(h(\bar{x}))\left[f_i(\bar{x}) - \hat{f}_i(\bar{x})\right] + \left[f_{i'}(h(\bar{x})) - \hat{f}_{i'}(h(\bar{x}))\right]\hat{f}_i(\bar{x})\right]\hat{f}_i(\tilde{x})\right| \\
&\leq k_i^*(x)D(f,\hat{f})
\end{aligned}$$

for some constant $0 < k_i^*(x) < \infty$ as all f are bounded everywhere.

If on the other hand we face $\Lambda_i(x) < \hat{\Lambda}_i(x)$ and we find

$$\left|T'f_i(x) - T'\hat{f}_i(x)\right| \leq \frac{1}{\Lambda^x \hat{\Lambda}_i(x)} \left|\hat{\Lambda}_i(x) C_i(x) - \Lambda^x \hat{C}_i(x)\right|$$
$$\leq \frac{1}{\Lambda^{L^2}} \left|\hat{\Lambda}_i(x) C_i(x) - \Lambda^x \hat{C}_i(x)\right|$$

But this also implies $\hat{\Lambda}_i(x) C_i(x) \geq \Lambda^x \hat{C}_i(x)$. Now we can create a sequence $\{z_i\}_{i=0}^M$ so that $z_0 = 1$, $z_M \Lambda^x = \hat{\Lambda}_i(x)$ and $|z_i - z_{i-1}| \Lambda^x \leq D(f, \hat{f})$. Then

$$\left|T'f_i(x) - T'\hat{f}_i(x)\right| \leq \frac{1}{\Lambda^{L^2}} \left|\hat{\Lambda}_i(x) C_i(x) + \sum_{i=0}^{M} (z_i - z_i) \Lambda^x \hat{C}_i(x) - \Lambda^x \hat{C}_i(x)\right|$$
$$= \frac{1}{\Lambda^{L^2}} \left|\hat{\Lambda}_i(x) C_i(x) - z_M \Lambda^x \hat{C}_i(x) + \sum_{i=1}^{M} (z_i - z_{i-1}) \Lambda^x \hat{C}_i(x) + (z_0 \Lambda^x \hat{C}_i(x) - \Lambda^x \hat{C}_i(x))\right|$$
$$\leq \frac{1}{\Lambda^{L^2}} \left|\hat{\Lambda}_i(x) C_i(x) - z_M \Lambda^x \hat{C}_i(x) + \sum_{i=1}^{M} (z_i - z_{i-1}) \Lambda^x \hat{C}_i(x) + (z_0 \Lambda^x \hat{C}_i(x) - \Lambda^x \hat{C}_i(x))\right|$$
$$\leq k_i^{**}(x) D(f, \hat{f})$$

for the same reasons as above. But this holds for all (i, x). Then

$$\left|T'f - T'\hat{f}\right| \leq k D(f, \hat{f})$$

for $0 < k < \infty$ and the result follows similarly as in Lemma 22. □

We can finally establish that T' is indeed a contraction.

Proposition 10. *Let $\mathbb{F}$ be the space of real-valued, bounded functions defined on $[0,1] \times \mathbb{R}$ which are continuous in x with elements $f, \hat{f}$, and $D : \mathbb{F} \times \mathbb{F} \to \mathbb{R}^+$ be the supnorm metric in the sense of $D(f, \hat{f}) = \sup_{i \times x \in [0,1] \times \mathbb{R}} \left\{\left|f_i(x) - \hat{f}_i(x)\right|\right\}$. Further, define T' by equation (21). In particular,*

$$T'(f_i(x)) = \frac{1}{\Lambda_i^r + \Lambda_i^c(x) + \Lambda^x} \left(\Lambda^x g(x) + \int_0^1 \int_{\mathbb{R}} f_{i'}(h_{i,i'}(\bar{x}, x)) f_i(\bar{x}) \times \right.$$
$$\left. \left(\lambda_{i'}^i(h_{i,i'}(\bar{x}, x)) + \lambda_i^{i'}(x)\right) \sqrt{1 + H_{i,i'}(\bar{x}, x)^2} d\bar{x} di'\right)$$

Then $T'f$ is a global contraction. Further, f is a well-defined, nondegenerate and continuous density function.

Proof: Lemma 26 shows that the subset $\mathbb{F}^d \subset \mathbb{F}$ whose functions f map into the nonnegative real line and integrate to 1 is closed under $\mathbb{F}$. Hence, $\mathbb{F}^d$ is complete under the supnorm. We then establish that $T' : \mathbb{F} \to \mathbb{F}$. Then we show that $T'f$ is a local radial contraction and in turn a global contraction very similar to the contraction proof for the value function.

From the assumptions and the properties of continuous functions it is easy to see that T' maps from $\mathbb{F}$ into the space of real-valued, bounded and continuous functions. Further, nonnegativity is also straightforward. Integration to one can be established by following the derivation of (21) and the fact that no leakage allows f to shrink or to grow.

For a global contraction we further require that for some f_0 and $T'f_0$ there exists a graph of finite length. By construction $g_{f_0,T'}(i) = if_0 + (1-i)T'f_0$ is a graph of finite length because $\mathbb{F}^d$ consists of bounded functions, and V and g are bounded and continuous. Nondegeneracy follows from the fact that $g(x)$ is nondegenerate. $\square$

Corollary 11. *f is even given V and g are even in x, and the assumptions of Proposition 10.*

Proof: We know by Lemma 25 that even functions are in a closed set. Lets see if T' maps a symmetric function into a symmetric function: First, note that $h(\bar{x}, x)$ is odd in x. A composition of an even and an odd function is even, and the product and sum of even functions is even. Therefore, T' maps an even function into an even function. $\square$

A.5 Comparative Statics

Proposition 12. *$\lambda_i^{i'}(x)$ is strictly decreasing (increasing) for $x < 0$ ($x > 0$) and has a unique minimum at $x = 0$, or agents further away from their target search more given V and f are even and the assumptions of Proposition 4.*

Proof: Lemma 20 shows that $ES_i^{i'}(x)$ is strictly decreasing (increasing) for $x < 0$ ($x > 0$) and has a unique minimum at $x = 0$. The result follows from the monotonicity of λ in ES(x). $\square$

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