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Coordinated Intermediation in the Federal Funds Market

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Abstract

Intermediation motivates why some banks move further away from their implied target level during the fed funds trading window. I use partially directed search (PDS) where search decisions coordinate trade among two groups that are possibly identical otherwise. Middlemen behaviour evolves endogenously and is illustrated in a small scale model. Robustness is discussed. A large scale model based on stylized facts of 2006 replicates the puzzle where undirected search (US) comes short. PDS is flexible enough to address another puzzle about inflated variances of fed funds rates in US models.

Keywords: Intermediation, Partially Directed Search, Fed Funds Market

Resumen

La intermediación motiva por qué algunos bancos se alejan más de su nivel objetivo de fondos requeridos en el mercado interbancario de Estados Unidos. Este modelo utiliza un modelo de búsqueda parcialmente dirigido (PDS) donde las decisiones de búsqueda coordinan el intercambio entre dos grupos que posiblemente sean idénticos de lo contrario. El comportamiento de los intermediarios evoluciona endógenamente y se ilustra en un modelo a pequeña escala. Se discute la robustez. Un modelo a gran escala basado en hechos estilizados de 2006 replica el enigma donde la búsqueda no dirigida (US) se queda corta. El modelo de búsqueda parcialmente dirigida (PDS) es lo suficientemente flexible para abordar otros enigmas sobre las variaciones infladas de las tasas del mercado interbancario en los modelos de búsqueda no dirigida (US).

Palabras claves: Intermediación, búsqueda parcialmente dirigida, mercado de *Fed Funds*

Coordinated Intermediation in the Federal Funds Market

Alexander Dentler*

Revised: March 28, 2014

Abstract

Intermediation motivates why some banks move further away from their implied target level during the fed funds trading window. I use partially directed search (PDS) where search decisions coordinate trade among two groups that are possibly identical otherwise. Middlemen behaviour evolves endogenously and is illustrated in a small scale model. Robustness is discussed. A large scale model based on stylized facts of 2006 replicates the puzzle where undirected search (US) comes short. PDS is flexible enough to address another puzzle about inflated variances of fed funds rates in US models.

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1 Introduction

Why do some banks that are far below their minimum reserve requirements trade away federal funds? Why do some banks that are far above their requirements buy more funds that have no or little return? In the former case these banks run the chance of having to borrow money from the central bank at a punitive interest rate while in the latter they forgo other investment opportunities that carry higher returns. The minimum reserve

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requirement can therefore be considered an implicit target level. This is an empirical puzzle described by Afonso and Lagos (2012a). They document trading of US federal funds during the active trading period between 16:00 and 18:30 on bank business days. At any given point some banks in the top 10-percentile of their historical distribution of balance holdings enter bilateral trade agreements that increase their balances. Similarly, some banks in the bottom 10-percentile decrease their balances by trade agreements.[1]

I propose that intermediation is the motivation for this behaviour. Banks do not have access to a centralized market mechanism and do not know exact the exact balances of other banks in this over-the-counter (OTC) market. Some banks serve as middlemen and are known to do so. They are contacted more frequently than other, more peripheral banks. It can be socially optimal that a middlemen bank trades with a bank that is

1. less likely to create another trading opportunity than the middlemen bank and

2. on the same side of her minimum reserve requirements as the middlemen bank.

This trade moves the middlemen bank away from her target level and the peripheral bank towards her target level.[2] The pattern of all trades disperses the distribution of balance holdings of the group of middlemen banks relative to the group of peripheral banks. In the future it is more likely to find a trading partner that is away from her target level and willing to trade in the group of middlemen banks. At the same time, peripheral banks exactly on or near their target will not want to trade. Therefore, a middlemen bank is frequented more often in the future and is more likely to create another trading opportunity than a peripheral bank. The dynamic realization of this sequential game reinforces this coordination.

I offer a modification of the conventional search approach where banks are assigned labels. This identifies them as members of a subgroup and allows for partially directed search (PDS) where banks can choose to only search among one subgroup. The labels become a coordination device where members of one group endogenously choose to take on the role of middlemen. The behaviour associated with these roles successfully replicates a pattern that is observational equivalent to the puzzle described above. Another puzzle

[1] They describe this fact in section 6, and figure 23 and 24 nicely illustrate this puzzle.

[2] Either both were in need of funds and the peripheral obtained funds from the middlemen bank or both had balances above their minimum reserve requirements and the peripheral bank sold funds to the middlemen bank. If two banks on opposite sides of their respective target level meet they both move towards their target.

Afonso and Lagos (2012b) observe is that the theoretical variance of federal fund rates is much higher than the empirical one when using undirected search (US). Modifying ex ante distributions with well identified groups to comply with public knowledge[3] allows me to reduce the variance of the federal fund rate in a separate application of PDS.

Future work on this topic with the proposed model attempts to match empirical moments. Possible objects are the US fed funds market or the Swiss Repo market that serves the same purpose. An appropriate dataset exists. Two problems arise: these moments are hard to come by as it involves proprietary data. Sources of secondary data include literature with empirical moments or publicly available statistics provided by central banks. The second complication is that the model proposed here is computationally more intensive than comparable models such as Afonso and Lagos (2012b).

I explain the market and motivate the economic mechanisms chosen and relate them to the literature. A small scale simulation allows me to illustrate microeconomic considerations and economywide effects where members of one group become middlemen. I proceed to address concerns about multiplicity of equilibria and show that mild and realistic modifications of basic parameters allow for a single equilibrium where one group becomes the middlemen group and the other accepts their services. These modifications are all associated with big banks. A large scale model based on stylized facts of the federal fund market in 2006 allows me to replicate the behaviour of banks explaining the main puzzle addressed in this paper. PDS matches empirical moments better than the US counterpart without calibration. The variance of the federal funds rates is addressed in a small scale simulation. A slight modification using stylized facts documented by Furfine (1999) addresses the issue of the large variance of federal fund rates in an US environment which can be reduced using PDS.

2 Money Markets and Modelling Choice

Money markets like the federal fund market solve the minimum reserve requirement problem of banks. In particular, the agent responsible for minimum reserve compliance observes trading by other business departments within her bank during the day and knows her minimum reserve requirements before the fed funds market starts at around 16:00 EST. She realizes she either is in need of funds or has too much. The former means if she

[3] In particular, it is common place that a few big banks enter the market and are on average in need of funds while they face a mass of small banks who are on average net seller of funds.

does not obtain sufficient funds she will approach the central bank where she can borrow lacking funds at a punitive interest rate at the end of the day. The latter means she receives little or no interest on her excess balances.[4] To avoid this misallocation banks trade overnight loans among each other to meet their minimum reserve requirements or to obtain interest by lending funds out before the maintenance period at 18:30.[5] Without a centralized mechanism in place trading is executed bilaterally in an OTC environment. The weighted interest rate paid on these loans is the federal funds rate, the main policy target of the Federal Open Market Committee. It is therefore of interest to learn about the workings of this market.

The model I propose is closely related to Afonso and Lagos (2012b). They construct a model that captures the main features of the federal fund market. In their model banks experience an exogenous liquidity shock at the beginning of the trading period and then meet randomly capturing the OTC environment with an US approach. The model proposed here adds a search decision to accompany the fact that US seems unrealistic in this environment. US implies that a match is created by pure randomness and is in particular not the result of choice. While it is true that the environment is not designed to reveal exact information about potential trading partners banks know basic characteristics about each other in distribution. In other words, banks that are always intermediating can be grouped under one label. A bank will call a member of that particular group in case it is more appealing to her to contact a bank that always intermediates than to contact a bank which is known to only trade based on her own needs. Or if one group of banks is on average in need of funds then a bank who would like to sell will be able to direct her search efforts towards a member of that group as opposed to any bank. This environment is more closely related to directed search as in Corbae et al. (2003) where search decisions are modelled explicitly as part of an equilibrium rather than some parameter. The difference is that in their directed search environment one always finds another trading partner in a particular physical state. In the model proposed here this only happens with chance. This is the main difference between fully directed search and PDS.

Models of PDS are relatively new and two prominent ones are applied to the labour market. Menzio (2007) investigates a model with of job ads. The wage announcement is

[4]Excess balances are balances above her minimumm reserve requirements.

[5]The minimum reserve requirements must actually only be met on average over a two week monitoring period. Ashcraft and Duffie (2007) find no evidence that these intervals affect intraday behaviour.

correlated with the actual wage and allows for communication between firm and worker before any contractually binding agreement is reached. The PDS I propose does not allow for direct signals between two parties. The search decisions are based on expectations about the distribution of holdings and conjectures of future meeting probabilities of banks. Menzio observes that a PDS depends on a balance of workers and firms. The PDS equilibria associated with intermediation proposed here depend on a more or less symmetric distribution of goods around a target level.

Another interesting application of PDS is Lester (2010). He allows for an endogenous distribution of firm sizes when matching multiple workers to firms having multiple open vacancies. Similar to the model proposed here he finds the endogenously defined matching to be more efficient the more concentrated the market becomes.

While most banks are not active in the federal fund market on a day to day basis, there is ample evidence that some of the more active ones are actively intermediating. Furfine (1999) and Afonso and Lagos (2012a) document a very skewed distribution of the number of counterparties banks have. Furfine (1999) in particular relates the number of counterparties to be positively related with the banks size. While it seems plausible that big banks are more likely to intermediate than smaller banks the model I propose here does not depend on the size of the bank. Nonetheless, the sufficiency conditions to pin down belief-robust equilibria discussed in the small scale model point to the connection between big banks and the intermediating group.

Rubinstein and Wolinsky (1987) provide a first analysis about intermediation in search environments. Buyer and seller can only meet with some chance and an always accessible middleman can buy and then sell the good with a markup. While I identify two markups the intermediating group enjoys I let the intermediator be a regular market participant that needs to fulfil their minimum reserve requirements as any other bank. This is in line with Ashcraft and Duffie (2007) who documents that only 27% of all trades in 2005 are brokered where the intermediating party carries no own risk.

Gehrig (1993) discusses implication for middlemen in financial markets, in particular what competition among middlemen does to welfare. While the model proposed here is mainly empirically motivated I can contrast the results. He uses price posting to impose Bertrand competition among the intermediators and finds that at no surprise this benefits market participants. Using PDS here increases welfare of all banks, but decreases the welfare of peripheral banks. Concentrating the group of middlemen increases overall welfare but keeps the welfare experienced by peripheral banks constant. The model here does not have the opportunity to post prices. Anecdotal evidence from a monitoring agent of the Swiss National Bank where price posting is possible allows me to conclude that this opportunity is rarely taken.

This environment comes with an empirical pattern derived from a melange of an OTC environment (dispersion of prices because of search friction) and bargaining power (dispersion of prices due to asymmetric bargaining power). E.g. Kraenzlin and von Scarpatetti (2011) find price setting behaviour influenced by a lagged measure of centrality. The bargaining mechanism used here does not require for explicit bargaining power. Implicit bargaining power only comes through a higher threat to leave the negotiation a la Nash and is therefore strategically derived.

There is a range of empirical network analysis pointing to a concentration in these markets. Compare Iori et al. (2008) among others and Gofman (2011) for an application of network theory. The line of research here is essentially search-theory driven, but can also be seen as a first step away from search to a network based model where links between agents hinge on their group membership and are created by the search decision.

NEED TO ADD COMPETITIVE SEARCH LIT AND MOTIVATION.

3 Model

3.1 Basic Environment

There is a unit mass of banks of two different types with label i $\in \mathbb{I} = \{p, m\}$ where the proportion of types is given by $p_p = 1 - p_m \in (0, 1)$. The label is fixed to a bank and does not change. Unless two banks are matched to bargain with each other no bank knows anything about any other bank except for their label i. To attach some meaning to the letters I chose p for peripheral banks and m for middlemen banks but the strategies associated with these roles are accepted endogenously.

Time is discrete and finite within a fed funds trading window at the end of the day (say from 4:00 pm to 6:30 pm). I take (trading) rounds as my time unit and label them with t, starting at t=0 and ending with t=T where T $\in$ N, and I denote the set $\mathbb{T} = [0, T]$. So a trading window as described above with 10 trading rounds leaves us to believe a trading round takes 15 minutes. For notational ease later I introduce τ=T-t that displays rounds left for the day (including the current one). A bank discounts a future return with a yearly discount rate of r, and $\beta = \left(\frac{1}{1+r}\right)^s$ is the period discount rate where s is the fraction of the year a trading period represents.

In the beginning of each round τ a bank has a balance holding $k^\tau \in \mathbb{K} = \{0, 1, ..., K\}$ where K $\in \mathbb{Z}$. Note that we can reinterpret $\mathbb{K}$ so that a segment below some k_{0i} represents negative balance holdings. All banks start out with some exogenously given $k^T \in \mathbb{K}$ at the beginning of the day.

Balances k^τ give a bank instantaneous return $u(k^\tau, i)$ at the beginning of a trading round τ. u() does not change over the day (k might of course), but rather reflects an interest rate schedule based on excess balances (or fees on overdrafts)[6]. A bank that is below her allowed overdraft limit pay an overdraft fee.[7] I denote this lower limit by $\underline{k}_i$ for a bank with label i.

$$u(k^\tau, i) = \begin{cases} (k^\tau - \overline{k})r^{e^s} & \text{if } k^\tau > \overline{k} \\ (\underline{k}_i - k^\tau)r^{o^s} & \text{if } k^\tau < \underline{k}_i \\ 0 & \text{otherwise.} \end{cases}$$

where r^e is the return on excess balances and r^o is the overdraft fee. Note that $\underline{k}_i \leq k_{0i} \leq \overline{k_i}$ in general where the first variable is the overdraft limit, the second is the absolute zero balance a bank can have and the latter is is the minimum reserve requirement of a bank with label i.

A bank of label i will receive a penalty if her balance holding is below the minimum requirement threshold $\overline{k_i}$ by the end of the day ($\tau = 0$). The bank can avoid this penalty by using an emergency overnight loan offered by the central bank who acts as a lender of last resort.[8] I can normalize $\overline{k_i} = \overline{k}$ by simply adding $\overline{k_m} - \overline{k_p}$ to each observation of balance holding associated with the label p.[9] This allows me to describe end-of-day

[6]The US Fed did not pay interest prior to October 9th 2008 and the SNB currently pays no interest on any deposits. A bank above her minimum reserve requirement $\overline{k_i}$ might receive a return, or she might have outside opportunities, e.g. a bank might transfer money to branches in other countries. In the US daylight overdrafts are allowed and carry no fee as long as they do not exceed a threshold determined by the net debit cap and deposited collateral.

[7]A bank that has collateral also does not pay any fees, but as she forgoes the interest on her collateral the discount factor endogenizes this tradeoff. This problem is omitted here.

[8]The interest rate the central bank offers is above an average intrabank lending rate but below any penalty otherwise imposed. In the US this is the Primary Credit Facility and in Switzerland the appropriate instrument is known as Liquidity-Shortage Financing Facility. In fact, there are different policies in place that make getting a loan from the central bank the most attractive alternative. For sake of ease I forgo these off equilibrium path actions.

[9]Similar modifications are done to $\underline{k}_p$ and $\overline{k_p}$ and I will take that as given for the remainder of the

return on balance holding k^0 by

$$U(k^0) = \begin{cases} k^0 & \text{if } k^0 \geq \overline{k} \\ \overline{k} - \beta^{\Delta}(1+r^p)^{s\Delta}(\overline{k} - k^0) & \text{otherwise.} \end{cases}$$

Again, for notational ease I ascribe the letter Δ the meaning of the difference between the actual market closing time T and the time the funds need to be repaid measured in round length. $r^p s$ is the interest rate of the central bank lending to financial institutions with reserve requirements at the end of the day multiplied by the fraction a period represents in years s.

Before I denote lifetime utility I need to pin down some distributional definitions and technicalities. The mass of banks with label i and holding k in the beginning of τ can be described by $f_i^\tau(k)$ where i $\in$ I, $\tau \in \mathbb{T}$ and k $\in \mathbb{K}$. Note that $f_i^\tau(k) \geq 0$ and $\sum_{l \in \mathbb{K}} f_i^\tau(l) = 1$, of course. I let F_i^τ be a vector with elements $f_i^\tau(k)$ for period τ and label i, and $F^\tau = \left[p_p F_p^{\tau\prime} \; p_m F_m^{\tau\prime}\right]'$. The sequence of money-holding distributions describing the distribution over the course of the day is denoted by $E = \{F^\tau\}_{\tau=0}^T$. Let that set of all possible (intraday) distributions be defined by $\mathbb{F}$ and the set of all sequences is $\mathbb{E}$.[10]

The central bank sets its lending interest rate r^p well above the average interbank rate observed during the day, so that banks are at least weakly better off trying to cover the gap between holdings and requirements among themselves. On the other hand, holding money bears no interest. Therefore, a bank tries either to borrow the required difference ($k^T < \overline{k}$) or to lent unnecessary funds ($k^T > \overline{k}$) during the day. She will attempt to establish a bilateral overnight credit arrangement during every period $\tau \in \mathbb{T}^+$ that results in a loan B^τ towards its counter party at the end of period τ (and away from her own balance) and an outstanding position R^τ due at time δ the next day. Without any central mechanism she can only do so by searching for a trading partner.

Therefore, maximum lifetime utility at the beginning of the day is described by

---

paper with a slight abuse of notation.

[10]The total of money held is described by $\bar{K} = p_p \sum_{k^\tau \in \mathbb{K}} (k^\tau - k_{0p}) f_p^\tau(k^\tau) + p_m \sum_{k^\tau \in \mathbb{K}} (k^\tau - k_{0m}) f_m^\tau(k^\tau)$. The total of minimum reserve requirements $\bar{Q} = p_p(\bar{k}_p - k_{0p}) + p_m(\bar{k}_m - k_{0m})$ and naturally neither changes over τ as I exclude default, free disposal or any other friction.

$$J(k^T, i, F^T) = E\left[\sum_{l=0}^{T-1} \beta^l \left(u(k^{T-l}, i) + \beta^{\Delta+T-1} R^{T-l*}\right) + \beta^T U_i(k^0)|X\right] \tag{1}$$

$$\text{s.t. } k^{T-l-1} = k^{T-l} - B^{T-l*} \; \forall \; l \in \{0, 1, ..T-1\}$$

$$k^T \text{given}$$

for all $k^T \in \mathbb{K}$, $i \in \mathbb{I}$ and $F^T \in \mathbb{F}$. $(B^{\tau*}, R^{\tau*})_{\tau=0}^{T-1}$ display optimal decisions in search and bilateral bargaining. Note that I take expectations over X which represents information about initial distribution of balances and optimal decisions of all other banks.

I can rewrite the maximum lifetime utility to the more handy indirect utility function (compare appendix (B)). Then

$$W^\tau_{F^\tau}(k^\tau, i) = u(k^\tau, i) + E[\beta^{\Delta+\tau} R^{\tau*} + \beta W^{\tau-1}_{F^{\tau-1}}(k^\tau - B^{\tau*}, i)|F^\tau] \tag{2}$$

for $\tau > 0$ and

$$W^0_{F^0}(k^0, i) = U(k^0) \tag{3}$$

for $\tau = 0$.

I drop one state variable, and all policies only depend on the balance position at τ. Denote by W^τ_i a vector with elements $W^\tau_{F^\tau}(k, i)$. Let $W^\tau = \left[W^{\tau\prime}_p \; W^{\tau\prime}_m\right]'$ be a vector. Then let $W = \{W^\tau\}_{\tau \in \mathbb{T}}$ be the sequence thereof. The set of all sequences is labelled $\mathbb{W}$.[11]

The only restrictions for (1) and (2) & (3) to be identical is that optimal policy functions only depend on the current distribution and future indirect utility. This holds since search and bargain will only depend on the current distribution and future indirect utility.

[11] Note that this implies that I abstract from credit default risk here. This seems appropriate in the Swiss money market where loans are 100% collateralized, but in markets where this is not the case default can seriously hamper trading mechanisms. This represents an interesting extension.

3.2 Trading Rounds

The exchange has the following sequential logic given by the OTC environment: At the beginning of each round nature draws a fraction of banks $y \in (0, \frac{1}{2}]$. This fraction is uniformly distributed across all banks and I refer to them as "callers" (as we can think of these banks as the party that picks up the telephone and dials). These banks decide to either search for a label-p bank, a label-m bank or mix which translates into a call to a bank of that label in a competitive sense ALA.... . A possible match with a randomly assigned bank of the desired label is proposed (or not) subject to rationing through queuing and a meeting technology of the called bank. This meeting technology represents available resources to answer a call and is captured in the constant parameter α_{ij}. Then the bank that is called decides to accept (or not). Only then complete information over the trading partners balance holding is exchanged as part of the bargaining process. The terms of trade are defined. Trade is executed, and banks depart losing all information about each other until resettlements occur the next day.

Note that this implies that banks are memoryless. Banks cannot reconnect with a trading partner of the same day even when the trade was done under complete information. While this seems like a simplification of reality it not only greatly facilitates the environment but can be supported by an economic argument: a settlement at the end of round τ indicates that nothing can be added to a (socially optimal) exchange. Hence the desire to reconnect with that particular bank should be small. Given that both spend time away from each other for at least one round (and cannot monitor each other) I assume reconnecting afterwards is similar to connecting with an arbitrary (other) bank.

Note that the search decision is done simultaneously and in a competitive environment. No bank knows until she decides to negotiate with the bank proposing a match what the bargaining partner's holding is. This precludes any form of screening and selection beyond the labels and expected values. This also explicitly excludes any form of cooperative equilibrium decisions where agents simultaneously deviate to form a new (more attractive) coalition.

3.2.1 Terms of Trade

The terms of trade in Nash's (1950) bargaining solution yield socially optimal result. They depend on the threat points which are defined by the indirect utility an agent gets when she would not participate in the trade. This not only defines the social surplus but also ensures individual rationality of the solution. For now I conjecture that indirect utility functions are well defined and non-decreasing.

In case two banks meet the respective individual rationality requirements are

$$\beta^{\Delta+\tau}R + \beta W^{\tau-1}_{F^{\tau-1}}(k^{\tau} - B, i) \geq \beta W^{\tau-1}_{F^{\tau-1}}(k^{\tau}, i) \tag{4}$$

$$-\beta^{\Delta+\tau}R + \beta W^{\tau-1}_{F^{\tau-1}}(k^{\tau\prime} + B, i') \geq \beta W^{\tau-1}_{F^{\tau-1}}(k^{\tau\prime}, i'). \tag{5}$$

The prime typography is used to distinguish banks. (4) and (5) determine (among others) the domain of all possible terms of trade, $\Pi^{\tau}_{W^{\tau-1}}(\chi, \chi')$ where $\chi \in \mathbb{K} \times \mathbb{I}$. Note that $R \geq 0$ is the repayment of the next day in favour of the first bank and $B \geq 0$ is the funds transferred to credit the second bank instantanuously which makes the first bank the lender and the second the borrower. Negative values invert the roles.

Additionally I require feasibility, or

$$(k^{\tau} - B) \in \mathbb{K} \tag{6}$$

$$(k^{\tau\prime} + B) \in \mathbb{K} \tag{7}$$

$$|R| < \overline{R} < \infty \tag{8}$$

where the last inequality is just an arbitrary modification of $R \in \mathbb{R}$ as I can set $\overline{R}$ arbitrarily large. Similar considerations apply to the first two constraints with K. This guarantees a solution exists for the Nash bargaining problem.[12] Namely, let the domain be described by

$$\Pi^{\tau}_{W^{\tau-1}}(\chi, \chi') = \{(B, R) \text{ s.t. (4), (5), (6), (7) and (8) holds}\} \tag{9}$$

I use Nash bargaining as a benchmark trading mechanism, so that the terms of trade $(B^{\tau *}_{W^{\tau-1}}(\chi, \chi'),\ R^{\tau *}_{W^{\tau-1}}(\chi, \chi'))$ are the results of the following problem:

[12] Note that the objective function below is continuous for R and well-defined over a compact set of integers.

$$
\begin{aligned}
&(B^{\tau *}_{W^{\tau-1}}(\chi,\chi'),\ R^{\tau *}_{W^{\tau-1}}(\chi,\chi')) \qquad (10)\\
= &\underset{(B,R)\ \in\ \Pi^{\tau}_{W^{\tau-1}}(\chi,\chi')}{\operatorname{argmax}} [\beta^{\Delta+\tau} R + \beta W^{\tau-1}_{F^{\tau-1}}(k^{\tau}-B,i) - \beta W^{\tau-1}_{F^{\tau-1}}(k^{\tau},i)]^{\frac{1}{2}}\\
&[-\beta^{\Delta+\tau} R + \beta W^{\tau-1}_{F^{\tau-1}}(k^{\tau\prime}+B,i') - \beta W^{\tau-1}_{F^{\tau-1}}(k^{\tau\prime},i')]^{\frac{1}{2}}
\end{aligned}
$$

It can be shown that either both (4) and (5) or none of them bind.[13] Therefore either an interior solution for $R^{\tau *}_{W^{\tau-1}}(\chi,\chi')$ exists and I can ignore those constraints, or the solution is defined by both constraints. In the first case I can use first order conditions and plug in a reduced form of $R^{\tau *}_{W^{\tau-1}}(\chi,\chi')$ while in the latter I just plug in the constraints into the original maximization problem which leaves me with

$$
\begin{aligned}
B^{\tau *}_{W^{\tau-1}}(\chi,\chi') = &\underset{B \text{ s.t. } (6)\ \&\ (7)}{\operatorname{argmax}} [\beta W^{\tau-1}_{F^{\tau-1}}(k^{\tau}-B,i) - \beta W^{\tau-1}_{F^{\tau-1}}(k^{\tau},i) + \qquad (11)\\
&+\beta W^{\tau-1}_{F^{\tau-1}}(k^{\tau\prime}+B,i') - \beta W^{\tau-1}_{F^{\tau-1}}(k^{\tau\prime},i')]\\
&= \underset{B \text{ s.t. } (6)\ \&\ (7)}{\operatorname{argmax}} \beta Z^{\tau}_{W^{\tau-1}}(\chi,\chi',B)
\end{aligned}
$$

which is a simple search over a grid defined over integers in (6) & (7). Note that $\beta Z^{\tau *}_{W^{\tau-1}}(\chi,\chi')$ is the social surplus of trade.

It can be shown that the set $B^{\tau *}_{W^{\tau-1}}(\chi,\chi')$ is convex set of integer numbers for weakly concave $W^{\tau-1}$, but I might not have a unique solution. Therefore, I impose as a selection program a uniform distribution over all elements of $B^{\tau *}_{W^{\tau-1}}(\chi,\chi')$. Let me denote the smallest element with $\underline{B}^{\tau *}_{W^{\tau-1}}(\chi,\chi')$ and $\overline{B}^{\tau *}_{W^{\tau-1}}(\chi,\chi')$ is the largest element. I can also denote the size of $B^{\tau *}_{W^{\tau-1}}(\chi,\chi')$ by $|B^{\tau}_{W^{\tau-1}}(\chi,\chi')|$ so that the inverse gives me a measure for each element $b^{\tau *}_{W^{\tau-1}}(\chi,\chi')$ of $B^{\tau *}_{W^{\tau-1}}(\chi,\chi')$.

The reduced form solution for elements $r^{\tau *}_{W^{\tau-1}}(\chi,\chi') \in R^{\tau *}_{W^{\tau-1}}(\chi,\chi')$ given $b^{\tau *}_{F^{\tau-1}}(\chi,\chi')$ always is

[13] If one binds I can change R a bit so that the objective function becomes strictly positive. If both bind I find $W^{\tau-1}_{F^{\tau-1}}(k^{\tau}-B,i) - W^{\tau-1}_{F^{\tau-1}}(k^{\tau},i) + W^{\tau-1}_{F^{\tau-1}}(k^{\tau\prime}+B,i') - W^{\tau-1}_{F^{\tau-1}}(k^{\tau\prime},i') = 0$ for some (B,R) $\in \Pi^{\tau}_{W^{\tau-1}}(\chi,\chi')$ (e.g., two identical agents meet) and I can blissfully continue with plugging in the (binding) constraints as described above.

$$
\begin{aligned}
&r^{\tau *}_{W^{\tau-1}}(\chi,\chi') \qquad (12)\\
&= \frac{1}{2}\beta^{1-\Delta-\tau}[-W^{\tau-1}_{F^{\tau-1}}(k^\tau - b^{\tau *}_{F^{\tau-1}}(\chi,\chi'),i) + W^{\tau-1}_{F^{\tau-1}}(k^\tau,i) + \\
&\quad W^{\tau-1}_{F^{\tau-1}}(k^{\tau\prime} + b^{\tau *}_{F^{\tau-1}}(\chi,\chi'),i') - \beta W^{\tau-1}_{F^{\tau-1}}(k^{\tau\prime},i')]
\end{aligned}
$$

I can rewrite the surplus from trade going to bank of type i with k when she meets χ' by plugging the solution of $r^{\tau *}_{W^{\tau-1}}(\chi,\chi')$ into (4)

$$
\begin{aligned}
&\beta^{\Delta+\tau} r^{\tau *}_{W^{\tau-1}}(\chi,\chi') + \beta W^{\tau-1}_{F^{\tau-1}}(k^\tau - b^{\tau *}_{W^{\tau-1}}(\chi,\chi'),i) - \beta W^{\tau-1}_{F^{\tau-1}}(k^\tau,i) \qquad (13)\\
&= \frac{1}{2}\beta[W^{\tau-1}_{F^{\tau-1}}(k^\tau - b^{\tau *}_{W^{\tau-1}}(\chi,\chi'),i),i) - W^{\tau-1}_{F^{\tau-1}}(k^\tau,i) + \\
&\quad + W^{\tau-1}_{F^{\tau-1}}(k^{\tau\prime} + b^{\tau *}_{W^{\tau-1}}(\chi,\chi'),i),i') - W^{\tau-1}_{F^{\tau-1}}(k^{\tau\prime},i')]\\
&= \frac{1}{2}\beta Z^{\tau *}_{W^{\tau-1}}(\chi,\chi')
\end{aligned}
$$

This verifies the idea that independent of the terms of trade chosen the expected surplus from meeting χ' remains constant and can be expressed as a fraction of total surplus. Note that this solution is identical to a solution using the proportional bargaining approach a la Kalai (1977).

Denote by $\underline{B}^\tau(i,i')$ a (K+1)x(K+1) matrix with elements $\underline{B}^\tau_{W^{\tau-1}}((k^\tau,i),(l^{\tau\prime},i'))$. I define $\overline{B}^\tau(i,i')$ similarly. Then let $B^\tau(i,i') = [\underline{B}^\tau(i,i'); \overline{B}^\tau(i,i')]$. Let $B^\tau = [B^\tau(p,p);\ B^\tau(p,m);\ B^\tau(m,p);\ B^\tau(m,m)]$ and B describes the sequence thereof. Note that I define R in a similar fashion.

3.2.2 Search and Match

A bank of type and holding χ becomes a caller with probability y. She looks into her phonebook and sees only names and numbers. She can uniquely assign to each number a label. She now takes expectations of potential trading outcomes $\frac{1}{2}\beta Z^{\tau *}_{W^{\tau-1}}(\chi,\chi')$ over k^τ for each label as she does not know what exact fed funds holding the trading partner will have. She then tries to reach a bank of a label that best suits her needs considering the probability to be matched with a bank of that label. As all banks of a certain label are identical in expectations she can randomly pick a candidate and dial.

There are three additional aspects of calling up a candidate. First, the phone needs to be attended to. The calling bank will revise the candidate she called if nobody picks up. Attendance is reflected in the parameter α_i that can be specific for a group but is constant across time and balance levels. It can be thought of on how well a money desk in a particular group of banks is managed. Second, the candidate cannot dial out in the same round. At any given point of time (1-y) of all banks call and will therefore not pick up. A calling bank will again revise the candidate she calls. Last but by no means the least, the candidate cannot be busy bargaining with another bank. This is caught by using competitive search. In this case the calling bank will hear a busy signal and call another candidate from her pre chosen pool while the candidate

Some words to resolve some technical issues leading to competitive search, motivate the labels further and display the narrative value of this approach: in case nobody picks up her call or she receives a busy signal a calling bank can easily find another potential partner to trade in her phone book. She then redials until she either reaches someone or the trading round ends. If she has not reached a trading partner by then she is rationed and will not bargain with anyone in this round. In particular, she will not change her mind and start calling banks of another label but only update her decision in the next round. This can be motivated by the fact that discovering unavailability takes actual (calling) time.

Banks who are not callers have no outside option, so that they always accept any caller and start bargaining with the first that calls. In particular, they will not hang up because of the identity or holding level of the other (calling) bank and wait for the next caller. A bank does not hang up once the name is revealed (and therefore the label known) because of social norms: as fed funds trading is done day after day any bank will be shunned that just quits the phone conversation after a brief hello. Further, the balance holding of each bank is only revealed to the other bank at the end of the phone conversation which, again, takes up the remaining time of the trading round.

In other words I define the set over which a calling bank searches $\bar{S}^{\tau}_{W^{\tau-1},F^{\tau},\bar{S}^{\tau}(-\chi)}(\chi)$ as a result of a competitive search game in the following sense. Each bank enters the stage knowing the rounds left τ, her holding k^{τ}, her label i, the distribution F^{τ}, the social surplus she can expect meeting another bank of label and holding χ', $\frac{1}{2}\beta Z^{\tau}_{W^{\tau-1}}(\chi,\chi')$, and the search decisions of others $S^{\tau *}(-\chi)$ in equilibrium given the search technology $g(\theta)$.[14] The calling bank then needs to make a decision that maps from this space to a strategy mapping into $\Delta\mathbb{I}$. To simplify notation the strategy $S^{\tau}_{W^{\tau-1},F^{\tau},S^{\tau}(-\chi)}(\chi)$ maps into the unit

[14] θ is usually referred to as market tightness in labor economics. Here it reflects the ratio of potential trading partners of a label that can be called over the mass of banks calling this particular group.

interval where 1 means she only searches for m-labels, 0 she only searches for p-labels and anything in between is a mixed strategy. In aggregation, the short side will always be fully matched.

The market tightness calling banks of label m is given by

$$\theta^{\tau}_{m,W^{\tau-1},F^{\tau},S^{\tau*}(-\chi)} = \frac{\sum_{k\in\mathbb{K}}(1-y)\alpha_m p_m f^{\tau}_m(k)}{\sum_{i\in\mathbb{I}}\sum_{k\in\mathbb{K}} y p_i f^{\tau}_i(k) S^{\tau*}_{W^{\tau-1},F^{\tau},S^{\tau*}(-\chi)}(\chi)} \tag{14}$$
$$= \frac{(1-y)\alpha_m p_m}{\sum_{i\in\mathbb{I}}\sum_{k\in\mathbb{K}} y p_i f^{\tau}_i(k) S^{\tau*}_{W^{\tau-1},F^{\tau},S^{\tau*}(-\chi)}(\chi)} = \frac{\theta_m}{\theta^{\theta}_{m,S}}.$$

Note that the numerator is defined by the sum of banks not calling (1-y) and attending (α_m) times the frequency of banks of label m and k available. The denominator gives the mass of banks calling banks of label m. It is defined by the probability to be a caller (y), the frequency of each label (p_i), holding ($f^{\tau}_i(k)$) and their effective equilibrium search decision ($S^{\tau*}_{W^{\tau-1},F^{\tau},S^{\tau}(-\chi)}(\chi)$). As there are more callers than potential m-bank candidates to call $\theta^{\tau}_{m,W^{\tau-1},F^{\tau},S^{\tau*}(-\chi)}$ goes to 0. Similarly, if nobody decides to search for m-banks θ goes to infinity.

Similarly, the market tightness for calling label p is given by

$$\theta^{\tau}_{p,W^{\tau-1},F^{\tau},S^{\tau*}(-\chi)} = \frac{\sum_{k\in\mathbb{K}}(1-y)\alpha_p p_p f^{\tau}_p(k)}{\sum_{i\in\mathbb{I}}\sum_{k\in\mathbb{K}} y p_i f^{\tau}_i(k)(1-S^{\tau*}_{W^{\tau-1},F^{\tau},S^{\tau*}(-\chi)}(\chi))} \tag{15}$$
$$= \frac{(1-y)\alpha_p p_p}{\sum_{i\in\mathbb{I}}\sum_{k\in\mathbb{K}} y p_i f^{\tau}_i(k)(1-S^{\tau*}_{W^{\tau-1},F^{\tau},S^{\tau*}(-\chi)}(\chi))} = \frac{\theta_p}{\theta^{\theta}_{p,S}}.$$

Let g denote the function that a calling bank is matched with the functional from

$$g(\theta) = \tag{16}$$

Note that $g(\theta) \to 0$ as $\theta \to 0^+$ and The problem a calling bank of type χ faces is to maximize her expected utility by choosing her search direction among the two groups given the (optimal) search decisions of others. In particular, $S^{\tau}_{W^{\tau-1},F^{\tau},S^{\tau*}(-\chi)}(\chi)$ is the solution to the following problem:

$$S^{\tau *}_{W^{\tau-1},F^{\tau},S^{\tau *}(-\chi)}(\chi)$$

$$= \underset{s\in[0,1]}{\operatorname{argmax}}\ s\Big\langle g(\theta^{\tau}_{m,W^{\tau-1},F^{\tau},S^{\tau *}(-\chi)})E[\ \frac{1}{2}\beta Z^{\tau *}_{W^{\tau-1}}(\chi,(k^{\tau\prime},m))|F^{\tau}]\Big\rangle$$

$$+(1-s)\Big\langle g(\theta^{\tau}_{p,W^{\tau-1},F^{\tau},S^{\tau *}(-\chi)})E[\ \frac{1}{2}\beta Z^{\tau *}_{W^{\tau-1}}(\chi,(k^{\tau\prime},p))|F^{\tau}]\Big\rangle$$

$$= \underset{s\in[0,1]}{\operatorname{argmax}}\ C^{\tau}_{W^{\tau-1},F^{\tau},S^{\tau *}(-\chi)}(\chi,s)$$

so that $C^{\tau}_{W^{\tau-1},F^{\tau}}(\chi,s)$ denotes the expected surplus of being a caller with search strategy s. $C^{\tau *}_{W^{\tau-1},F^{\tau}}(\chi)$ denotes the maximum surplus from trade a bank gets conditional on being a designated caller.

Note that in case of a mixed strategy $S^{\tau *}_{W^{\tau-1},F^{\tau},S^{\tau *}(-\chi)}(\chi)$ we observe indifference, or

$$g(\theta^{\tau}_{m,W^{\tau-1},F^{\tau},S^{\tau *}(-\chi)})E[\ \frac{1}{2}\beta Z^{\tau *}_{W^{\tau-1}}(\chi,(k^{\tau\prime},m))|F^{\tau}]$$
$$= g(\theta^{\tau}_{p,W^{\tau-1},F^{\tau},S^{\tau *}(-\chi)})E[\ \frac{1}{2}\beta Z^{\tau *}_{W^{\tau-1}}(\chi,(k^{\tau\prime},p))|F^{\tau}]$$

Now

$$S^{\tau *}_{W^{\tau-1},F^{\tau}}(\chi) = \begin{cases} 0 & \text{if } C^{\tau}_{W^{\tau-1},F^{\tau}}(\chi,p) > C^{\tau}_{W^{\tau-1},F^{\tau}}(\chi,m) \\ p_m & \text{if } C^{\tau}_{W^{\tau-1},F^{\tau}}(\chi,p) = C^{\tau}_{W^{\tau-1},F^{\tau}}(\chi,m) \\ 1 & \text{if } C^{\tau}_{W^{\tau-1},F^{\tau}}(\chi,p) < C^{\tau}_{W^{\tau-1},F^{\tau}}(\chi,m) \end{cases} \tag{17}$$

I can denote the S^{τ}_{i} the $(K+1)\times 1$ vector with elements $S^{\tau}_{W^{\tau-1},F^{\tau}}(\chi)$. Then $S^{\tau} = \left[S^{\tau\prime}_{p}\ S^{\tau\prime}_{m}\right]$, and the sequence is described by S which is in the set $\mathbb{S}$.

3.2.3 Transition Matrix

I denote with $l^{\tau}_{F^{\tau},S^{\tau}}(\chi,\chi')$ the probability that a bank of type χ' who is a designated caller calls a bank of type χ, a match is established and they start bargaining. In particular,

$$l^{\tau}_{F^{\tau},S^{\tau}}(\chi,\chi') = \alpha_{i'}\ f^{\tau}_{i'}(k')\ \{1 - S^{\tau}_{W^{\tau-1},F^{\tau}}(\chi)\} \text{ if } i' = p \tag{18}$$

$$l^{\tau}_{F^{\tau},S^{\tau}}(\chi,\chi') = \alpha_{i'}\ f^{\tau}_{i'}(k')\ S^{\tau}_{W^{\tau-1},F^{\tau}}(\chi) \text{ if } i' = m \tag{19}$$

where the matching eventual hinges on the technological parameter $a_{s'}$, the mass of agent χ' ($f^{\tau}_{i'}(k')$) and the search choice ($S^{\tau}_{W^{\tau-1},F^{\tau}}(\chi)$).

Now I need to define the mass of agents in each group i that is successfully called to bargain by a χ' type. For that denote by

$$n^{\tau}_{F^{\tau},S^{\tau}}(p,\chi') = \frac{1}{p_p(1-y)} y \ \alpha_p \ p_{i'} \ f^{\tau}_{i'}(k') \ \{1 - S^{\tau}_{W^{\tau-1},F^{\tau}}(\chi')\} \tag{20}$$

$$n^{\tau}_{F^{\tau},S^{\tau}}(m,\chi') = \frac{1}{p_m(1-y)} y \ \alpha_m \ p_{i'} \ f^{\tau}_{i'}(k') \ S^{\tau}_{W^{\tau-1},F^{\tau}}(\chi') \tag{21}$$

the probability of agents of type i contacted successfully by agents of type and holding χ'. In particular this is defined by the mass being assigned the caller role (y), the probability that such a match happens given technology (α_i), the mass of agents of type i' ($p_{i'}$) times the mass of agents with that label holding k' ($f^{\tau}_{i'}(k')$). The search decision is between 0 and 1 as described above. Note that all calls in a particular group need to be uniformly distributed among non-callers ($\frac{1}{p_i(1-y)}$).

The expected surplus from bargaining conditional on not being a caller is

$$D^{\tau}_{W^{\tau-1},F^{\tau},S^{\tau}}(\chi) = \tag{22}$$

$$\sum_{k' \in \mathbb{K}} \frac{1}{2} \beta \{ n^{\tau}_{F^{\tau},S^{\tau}}(i,(k',p)) Z^{\tau}_{W^{\tau-1}}(\chi,(k',p)) + n^{\tau}_{F^{\tau},S^{\tau}}(i,(k',m)) Z^{\tau}_{W^{\tau-1}}(\chi,(k',m)) \}$$

Therefore the expected utility at the beginning of round τ is

$$W^{\tau}_{F^{\tau}}(k^{\tau},i) = u(k^{\tau},i) + yC^{\tau}_{W^{\tau-1},F^{\tau}}(\chi) + (1-y)D^{\tau}_{W^{\tau-1},F^{\tau},S^{\tau}}(\chi) + \beta W^{\tau-1}_{F^{\tau-1}}(k^{\tau},i), \tag{23}$$

or the instantaneous return plus the probability of being a caller times the expected surplus from being a caller plus the probability of not being a caller times the expected surplus from being called plus the continuation value of current balance holdings.

Define by $q^{\tau}_{F^{\tau},S^{\tau},B^{\tau}}(i,k,l)$ the probability of a label i bank with holding k to have l in the next period. Naturally, $q^{\tau}_{F^{\tau},S^{\tau},B^{\tau}}(i,k,l) \geq 0$ and $\sum_{l \in \mathbb{K}} q^{\tau}_{F^{\tau},S^{\tau},B^{\tau}}(i,k,l) = 1$. Denote by Q^{τ}_i a KxK matrix with elements $q^{\tau}_{F^{\tau},S^{\tau},B^{\tau}}(i,k,l)$, i.e. the transition matrix for label i at time τ. I can expand this concept to a transition matrix $Q^{\tau} = [Q^{\tau}_p \ \mathbf{0}; \ \mathbf{0} \ Q^{\tau}_m]$ which is block diagonal. Denote by $Q = \{Q^\tau\}_{\tau \in \mathbb{T}}$. Note that the states p and m do not communicate. All elements are nonnegative and the rows sum to one.

Note that elements of Q^τ can be calculated in the following fashion: First, consider the elements of Q^τ_p $\forall\ l \neq k$

$$
\begin{aligned}
q^\tau_{F^\tau,S^\tau,B^\tau}(i,k,l) = & \sum_{k' \in \mathbb{K}} \sum_{i' \in \mathbb{I}} \sum_{b \in B^\tau_{W^{\tau-1}}(\chi,\chi')} \frac{y}{|B^\tau_{W^{\tau-1}}(\chi,\chi')|}\ l^\tau_{F^\tau,S^\tau}(\chi,\chi') 1\left[k-b=l\right] \\
& + \sum_{k' \in \mathbb{K}} \sum_{i' \in \mathbb{I}} \sum_{b \in B^\tau_{W^{\tau-1}}(\chi,\chi')} \frac{1-y}{|B^\tau_{W^{\tau-1}}(\chi,\chi')|} n^\tau_{F^\tau,S^\tau}(i,\chi') 1\left[k-b=l\right]
\end{aligned}
\tag{24}
$$

where the first row takes into consideration that a $\chi = (i,k)$ bank is being assigned the caller role (y) times the probability that she meets χ' successfully. Then I take a random draw over all elements of $B^\tau_{W^{\tau-1}}(\chi,\chi')$ with uniform weight. The current holding of k minus the loan needs to fit the future holding l. The second line sums up the effect of being called in a similar way.

Now

$$
q^\tau_{F^\tau,S^\tau,B^\tau}(i,k,k) = 1 - \sum_{l \in \mathbb{K}} q^\tau_{B^\tau,S^\tau,B^\tau}(i,k,l). \tag{25}
$$

Note that the transition matrix has two handy functions:

$$
F^{\tau-1\prime} = Q^{\tau\prime} F^{\tau\prime}\ \forall\ \tau > 0 \tag{26}
$$

describing the law of motion for the distribution of holdings where the prime indicates a transposed vector here. Similarly

$$
W^\tau = [u(K',p)\ u(K',m)]' + \beta Q^\tau W^{\tau-1}\ \forall\ \tau > 0 \tag{27}
$$

describes the law of motion for indirect utility in a concise manner.

Note that the first statement is basic accounting given search and transfer decisions and the second from the same notion of rational expectations.

3.3 Equilibrium

3.3.1 Definitions

Definition 1. *An* ***equilibrium*** *is described by the value function W^e, policy functions S^e, B^e, R^e, the Markov transition matrix sequence Q^e and the sequence of distribution E^e with a given initial distribution F^T. In particular,*

1. *$\forall\ b^{\tau e}_{W^{\tau-1}}(\chi,\chi') \in\ B^{\tau e}$ solve (10) given $W^{\tau-1e}$. $\forall\ r^{\tau e}_{W^{\tau-1}}(\chi,\chi') \in\ R^{\tau e}$ solve (12) respectively.*

2. *$S^{\tau e}_{Z^{\tau e},F^{\tau e}}(\chi)$ solves (17) given $W^{\tau-1e}$ and $F^{\tau e}$.*

3. *$q^{\tau e}_{F^{\tau e},S^{\tau e},B^{\tau e}}(i,k,l)$ can be derived using (24) and (25) given $F^{\tau e}$, $S^{\tau e}$ and $B^{\tau e}$.*

4. *$W^{\tau e}_{F^{\tau e}}(k,i)$ can be derived using (27) given $W^{\tau-1e}$ and $Q^{\tau e}$.*

5. *$F^{\tau-1e}$ can be derived using (26) given $Q^{\tau e}$ and $F^{\tau e}$.*

First I define an equilibrium where no agent intends to intermediate.

Definition 2. *Let W^e, S^e, B^e, R^e, Q^e and E^e satisfy the conditions in 1. When all agents displays a trading pattern that is independent off labels we observe a* ***no-coordinated-intermediation-equilibrium (NCIE)****.*

I define an coordinated-intermediation-equilibrium where members of one group deviate from their target level when meeting a member of the other group.

Definition 3. *Let W^e, S^e, B^e, R^e, Q^e and E^e satisfy the conditions in 1. When members of the m-labelled (p-labelled) group deviate from target only for members of the p-labelled (m-labelled) group we observe an* ***m-induced coordinated-intermediation-equilibrium (MICIE) (p-induced coordinated-intermediation-equilibrium (PICIE))****.*

3.3.2 Existence

Next, I propose existence of these equilibria.

Proposition 1. *An equilibrium consisting of the tupel $\{W^*, S^*, B^*, R^*, Q^*, E^*\}$ exists.*

A sketch of the proof can be found in the appendix.

Proposition 2. *NCIE and MICIE (PICIE) consisting of the tupel* $\{W^*, S^*, B^*, R^*, Q^*, E^*\}$ *exist.*

I construct a MICIE below and discuss robustness with respect to the environment and conjectures agents form. Similar considerations apply to PICIEa. In the robustness study I construct a NCIE.

The next section describes the routine and some empirical facts this process can describe.

4 Simulation Results

4.1 Description of Routine

Inspired by the proof I write a program following the sequence.

1. Decide on a proposition for $S^{\phi}(\chi) \in [0\ 1]\ \forall \tau \in \mathbb{T}, \chi \in \mathbb{K} \times \mathbb{I}$. Mixing like $S^{\phi}(\chi) = 1$ is a good guess to nudge beliefs to an m-equilibrium.

2. Provide a guess for E. Using $F^{\tau} = F^{T}\ \forall \tau \in \mathbb{T}$ is a good guess.

3. Calculate for $\tau = 1$ B^0, R^0. Q^0 and W^0 using W^0, F^{0p} and S^{0p}.

4. Redo the last step for $\tau = 2 : T$ and collect Q^{τ} in $\mathbf{Q}$.

5. Update the sequence E starting with F^T using $\mathbf{Q}$.

6. Redo until $\mathbf{Q}$ converges.

7. Verify and revise S going forward. Set τ=T and repeat the following two step program until τ=1:

   (a) Using C^{τ} verify S^{τ}. If nothing changes set $\tau = \tau - 1$ and repeat this step. If something changes go to the following step.

   (b) Update B, R, S, Q and W from period τ to T backwards, update E forwards and repeat until Q converges like above. Then go back to the step above.

4.2 Proposed Search, Identical Groups and All Above Water

Note that the current environment yields a different scenario than random search. Search here is directed by the differences among groups. The different (current) distributions of money holdings and different meeting technologies stick out as unchangeable facts. Added to these are conjectures of agents that can have an influence on future indirect utility, bargaining solutions and therefore (economy wide) distributions. This in turn influence search decisions and the argument closes as future search decisions influence future indirect utility. This not only drives the result but can result in multiple equilibria that are not robust with respect to out of equilibrium beliefs.

In particular, say everybody always searches for an m-bank and this is common knowledge. Then any unnecessary balances that are found when an m-bank and a p-bank meet can be sold with a higher probability to a third bank when the m-bank takes the balances with her. Similarly, when funds are missing for the combined minimum requirements it is socially optimal to give the necessary balances to the p-labelled bank as her probability to be contacted in the future is smaller. This information is reflected in $W^{\tau-1}$. Therefore m-banks will have a less concave indirect utility function making trade with banks of that label more attractive now. The bargaining solution will therefore let the p-group converge weakly while this is not necessarily true for the distribution of holdings of the m-group. Now going forward, banks will search in groups where they believe to find more people on the opposite side of their target level (excluding banks exactly on target). So future search conjectures realize. This implies equilibrium selection can take place based on beliefs. The initial fix of $S^{\phi}(\chi)$ allows me to create common beliefs that can lead the m-labelled group to become middlemen as described by MICIE. Keeping both groups identical ex ante in holding distribution in the beginning of the day and contacting rate ($\alpha_p = \alpha_m$) therefore can verify that a MICIE exists even under mild conditions and can solely be based on beliefs.

Note that up to now I have labelled one group middlemen, but no primitive necessarily makes a particular group the intermediation group. For the sake of ease in exposition, the group with the m-label is obviously used to demonstrate middlemen behaviour. But I could similarly apply the argument to the p-group. Later I look for sufficient conditions making one group the intermediation group naturally focussing on MICIEa.

Also, if all banks are above water (not in need of funds) in the beginning of the day, no bank above their target level has an incentive to trade or even to intermediate. This is not necessarily the case if a "small" fraction is under water. If two (otherwise identical) banks with sufficient funds meet they increase the social surplus by splitting the excess funds between them therefore maximizing the probability that those funds are eventually sold to a bank in need.

4.3 When Everybody Believes in Middlemen

For this simulation analysis I set K=2, so that $\mathbb{K} = \{0, 1, 2\}$. I let $\bar{k} = 1$, or all banks have a target level of 1. Trade takes place between 16:00 and 18:30 similar to Afonso and Lagos (2012b). Trading rounds are 75 minutes long so that T=2. I leave y (fraction that is deemed a caller) at 0.05 throughout. Note that Afonso and Lagos (2012b) calibrate their meeting probability so that 10.4 meets per bank happen in this window while the parameters here just allow for T*y*2=0.2 meets per bank on average. But this is just exemplary as T can always be increased arbitrarily. I set the discount rate at r=0.0001. The interest rate to get an emergency loan at the discount window is set to $r^p = 0.0596$ which mimics the US average of 2006. Loans need to be repaid at 16:30 the next day.

For a better exposition I allow only for a single term of trade for each meeting constellation. I only keep the smallest absolute amount of exchange that is socially optimal. Disregarding exchanges that are not necessary (say a bank with holding 1 meets a bank with holding 2 in the last round could trade 0 or 1 units) simplifies explanations without loosing fundamental patterns. I later allow for this in a robustness analysis and find no fundamental changes. In particular, the puzzle explained in this paper is only explained by PDS and not by US when I allow for arbitrary trades. Further, to induce MICIEa in the reference simulation I fix $S^{\phi}(\chi) = 1$ everywhere. A robustness analysis shows that for identical groups the belief alone can drive which equilibrium I end up with.

I set $\alpha_p = \alpha_m = 1$ for the reference simulation. I leave the groups to be identical in distribution in a reference simulation. In particular, they start the day as a random draw from a distribution assigning 0.25 mass to $k^2 \in \{0, 2\}$ and 0.5 so that $k^2 = 1$. This leaves 25% of all banks in need for funds to meet the reserve requirements. Further, agents do not receive instantaneous utility in this reference environment, or $u(k^{\tau}, i)$=0 always as $\underline{k}_i = 0, 0 \leq k_{0i} \leq 1 \ \forall \ i$ arbitrarily and $r^e = r^o = 0$. The current setup will serve as a reference scenario for the sufficiency conditions where I change meeting technology, ex ante distribution of holdings and instantaneous utility to single out MICIEa.

In order to contrast PDS I run similar simulations with US. I then compare the result from PDS with US where I let the size of the m-labelled group vary from 0.1 to 0.5. Note that as we let $p_m \to 2 \times y$ we approach complete matching if everybody searches for an m-bank.

In order to shorten an otherwise lengthy description I define the set H with elements $\{h\}$ to consist of tuples $\mathbb{I} \times \mathbb{I} \times \mathbb{T}$ describing all possible meeting constellations. In this two period economy I find $H = \{(p,p,2),(p,m,2),(m,m,2),(p,p,1),(p,m,1),(m,m,1)\}$ as I omit symmetric cases. Partitions of H will refer to bargaining matrices. E.g., I denote a partition of H associated with trading pattern A in an US environment with H^A_{US}, a partition with trading pattern B in a PDS environment by H^B_{PDS} and so fourth.

4.3.1 Policies

Undirected search (US) yields a no-coordinated-intermediation equilibrium (NCIE). When both groups cannot direct their search the policies evolving from the bargaining stage are simple: In every period I only observe one form of contract, namely a bank with holding 2 and a bank with holding 0 will have a meaningful exchange of funds resulting in both having 1 unit. All other constellations will result in a meeting without an exchange of funds. Table (1) displays the trading pattern explicitly so that $H^A_{US} = \{(p,p,2),(p,m,2),(m,m,2),(p,p,1),(p,m,1),(m,m,1)\}$.

$B^{\tau e}_{W^{\tau-1e}}(\chi,\chi')$		bank of label i'		
		$k^{\tau\prime}=0$	$k^{\tau\prime}=1$	$k^{\tau\prime}=2$
bank of label i	$k^\tau=0$	0	0	-1
	$k^\tau=1$	0	0	0
	$k^\tau=2$	1	0	0

Table 1: Bargaining pattern A

In the partially directed search (PDS) environment I observe an m-induced coordinated-intermediation equilibrium (MICIE). Allowing for explicit search does not change end of day considerations where only banks off their target trade with each other, so that $\{(p,p,1),(p,m,1),(m,m,1)\} \in H^A_{PDS}$. This in turn makes any bank that has $k^1 = 2$ ($k^1 = 0$) search among the subgroup i where $f^1_i(k^1 = 0)$ ($f^1_i(k^1 = 2)$) is largest. It turns out that the m-group has the most dispersed distribution in the last group as can be seen in the decisions made by both types of groups when they are off their target level and in the last round ($\tau = 1$, compare table (3)). But this in turn allows m-banks to make a trading opportunity in (p,m,2) that was meaningless before to a social optimal transaction: In the first round ($\tau = 2$) an m-bank with $k^2 = 1$ will exchange funds with a p-bank that is off target ($k^2 \in \{0,2\}$) so that the p-bank satisfies the minimum reserve requirements exactly $k^1 = 1$ even when this means the m-bank is now missing or has too much funds (compare table 2). This is a socially optimal solution as the whole group of m-banks disperses in distribution making it more attractive to search among this group later in the day. Therefore $(p,m,2) = H^B_{PDS}$.

A p-bank and an m-bank will trade according to pattern A with their own kind in the first round, so that $\{(p,p,2),(m,m,2)\} \in H^A_{PDS}$

$B^{2e}_{W^{1e}}(\chi,\chi')$		bank of label m		
		$k^{2\prime}=0$	$k^{2\prime}=1$	$k^{2\prime}=2$
bank of label p	$k^2=0$	0	-1	-1
	$k^2=1$	0	0	0
	$k^2=2$	1	1	0

Table 2: Bargaining pattern B

In the first round a p-bank that is off her reserve requirements ($k^\tau \in 0,2$) directs her search to m-banks (compare table (3)). Note that a chance of a trade is higher when a p-bank looks for an m-bank because of the trading pattern B: not only the complimentary position of the p-bank will trade with her but an m-bank that meets exactly her requirements ($k^2 = 1$) will also trade and give a p-bank exactly her requirements while she will be off target for now. All m-banks in the first round look for p-banks to exploit the exact same trading opportunities they would not find among themselves. On the other hand, a p-bank that has exactly 1 unit will not trade in either pattern. Therefore her social surplus from bargaining is always 0 and she searches undirectedly as she is indifferent.

		$\tau=2$	$\tau=1$
$S^{\tau e}_{Z^{\tau e},F^{\tau e}}((k^\tau,p))$	$k^\tau=0$	1	1
	$k^\tau=1$	p_m	p_m
	$k^\tau=2$	1	1
$S^{\tau e}_{Z^{\tau e},F^{\tau e}}((k^\tau,m))$	$k^\tau=0$	0	1
	$k^\tau=1$	0	p_m
	$k^\tau=2$	0	1

Table 3: Optimal search decisions

In other words, an m-banks probability of contacting a trading partner in the final round that allows her to be on target again is higher than the probability of the same event for a p-bank in a PDS environment.[15] This makes m-banks deviate from her target

[15]Note that a p-bank that meets exactly her requirements does not create any observable trading patterns. This is in line with observations in the fed fund market: A relatively low participation rate compared to the amount of banks that are required to have minimum reserves and a relatively high concentration of number and volume of loans on a few banks.

level when given the opportunity.

4.3.2 Holding Distributions

In both rounds the distribution of p-banks will shrink as they will only trade from $k^2 \in \{0, 2\}$ and $k^1 \in \{0, 2\}$ to $k^1 = 1$ and $k^0 = 1$ respectively. This can be seen in the top left panel of figure (1) where the variance of p-banks is marked with triangles pointing up for $\tau = 1$ ($\sigma_p^2(k^1)$) and triangles pointing down for $\tau = 0$ ($\sigma_p^2(k^0)$). Increasing the fraction of m-banks contracts the distribution of p-banks as relatively more m-banks are willing to deviate from $k^2 = 1$. M-banks diverge from the target level initially in the same figure ($\sigma_m^2(k^1)$ has circles). In the second round the distribution of holdings contracts as only $k^1 \in \{0, 2\}$ trade to $k^0 = 1$ ($\sigma_m^2(k^0)$ has squares on the lines). Note that this end of day contraction among m-banks is monotonically increasing in the concentration of p_m. As we approach $p_m = 0.1$ we approach complete matching for m-banks in the second round.

The top right panel shows the total fraction of banks that are above their requirements in the last period ($\tau = 0$) for PDS and US. The fraction of all banks above minimum reserve requirements in PDS (solid line) is above the one in US (dotted line). We see the coordination of intermediation here: a larger fraction of banks meets the requirements independent of the size of the group. This is due to two effects. The believe that m-banks will be contacted in the second round frees up funds from m-banks with $k^2 = 1$ in the first that go to banks that have little chance of meeting another bank again in the future. The second is that banks that are off target in the last round (m-banks) will be contacted with a higher frequency resulting in a coordinated effort to solve the social problem of intermediation. This effect increases as we approach (almost) complete matching.

The bottom panel of figure (1) displays the coordination role that middlemen play in large decentralized markets: before the end of the day they are willing to take on riskier positions. Therefore their overall distribution spreads compared to other participants in the market and less m-banks end up meeting the requirement (dotted line with squares) compared to p-banks (dotted line). I turn to the terms of trade for this service.

4.3.3 Terms of Trade

Naturally, m-banks need to be compensated for the risk they take for leaving their target position. Figure (2) displays all observable fed fund rates for different terms of trade in $\tau =$

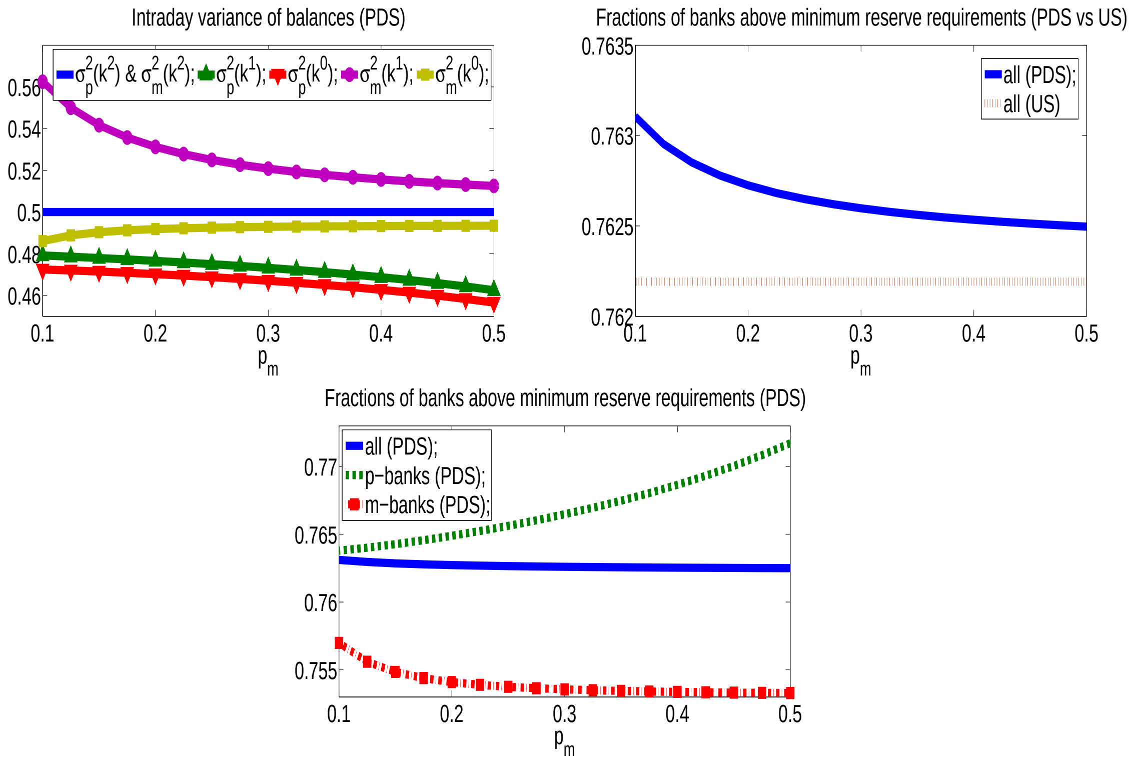

Figure 1: The top left panel shows how the variance of holdings behaves for both subgroups in UDS for each trading round for different concentrations of m-banks. Note that both groups have an identical distribution ex ante, therefore $\sigma_p^2(k^2) = \sigma_m^2(k^2)$. The top right panel shows the fraction of banks meeting minimum reserve requirements over different p_m in PDS and US. Note that m-banks and p-banks are identical ex post in the US environment as they are identical ex ante. The bottom panel shows the fraction of banks above minimum reserve requirements in the PDS environment only.

2.[16] The top left panel displays the interest rates when a p-bank switches from 0 to 1 while an m-bank switches from 1 to 0 (solid line) and from 2 to 1 (triangle pointing upwards). Note that the difference between the two lines is a the markup on intermediation services the m-bank provides by giving the p-bank one unit and taking the risk of being off target later.

The top right panel shows the rates where a p-bank can switch her position from 2 to 1, while her trading partner, an m-bank, switches from 0 to 1 (triangle pointing down) and from 1 to 2 (dotted line). The difference between the two lines is the markdown for the m-bank to go above her target. These intermediation markups and markdowns decrease in absolute terms as I increase the concentration (decrease p_m). The probability

[16]Trade in $\tau = 1$ only has one federal fund rate independent of the fractions of p-banks and m-banks as payoffs in the end of the day are certain.

for the m-bank to be matched in the last round goes to one and reduces her risk to remain off target.

Another markup and markdown an m-bank extracts from a p-bank is the difference between buying fed funds and selling them when both move towards their target levels. Graphically, it is the difference between the two lines with triangles in the bottom left panel of figure (2). Note that this label-markup increases in concentration as the m-banks outside option of meeting someone in the next round becomes higher.

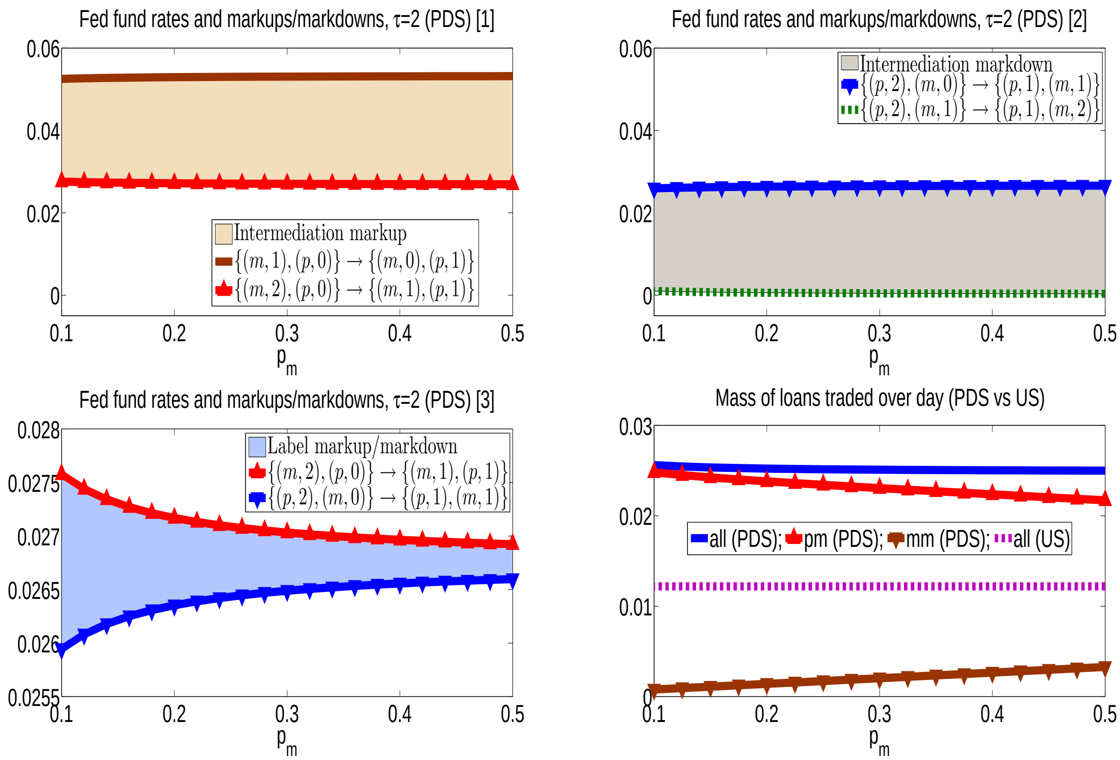

Figure 2: The top left, top right and bottom left panel show how different contracts vary as I change the concentration of m-banks (p_m). Note that the markups and markdowns are the shaded areas between the lines of the federal funds rates. The bottom right panel shows the mass of loans issued on the fed fund market for PDS and US for different p_m. Note that for PDS we also observe group-specific trade between p-banks and m-banks (pm) and between two m-banks (mm).

Another affect alluded to above can be seen in the bottom right panel of figure (2). The belief that m-banks are contacted later frees up balances for trade in PDS (solid line) compared to US (dotted line). This increases trade volume by an upward shift and is slightly increasing as I concentrate p_m. This can be reasoned by the complete matching as p_m approaches 0.1. Further, trade between m-banks (triangle pointing down) which only happens in the final round is substituted by trade between p-banks and m-banks (triangle pointing up) as the group of m-banks becomes smaller. There is no trade between two p-banks.[17]

4.3.4 Excess Fund Reallocation and Premium for Labels

As a final point of view I introduce figure (3). The top left panel sums up a measure of intermediation introduced by Afonso and Lagos (2012b). Let O^p_{at} be the cumulative dollar amount purchased as loans by a single bank a at time t. Similarly let O^s_{at} be the same measure for loans sold. The excess fund reallocation of a bank a is $X_a = \sum\limits_{t \in \mathbb{T}} |O^p_{at}| + |O^s_{at}| - |O^p_{at} - O^s_{at}|$. We obtain the proportional excess fund reallocation by dividing the sum of all banks excess fund reallocation by the total amount of funds bought and sold during the day, or $\iota = \frac{\sum X_a}{\sum\limits_{t \in \mathbb{T}} |O^p_{at}| + |O^s_{at}|}$. This measure is increasing in the concentration of m-banks (decrease of p_m) in PDS (solid line), and zero for US throughout (dotted line). This simply derives from the bargaining pattern A and B in respective environments, and is solely driven by m-banks in the PDS environment.

The top right hand side of figure (3) shows the average return of an average bank in a PDS environment (triangles pointing up) and the average return of being a bank in an US environment (triangles pointing down). It is strictly above the average return in an US environment and increases in the concentration of m-banks.

The bottom panel focusses on the subgroups in PDS. An average m-bank (solid line) always exceeds the return of an average bank in a PDS environment. An average p-bank (dotted line) is always below. In fact, her expected return is below that of an average bank in the US environment. Note that while the differences of welfare is in a relatively stable proportion across different concentration levels an individual m-banks profit increases in the concentration of p_m sharply.

4.4 Equilibrium Robustness

In this section I look at the robustness of the equilibria computed above. Two things were crucial for the computation: the proposed search direction and the restriction of Nash-bargaining solutions to the smallest absolute loan.

[17] It is not clear how the change of middlemen group size would influence the distribution of fed funds rates. E.g., a higher concentration of middlemen decreases intermediation markups/markdowns (so it reduces the span of all possible fed funds rates) but it also increases the trade volume between p-banks and m-banks which pools from this set.

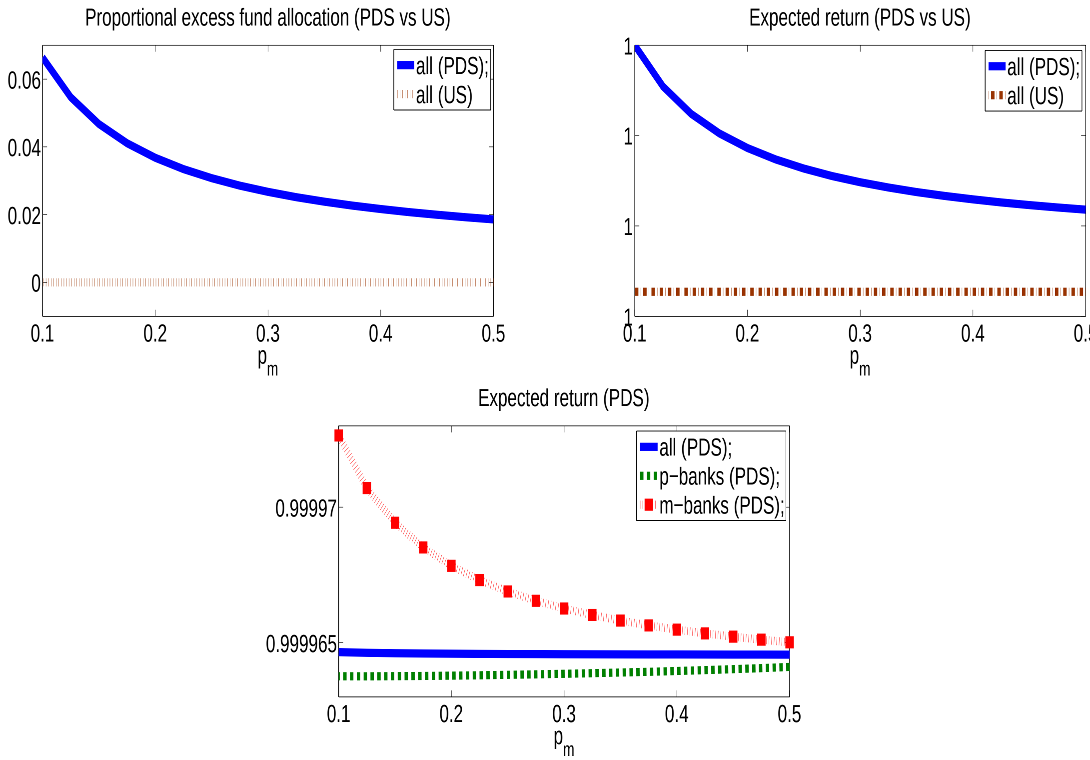

Figure 3: The top left panel shows the proportional excess fund reallocation vary as I increase the concentration of m-banks (decrease p_m). The top right panel shows the average expected return when I vary the concentration of m-banks for both PDS and US. The bottom panel shows the average expected return for different groups of banks in a PDS environment.

4.4.1 Robustness I: Trades under Indifference

I restricted the terms of trade to the smallest absolute transaction among socially optimal Nash solutions. The motivation had pure expositional purposes. But trades that I do not deem necessary could still be executed and change empirical patterns. I will contrast PDS with bargaining restrictions like in the reference simulation with an US version without this restriction on a larger scale. The analysis here develops the intuition for this relaxation.

The main changes can be found in the bargaining patterns displayed in table (4) and (5). Again, US leads all encounters to trade as displayed in table (4), or $H_{US}^{A+} = \{(p,p,2),(p,m,2),(m,m,2),(p,p,1),(p,m,1),(m,m,1)\}$. Note that this is a NCIE again.

Allowing for PDS does not alter search nor bargaining when compared to the US-unconstrained search model except in a meeting between a p-label and an m-label in the first round. Here the bargaining pattern is captured in table (5) which is identical to the above bargaining pattern (2). Therefore, $H^{A+}_{PDS} = \{(p,p,2),(m,m,2),(p,p,1),(p,m,1),(m,m,1)\}$ and $H^{B+}_{PDS} = (p,m,2)$. This is a MICIE.

The only difference to the reference simulation is that not-welfare improving trades happen, e.g. a bank with k=1 will sell her balances to a bank who has k=0 with probability 0.5 unless the first bank has a p-label, the second has an m-label and they are in the first round. In the latter case it is socially optimal that the m-bank carries the risk of being off target. Note that this change to pattern (4) allows for a p-bank to meet another p-bank holding 0 in the first round and trade from $k^2 = 1$ to $k^1 = 0$ and then again in the second round meet a bank holding 1 and trade from $k^1 = 0$ to $k^0 = 1$. This looks like she intermediated but does not change end of day reserve requirement fulfilment rates.

$B^{\tau e}_{W^{\tau-1e}}(\chi,\chi')$		bank of label i'		
		$k^{\tau\prime} = 0$	$k^{\tau\prime} = 1$	$k^{\tau\prime} = 2$
bank of label i	$k^\tau = 0$	0	[-1 0]	-1
	$k^\tau = 1$	[0 1]	0	[-1 0]
	$k^\tau = 2$	1	[0 1]	0

Table 4: Bargaining pattern A^+

$B^{2e}_{W^{1e}}(\chi,\chi')$		bank of label m		
		$k^{2\prime} = 0$	$k^{2\prime} = 1$	$k^{2\prime} = 2$
bank of label p	$k^2 = 0$	0	-1	-1
	$k^2 = 1$	0	0	0
	$k^2 = 2$	1	1	0

Table 5: Bargaining pattern B^+

The empirical effects can be summarized easily: Distributions of balances do not change as either agents within a label change position or an exchange between both groups is balanced by an equally likely exchange in the other direction. Therefore it does not change end of day holdings of groups as well.

The change of Nash-bargaining increases the mass of loans traded significantly. The top left panel of figure (4) summarizes the change and can be directly compared to the bottom right panel of (2). Note that I observe trade between p-labels who are indifferent in their search direction and therefore mix US-like in the first round. When these banks meet another p-bank with $k \in \{0, 2\}$ they will trade half the time.

The exact federal fund rate for a p-bank going below her target in $\tau = 2$ can be found in the top right panel of figure (4). Note that a p-bank receives an additional risk markup when deviating from her target compared to a similar trade of an m-bank. A p-bank has a smaller chance to meet someone in the final round. Note that the risk markup only changes because the fed fund rate of an m-bank going above her target changes.

Similarly, a p-bank receives a risk markdown when she accrues more funds than she eventually needs as displayed in the bottom panel of figure (4). The fed funds rates an m-bank faces trading with p-banks do not change compared to the reference scenario. Trade in period $\tau = 1$ now also includes not-welfare improving exchanges and there will not be one unique fed fund rate as in the reference simulation.

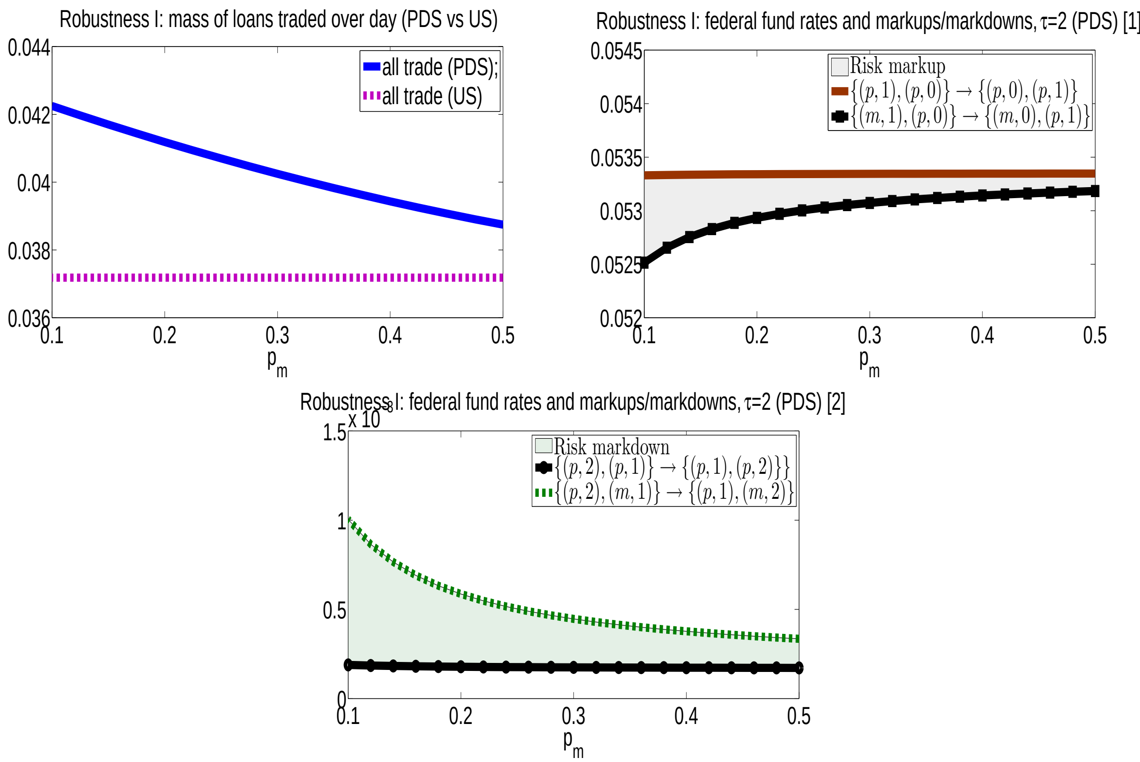

Figure 4: The top left panel shows the mass of loans issued on the fed fund market when I increase the concentration of m-banks allowing for trades under indifference. The top right and the bottom panel show interest rates p-banks and m-banks obtain for leaving their target level.

The change of behaviour over bargaining also allows for some p-banks to become intermediaries by chance which can be seen in the left panel of figure (5). Note that the amount of proportional excess fund reallocation was zero in the reference simulation above with US. The change of bargaining behaviour also increases the proportional excess fund reallocation in the PDS case for the same reason.

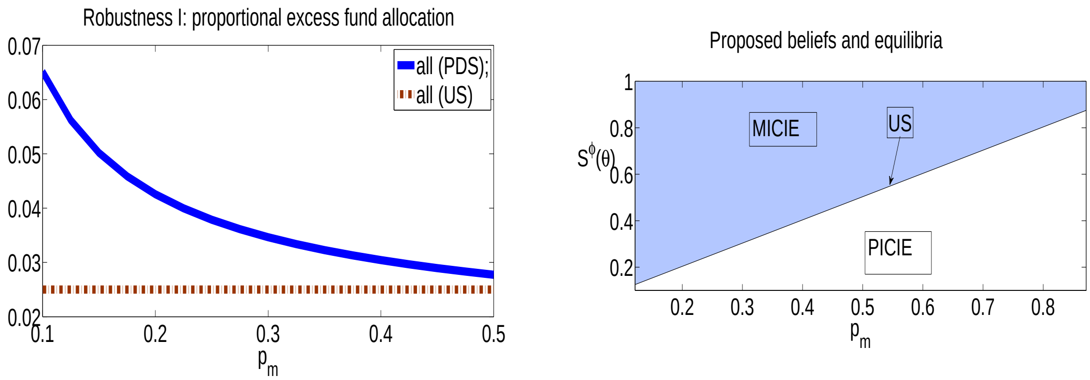

Figure 5: The left panel shows the proportional excess fund reallocation as I vary the concentration of m-banks when I allow for trades under indifference in PDS and US. The right panel shows convergence to equilibria for starting values of $S^{\phi}(\chi)$ as I vary p_m.

4.4.2 Robustness II: Out of Equilibrium Beliefs and Belief-Robustness Conditions

The computational procedure relied on a "proposed search direction towards the m-group" ($S^{\phi}(\chi) = 1$) to induce common beliefs. These search directions are later verified or modified and lead us to the expected result that members of the m-group became middlemen (MICIE). In other words, the observational equivalence of the directed search model (where all banks search as if indifferent) is not robust to the out of equilibrium belief where all banks belief the m-group is the more attractive group to search for. Therefore, it might be interesting to see how robust this result is when the agents initially believe less in the m-banks being middlemen.

The right panel of figure (5) plots $S^{\phi}(\chi)$ and p_m mapping into the space of equilibria as described by definition (1). In fact, all points $S^{\phi}(\chi) > p_m$ (the white area) lead to a MICIE where $H^A_{PDS} = \{(p,p,2),(m,m,2),(p,p,1),(p,m,1),(m,m,1)\}$ and $H^B_{PDS} = (p,m,2)$ similar to the reference model. Only points that are exactly corresponding to the size of the m-group lead to a behaviour that is observationally equivalent to US (the black line) where $H^A_{PDS} = \{(p,p,2),(p,m,2),(m,m,2),(p,p,1),(p,m,1),(m,m,1)\}$. Notice further that a proposed search direction below the size of the m-group actually lead to the p-group becoming middlemen (PICIE, the red area) where $H^A_{PDS} = \{(p,p,2),(m,m,2),(p,p,1),(p,m,1),(m,m,1)\}$ and $H^C_{PDS} = (p,m,2)$ (compare table (6)). This result is independent of the relative size of the m-group and displays the frailty of belief driven results.

This leaves open the question about what sufficiency condition such an economy must

$B^{2e}_{W^{1e}}(\chi,\chi')$		bank of label m		
		$k^{2\prime}=0$	$k^{2\prime}=1$	$k^{2\prime}=2$
bank of label p	$k^2=0$	0	0	-1
	$k^2=1$	1	0	-1
	$k^2=2$	1	0	0

Table 6: Bargaining pattern C

inherit so that a certain group can obtain the role of the middlemen for all types of initial beliefs. The next couple of sections introduce these conditions and reason their economic plausibility. It is supplemented with simulation-based results where the proposed belief is set to $S^{\phi}(\chi) = 0$. Further, middlemen behaviour is often associated with big banks.[18] It is therefore interesting to link the size of the bank with the condition that ensures that one group becomes a middlemen group.

Meeting Technology

One critical aspect of becoming a middlemen bank is the opportunity not only to search but also to be found more often in relative terms. Since α_i is in the unit interval I can similarly decrease the meeting technology of the other group and raise y and/or T.

Anecdotal evidence with a monitoring agent of the Swiss National Bank suggests that bigger banks have dedicated money desks while smaller banks do not for cost reasons. In the latter it is usually an employee whose main task is not to manage the fund market but who dedicates some time in the late afternoon to correct the banks balance position. This might only take a couple of minutes and he reverts his attention to his main duty. The remainder of the day he might be unavailable for federal funds trades which reduces the probability to be reached, α_p.

Figure (6) displays the result. The white area leads to a MICIE in line with the bargaining patters (1) and (2) described above, or $H^A_{PDS} = \{(p, p, 2), (m, m, 2), (p, p, 1), (p, m, 1), (m, m, 1)\}$ and $H^B_{PDS} = (p, m, 2)$. Search decisions of the p-banks are mostly in line with the reference simulation. Only m-banks that are off their target level now search for other m-banks in the initial period (compare table (7)). This can be reasoned by the lower prospect to be matched with a p-bank.

The red area on the other hand leads to a PICIE with $H^A_{PDS} = \{(p, p, 2), (m, m, 2),$

[18] The economic collapse in 2008 was widely associated with the lack of trust the market put into big banks and their intermediating role on the fund market.

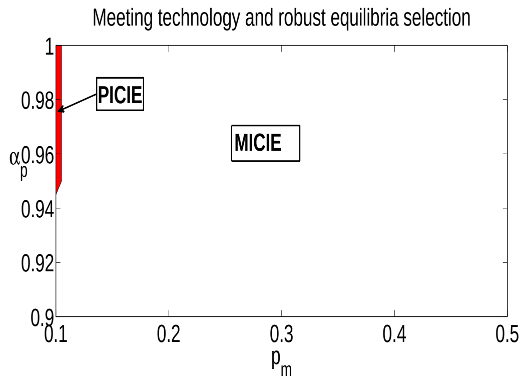

Figure 6: The figure shows how different equilibria evolve as the meeting probability of p-banks and the ratio of group sizes vary.

		MICIE			PICIE	
		$\tau = 2$	$\tau = 1$		$\tau = 2$	$\tau = 1$
$S^{\tau e}_{Z^{\tau e},F^{\tau e}}((k^\tau, p))$	$k^\tau = 0$	1	1		1	0
	$k^\tau = 1$	p_m	p_m		1	p_m
	$k^\tau = 2$	1	1		1	0
$S^{\tau e}_{Z^{\tau e},F^{\tau e}}((k^\tau, m))$	$k^\tau = 0$	1	1		1	0
	$k^\tau = 1$	0	p_m		p_m	p_m
	$k^\tau = 2$	1	1		1	0

Table 7: Optimal search decisions when I alter α_p

$(p, p, 1), (p, m, 1), (m, m, 1)\}$ and $H^C_{PDS} = (p, m, 2)$. These PICIEa are striking in one aspect: search is initially directed towards m-banks yet the p-banks take on the role of the middlemen and deviate from their target.

Liquidity Shocks

An essential feature is that search directions depend on the relative weight of the tails of the distribution of both groups. I therefore look at the distribution and how it affects the robustness of an m-equilibrium.

Furfine (1999) documents the difference of big banks and small banks in the 1998 Q1 federal fund market. He finds that the participation rate (defined as having at least one trade in a day) is highest among banks in the highest asset class and monotonically decreases for lower classes. This points to the idea that bigger banks are more likely to be middlemen in the fund market. They also also have the highest net changes. It is therefore reasonable to assume that the middlemen group has a more dispersed distribution at the beginning of the day. I increase the mass of m-banks with $k^2 = 0$ and $k^2 = 2$ and decrease the mass of m-banks with $k^2 = 1$ respectively before they start to trade and the ex ante distribution of p-banks remains identical to the reference scenario.

Figure (7) summarizes the equilibria results. The white area leads to stable MICIEa where $H^A_{PDS} = \{(p,p,2),(m,m,2),(p,p,1),(p,m,1),(m,m,1)\}$ and $H^B_{PDS} = (p,m,2)$ while the red area does not lead to the desired result of the m-bank being the intermediator, or we remain in PICIEa. Trading follows the same pattern as above with $H^A_{PDS} = \{(p,p,2),(m,m,2),(p,p,1),(p,m,1),(m,m,1)\}$ and $H^C_{PDS} = (p,m,2)$. Search leads to expected results (compare table (8)).

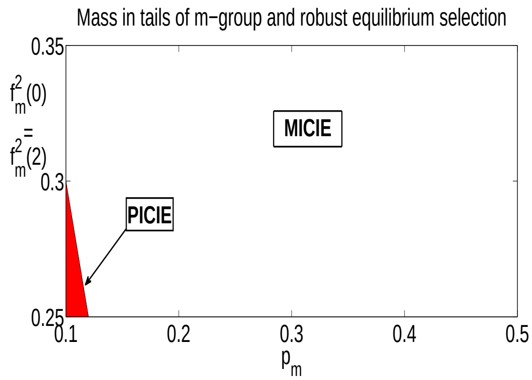

Figure 7: The figure shows how different equilibria evolve as I vary the liquidity shocks of the m-bank and the relative group sizes vary.

		MICIE			PICIE	
		$\tau = 2$	$\tau = 1$		$\tau = 2$	$\tau = 1$
$S^{\tau e}_{Z^{\tau e},F^{\tau e}}((k^\tau, p))$	$k^\tau = 0$	1	1		1	0
	$k^\tau = 1$	p_m	p_m		1	p_m
	$k^\tau = 2$	1	1		1	0
$S^{\tau e}_{Z^{\tau e},F^{\tau e}}((k^\tau, m))$	$k^\tau = 0$	1	1		1	0
	$k^\tau = 1$	0	p_m		p_m	p_m
	$k^\tau = 2$	1	1		1	0

Table 8: Optimal search decisions when I alter $f^2_m(0) = f^2_m(2)$

Intraperiod Utility

Banks can incur daylight overdrafts. This capacity is limited by the net debit cap which is equal to the capital measure times the cap multiple.[19] Holding overdrafts above

[19]The capital measure is equivalent to supervisory capital as defined by the capital adequacy guidelines

this limit incurs overdraft fees. As banks have different capital measures and the model proposed depends on the ability to hold overdrafts this is another candidate measure to pin down the group of intermediators. Naturally, small banks with less assets in their portfolio have a smaller capital measure.

On the other hand, banks that have excess reserves might be able to wire these funds to investment opportunities at their disposal even while the federal fund market freezes up non-fed funds trading in the United States. International opportunities can provide higher return vehicles. Big banks with branches abroad can take advantage of these opportunities which change their intraday utility derived from excess balances compared to the intraday utility of smaller local banks.

I therefore change $\underline{k}_p = 1$ which implies that p-banks pay overdraft fees for holding less than 1. This in turn means that their minimum reserve requirements coincides with their overdraft limits but it simplifies the exposition. Further, only m-banks get return on excess balances. So I set $r^e_m = r^o_m = r^o_p$ and $r^e_p = 0$. Note that in this setup only p-banks with k=0 pay the fee and only m-banks with k=2 receive a return. I keep $S^\phi(\chi) = 0$ as before.

Figure (8) summarizes the equilibria results. Note that only small storage fees and return (below half a percentage point for most ratios of bank groups) make a MICIE belief-stable. I find the usual collection of bargaining patterns, namely that $H^A_{PDS} = \{(p,p,2),(m,m,2),(p,p,1),(p,m,1),(m,m,1)\}$ and $H^B_{PDS} = (p,m,2)$. The search decisions are found in table (9).

The red area remain PICIEa as induced by the proposed search decision where $H^A_{PDS} = \{(p,p,2),(m,m,2),(p,p,1),(p,m,1),(m,m,1)\}$ and $H^C_{PDS} = (p,m,2)$.

		MICIE		PICIE	
		$\tau=2$	$\tau=1$	$\tau=2$	$\tau=1$
$S^{\tau e}_{Z^{\tau e},F^{\tau e}}((k^\tau,p))$	$k^\tau=0$	1	1	1	0
	$k^\tau=1$	p_m	p_m	1	p_m
	$k^\tau=2$	1	1	1	0
$S^{\tau e}_{Z^{\tau e},F^{\tau e}}((k^\tau,m))$	$k^\tau=0$	0	1	0	0
	$k^\tau=1$	0	p_m	p_m	p_m
	$k^\tau=2$	0	1	0	0

Table 9: Optimal search decisions when I alter $f^2_m(0) = f^2_m(2)$

by the federal financial regulatory agencies. The cap multiple derives from assessments of the individual banks.

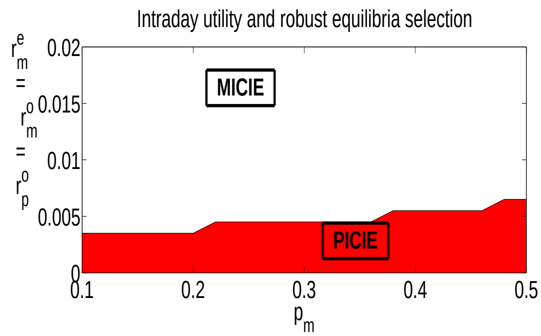

Figure 8: The figure shows how different equilibria evolve as I vary the level of fees for overdrafts of the p-banks and returns for excess holdings for m-banks and the ratio of group sizes.

Other Measures and Empirical Validity

All above measures are sufficient to pin down a MICIE that is belief-robust. Nonetheless, some parameter areas seem not robust so that we end up with PICIEa.

Several other measures that could serve to pin down MICIEa are reasonable. E.g.,small banks might suffer from a lack of public exposure when trying to lend money while big banks overcome this problem by their prominence. These asymmetric information leads to risk premia paid by small banks as overnight loans are usually not collateralized in the fed funds market. This leads to a different form of bargaining as Nash solution implies a full transfer of information. A second measure not exposed here is relationship banking. In this line of thought a big bank can overcome the asymmetric information for a (sub)group of small banks as she deals with the same small banks over and over again. Network models appear a reasonable approach but under mild assumptions the model proposed here is still applicable if the distributions of the subgroups of banks remains homogeneous.

Given the lack of data no measure can be validated easily. It might even be a range of measures that allow one group to be middlemen. Another line of thought must address that heterogeneous distribution over these measures can make a certain group middlemen.

4.5 Large Scale Simulation

In this section I contrast PDS with the US version a la Afonso and Lagos (2012b) for an environment replicating the federal fund market in 2006.

I scale the model up and set K=6. This is meant to milden the effect of constraint behaviour.[20] Further, I set $\bar{k} = 3$ and reinterpret $k_{0m} = k_{0p} = 2$ as a zero holding position. With some arbitrariness I define a mass point distribution [0 0.05 0.20 0.51 0.14 0.1 0] as the mass of holdings for $\mathbb{K} = \{0, 1, 2, 3, 4, 5, 6\}$ respectively when trading starts in $\tau = T$ for both subgroups. Note that this is in line with Afonso and Lagos (2012a) empirical findings that roughly 25% of all banks have nonpositive holdings at the beginning of the trading period before 2007 and that excess holding was about 4% across banks as documented by the Federal Reserves H3 Historical Table. Primary Credit Rate is set to 5.96, the yearly average of 2006. I set r=4.66 which is the return of a 4 week treasury bill in that period, or a riskfree alternative to invest any excess reserves.

To mark the physical differences between middlemen banks and peripheral banks I set $r^e_m = r^o_m = r^o_p = .0001$, $r^e_p = 0$, $\underline{k}_p = 2$ and $\underline{k}_m = 0$ to reflect the considerations above about international investment opportunities of big banks and lower capital measures of smaller banks. Further, I also let $\alpha_p = 0.5$ and $\alpha_p = 1$ to reflect the difference of dedicated personnel to the money desk. I propose $S^\phi(\chi) = 0$ as beliefs.

Trading takes place between 16:00 and 18:30 similar to Afonso and Lagos (2012b). I fix the number of rounds to 10, y=0.1 and $p_m = 0.2$.[21] Loans need to be repaid at 16:30 the next day.

The US model is not restricted in the size of the solutions to the bargaining problem. This allows for trades that might appear non-welfare improving and could contribute to the off-target behaviour described as the main empirical puzzle. In other words, I allow for two banks with holdings $k^\tau = 4$ and $k^{\tau\prime} = 4$ to change to $k^\tau = 3$ and $k^{\tau\prime} = 5$ if it is deemed socially optimal. This allows for some form of intermediation in the tails by indifference as described in the first robustness analysis. In the PDS model I only allow for the smallest sized loan to be a solution to the Nash bargaining. Note that here the trade from ($k^\tau = 4$, $k^{\tau\prime} = 4$) to ($k^\tau = 3$, $k^{\tau\prime} = 5$) can still occur but not between two banks of the same group.

[20] Banks with k=2 were physically unable to buy funds so far and I are primarily interested in their behaviour. A larger increase increases computational time immensely while this setup still allows me to solve the puzzle.

[21] This means there are 15 minutes per round and I allow for an average of 2 meets per bank

4.5.1 Intermediation in Tails

The empirical puzzle that US cannot reproduce is replicated in figures (9) by PDS. Some m-banks that are in the bottom 10-percentile sell loans to p-banks and move away from their own target level. Some m-banks in the top 10-percentile still purchase more funds from p-banks and move away from their target level. US with unconstrained bargaining cannot replicate this even though the subgroups are heterogeneous in parameters as in PDS. It is solely the search decision that increases the likelihood to meet another trading partner in the future. It allows the m-banks to trade away from their target level even though they already are far away. Note that the kink in the US-line at the end of the day comes from trading under indifference in the last round similar to the first robustness analysis above.

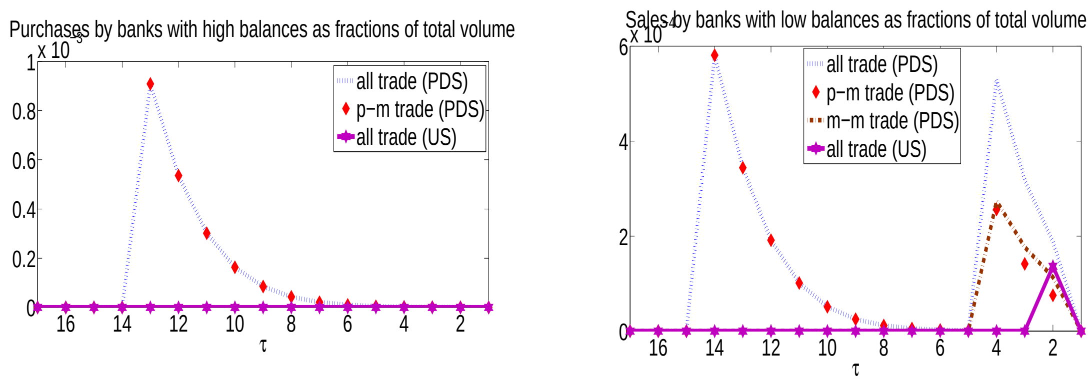

Figure 9: The left panel shows the volume of funds bought by banks that hold balances in the top 10-percentile as a fraction of total volume. The right panel shows the volume of funds sold by banks that hold balances in the bottom 10-percentile as a fraction of total volume.

4.5.2 Stylized Facts and Empirical Moments

The PDS model is not calibrated and rather small because of her computational demands. Nonetheless, a comparative analysis between PDS and US seems interesting.

Because of the unit mass of banks and the normalization along funds sizes I normalize the moments whenever possible. The first moment is the average loan size divided by the total volume traded by all banks which in turn is divided by the number of banks derived from the empirical study of Afonso and Lagos (2012a). The second is the ratio of funds borrowed by commercial banks from the Discount Window of the Federal Reserve over excess balances derived from the Federal Reserves online database FRED. Afonso and Lagos give statistics about median and mean number of trading partners and the proportional excess fund reallocation as described above. Last, the weighted federal funds rate is the FOMCs target rate. All empirical moments refer to 2006.

PDS matches some moments rather well or at least better than US as presented in table (10). The US moments are always further away than the PDS moments if they are on the same side. In one case where they are on opposite sides of the empirical moment PDS is closer in absolute terms. Overall, it appears that the current model does not allow for enough trading rounds.

	Empirical moments	PDS	US
Mean loan size / mean trade volume	0.16	3.05	6.01
Volume borrowed / excess balances	0.13	4.18	5.71
Median # trading partners	2	1	0
Mean # trading partners	6.19	1.06	0.44
Proportional excess fund reallocation	0.40	0.52	0.19
Weighted Fed Funds rate	4.96	4.91	4.89

Table 10: Some empirical and computed moments.

5 Heterogeneous Distributions Among Groups

5.1 Another Small Scale Simulation

The second puzzle Afonso and Lagos (2012b) find using US is that the theoretical variance of the federal funds rate is larger than its empirical counterpart. They reason that too many meets involve banks on the same side of their target level where each trade involves an intermediation markup (markdown) and a fraction of a risk markup (markdown).[22] In the reference simulations above the risk premium is lost by coordinated intermediation which in turn reduces the variance of federal funds rates (compare figure (10)). While associating the labels with a certain market role labels are fundamentally just assigning members to groups so that expectations can be formed.

Another approach to this dispersion problem is to acknowledge that the distributions of holdings for groups with public membership can be heterogeneous, and search is directed towards the group that is known to be on the opposite side of the target level. Note

[22]Remember that in their model no bank has a higher probability to meet another bank in the future. Therefore the total sum of markups (markdowns) is above (below) the intermediation markup (markdown) but below (above) the sum of intermediation and risk markup (markdown).

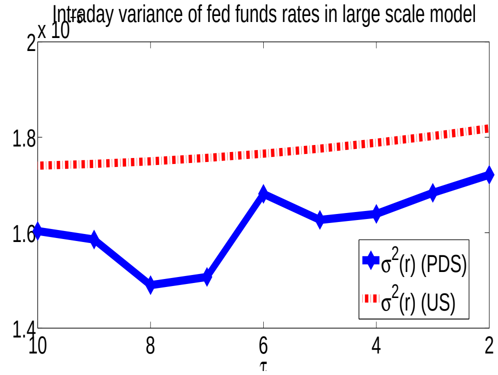

Figure 10: The figure shows the intraday variance of federal funds rates in the large scale model from section (4.5).

that this does not imply there is any form of intermediation and it should be looked at independent of the coordinated intermediation as described above. It is simply another application of PDS.

I therefore change the small scale simulation in the following way: the fraction of groups remain the same throughout ($p_m = p_p = 0.5$) while I let the p-group increase the ex ante mass of $k^2 = 2$ while I reduce the mass of $k^2 = 0$ leaving $k^2 = 1$ unchanged. The figure (11) incorporates this change along the horizontal axis. The left corner starts similar to the reference scenario ($F_p^\tau = [0.25\ 0.5\ 0.25]$) and ends at the right corner with ($F_m^\tau = [0\ 0.5\ 0.5]$). At the same time I change m-banks to be in need of funds in the similar fashion. Everything else remains as it was.

Figure (11) shows that the variance of federal funds rates decreases as I let more heterogeneous groups search rationally. Banks that are in need of funds search among p-banks while banks with excess reserves search for m-banks to sell their funds. The kink comes as agents in the furthest left corner search undirectedly and search with gusto as F_p^τ and F_m^τ change.

5.2 Big Banks and Small Banks

Furfine (1999) documents the statistical differences between big banks and small banks. He categorizes them according to their asset classes and confirms that big banks on average are net buyers while small banks are on average net sellers but hardly participate in the first place on a given day. He also finds that large banks have a larger number of counterparties. This public knowledge that is not only linked to intermediation but also to beginning-of-day distributions could solve the second theoretical puzzle Afonso and

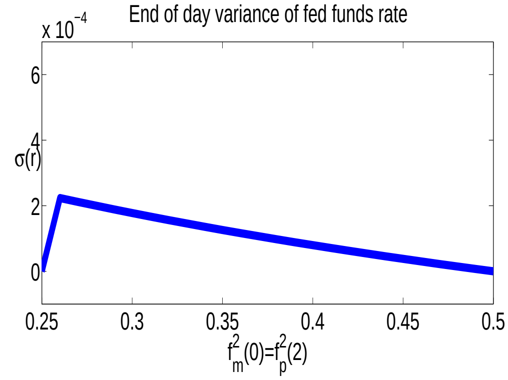

Figure 11: The figure shows how a change in the distribution can affects the end of day variance of the federal fund rate.

Lagos (2012b) encounter in their undirected search model.

I take a similar setup as above in the large scale simulation but I take guidance by Furfine (1999) to construct two different distributions for p-banks (taking the place of small banks) and m-banks (representing the big banks). I take his five categories and aggregate them in two groups in table (5.2).[23] I redefine k=4 as being exactly on target across any bank group and interpret k=2 as having zero funds. This leaves us roughly with [0.1 0 0 0 0.42 0.1975 0.2825] ([0.235 0.235 0 0 0.12 0.225 0.185]) as the mass of holdings for $\mathbb{K} = \{0, 1, 2, 3, 4, 5, 6\}$ for p-banks (m-banks). Note that we have 4% excess reserves with this distributions as before.

Label	Asset size of institution ($ million)	Frequency of being a net seller of funds	Frequency of being a net buyer of funds	Frequency of having no net changes in balances
p	0-10000	0.48	0.10	0.42
m	above 10000	0.41	0.47	0.12

Table 11: Aggregated data from FURFINE (1999), table 3.

Figure (12) shows the intraday variance of interest rates. Allowing the distributions to be heterogeneous and using PDS reduces the variance and can suggest which direction search theoretic methods might take when trying to fit empirical moments in general.

[23] While the distribution of net positions described by Furfine is ex post trading it should give a good approximation to its ex ante equivalent. Further one must note that the Furfine analysis focusses on the first quarter of 1998 while I use policy numbers of 2006. I assume the pattern that small banks are net sellers on average did not change.

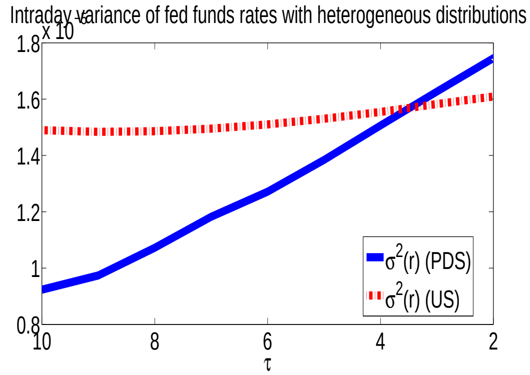

Figure 12: Intraday variance of interest rates for PDS and US in a large scale simulation with heterogeneous distributions.

6 Conclusion

I apply partially directed search to explain why some banks move further away from their target level. This line of modelling is relatively new. It allows agents not only to update a la Bayes but also allows coordination to solve an allocation problem in a decentralized market. The underlying modelling method is quite general as the extension about the variance of federal funds rates showed, and it shows a bridge between undirected search where agents are not thought to, well, search and directed search where agents know exactly about the physical states of potential trading partners.

A future of this research has two paths: a direct application of the proposed model using the method of simulated moments or indirect inference to estimate selected parameters is appealing. Two minor obstacles are obvious. For one, the data is rather hard to come by. Using publicly available and official statistics or moments mentioned in the literature like in Furfine (1999) might allow me to find appropriate target moments. Another approach is to get the data directly. In particular, the SNB has a very attractive dataset where terms of trades are explicitly given. The second obstacle is the computational demand when I expand the dimension of the model in time and balance levels. Unlike US the PDS needs to verify the search decisions and in case of a correction recalibrate whole sequences of distributions and value functions which in turn change policies. Obviously it is interesting to find conditions where intermediation of the middlemen group breaks down as it happened in 2008.

The second path this research can take is to ground intermediation and PDS in a more general framework and contrast it with US and directed search as in Corbae et al. (2003). The model here presents itself in its most abstract form as a storage problem in a search environment where agents belong to groups and their membership is publicly known. Multiplicity of equilibria can be addressed by meeting technologies, liquidity shocks (which are observationally equivalent to preference shocks about the target level), storage costs and transaction costs. The latter has not been incorporated in the robustness analysis above but should yield similar results. The one open question is how agents obtain a label. In the federal funds market I assumed that banks can assign membership of banks to the middlemen group or the peripheral group because the game is played repeatedly day after day. In the long run banks might want to enter the group of middlemen due to their higher expected return. Other theoretical considerations can address the quantity of partitions for labels (here we had only 2), multidimensional labels (where we have, say, a label for middlemen and peripheral banks and another for net buyers and net sellers) or stochastic labels (where searching for one label can with some probability lead to a member of a different group).

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B Reduction of Notation

I can rewrite the maximum lifetime utility in (1) to deal with the more handy indirect utility function. Then the end-of-day utility is given by $V(k^0, x^0, 0, i, F^0) = U_i(k^0) + \beta^{\Delta} x^0$ where x^0 is the (net) debit position at the end of the day. This yields a second law of motion. Additionally to the one above describing balance holdings I also need to acknowledge that x evolves according to $x^{\tau-1} = x^\tau + R^\tau \ \forall \ \tau \in \mathbb{T}^+$ and $x^T = 0$. Using backward induction I can derive indirect utility functions for earlier rounds, so that V(k^τ,x^τ,τ,i,F^τ)=u(k^τ)+β E[V$(k^\tau - B^{\tau *}, x^\tau + R^{\tau *}, \tau - 1, i, F^{\tau-1})$] for $\tau > 0$.

Note that if I write V$(k^\tau, x^\tau, \tau, i, F^\tau) = \beta^{\Delta+\tau} x^\tau + W^\tau_{F^\tau}(k^\tau, i)$ I can reformulate the indirect formula similar to Afonso and Lagos (2012b)

$$\begin{aligned}
&V(k^\tau, x^\tau, \bar{k}_i, \tau, i, F^\tau)\\
&= \beta^{\Delta+\tau} x^\tau + W^\tau_{F^\tau}(k^\tau, i)\\
&= u(k^\tau, i) + \beta E[\beta^{\Delta+\tau-1}(x^\tau + R^{\tau*}) + W^{\tau-1}_{F^{\tau-1}}(k^\tau - B^{\tau*}, i)|F^\tau]\\
&= u(k^\tau, i) + E[\beta^{\Delta+\tau}(x^\tau + R^{\tau*}) + \beta W^{\tau-1}_{F^{\tau-1}}(k^\tau - B^{\tau*}, i)|F^\tau]
\end{aligned}$$

which I can rearrange to equation (2) and equation (3).

C Sketches of Proofs

The proofs for existence and uniqueness are sketched out. I proceed by working out conjectures that relate equilibrium objects between rounds. I then sketch out a relation tying together conditions of the first round and the last.

C.1 Some Lemmata

Lemma 1. *$W^\tau_{F^\tau}(k, i)$ is bounded.*

First, I see that $W^0_{F^0}(k, i) = U(k)$ is bounded over the domain $\mathbb{K}$ for any F^0 and i. Say $W^{\tau-1}_{F^{\tau-1}}(k, i)$ is bounded, then $W^\tau_{F^\tau}(k, i) = \sum_{l\in\mathbb{K}} q^\tau(i, k, l)\beta W^{\tau-1}_{F^{\tau-1}}(l, i)$ where $q^\tau(i, k, l)$ is a discrete probability distribution over $\mathbb{K}$ and $\beta \leq 1$. Therefore $W^\tau_{F^\tau}(k, i)$ is bounded everywhere. The argument is easily extended to accompany (bounded) instantaneous utility functions.

Lemma 2. *A set $(B^{\tau*}_{W^{\tau-1}}(\chi, \chi'), R^{\tau*}_{W^{\tau-1}}(\chi, \chi'))$ solving (10) exists given that $W^{\tau-1}_{F^{\tau-1}}(k, i)$ is bounded, and $Z^{\tau*}_{W^{\tau-1}}(\chi, \chi')$ is unique.*

The objective function in (10) maps into a bounded (sub)set of $\mathbb{R}$ given that $W^{\tau-1}_{F^{\tau-1}}(k, i)$ is bounded over $\mathbb{K}$ for any given (finite) R. On the other hand, (8) and continuity over R imply that (10) always has a maximum for any B. Now I just need to find a maximum over a finite set of bounded values. Uniqueness of $Z^{\tau*}_{W^{\tau-1}}(\chi, \chi')$ is obvious.

Note that I circumvent the problem of a possible corner solution in R by arbitrarily increasing $\overline{R}$ in (8). Similarly, to avoid corner solutions in B I can increase $\mathbb{K}$ at the cost of computational time.

Lemma 3. *$(B^{\tau*}_{W^{\tau-1*}}(\chi, \chi'), R^{\tau*}_{W^{\tau-1*}}(\chi, \chi'))$ solves (10) if and only if $(B^{\tau*}_{W^{\tau-1*}}(\chi, \chi'), R^{\tau*}_{W^{\tau-1*}}(\chi, \chi'))$ solves (11) and (12).*

A proof that can be directly transferred is given by Afonso and Lagos (2012b).

Lemma 4. $S^{\tau*}_{Z^{\tau},F^{\tau}}(\chi)$ *is defined by (17) given* $Z^{\tau}_{W^{\tau-1}}(\chi,\chi')$ *and* F^{τ} *and it is unique.*

Note that $C^{\tau}_{W^{\tau-1},F^{\tau}}(\chi,s)$ is well-defined by (**??**) and unique for a given F^{τ} and $Z^{\tau*}_{W^{\tau-1}}(\chi,\chi')$ and maps onto $\mathbb{R}_{\geq 0}$. This defines $S^{\tau*}_{Z^{\tau*},F^{\tau*}}(\chi)$ uniquely.

Lemma 5. Q^{τ} *is uniquely given* F^{τ}*,* S^{τ} *and* B^{τ} *and the transition matrix has finite elements.*

Given F^{τ} and S^{τ} I can use (18), (19), (20) and (21) to find $l^{\tau}_{F^{\tau},S^{\tau}}(\chi,\chi')$ and $n^{\tau}_{F^{\tau},S^{\tau}}(i,\chi')$. Using (24) and (25) I find the elements of Q^{τ}. Note that as B^{τ} is bounded so are the transition steps.

Lemma 6. $Z^{\tau*}_{W^{\tau-1}}(\chi,\chi')$ *exists on* $\mathbb{R}_{\geq 0}$*.*

Note that I can always set B=0 which results in $Z^{\tau}_{W^{\tau-1}}(\chi,\chi',0)=0$. I find $Z^{\tau}_{W^{\tau-1}}(\chi,\chi',0)\leq Z^{\tau*}_{W^{\tau-1}}(\chi,\chi')$.

C.2 Proposition 1

Fix E^e, the sequence of distributions across periods. I start from a period $\tau-1$ and assume $W^{\tau-1e}$ is well defined and bounded. All other elements in τ are now well defined. Using lemmata (2) and (3) I find $B^{\tau e}$ (and $R^{\tau e}$). Lemma (2) also implies a unique $Z^{\tau e}$ exists.

$S^{\tau e}$ is unique using lemma (4) given $Z^{\tau e}$ and a (fixed) $F^{\tau e}$. This in turn allows me to derive $Q^{\tau e}$ using lemma (5). Using equation (27) and lemma (1) ensures we obtain a unique and bounded $W^{\tau e}$.

Now I need to acknowledge that W^{0e} is bounded and well defined. Given a fixed E^e we can derive a sequence W^e given the sequential logic just described. As this holds for an arbitrary E^e it also holds for an E^e with a well defined initial distribution.

Lemma (6) ensures that all potential matches are individual rational so that banks that are called have no incentive to decline an offer to bargain.