Competition and Bank Risk Taking under Differentiated Oligopoly*

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Abstract

We explore the relationship between the degree of competition in the deposit market and the risk taking behavior of banks in a framework where banks compete in differentiated deposit services. As has already been established in the extant literature, we find that an increased degree of competition, measured by greater degree of substitutability or by higher number of banks, induces the banks to take more risk in equilibrium. This result has been established under the assumption that the banks can invest their deposits directly to a risky project by choosing the optimal level of risk. Next, we show that when banks do not have direct control over the risk level of the projects chosen by their borrowers, the equilibrium level of risk depends neither on the characteristics of the deposit market nor on the number of banks in the financial sector, thereby challenging the so-called positive relationship between deposit market competition and risk already established in the extant literature. We further derive the implications of loan contracts on the equilibrium deposit rates.

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1 Introduction

When banks are able to raise deposits to invest in assets with uncertain returns, excessive deposit might induce banks to take more risk. Involvement in high risk activities is viewed as one of the principal causes of several instances of banking crises that the world economy has witnessed during the last few decades. The main goal of this paper is to analyze the role of deposit market competition in determining the risk taking behaviour of banks. In general, there exists a positive correlation between the degree of competition in the deposit market and the risk taking behavior of banks. When banks have the choice of investing their deposits among many risky projects available, lower intermediation margin, implied by greater deposit market competition, incentivizes banks to take more risk as riskier projects, if successful, yield higher returns. In order to re-examine the positive relationship between competition in the deposit market and risk taking, we consider a monopolistically competitive deposit market where a finite number of banks compete in differentiated deposit services. Thus, the degree of substitutability, together with the number of banks in the economy, measures the degree of competition in the deposit market. Banks mobilize deposits by simultaneously offering deposit rates, and then invest their deposits in one of the many risky projects available to them. Banks' choice of risk is not observable by the depositors, and banks are protected by limited liability in the sense that, when the project of a bank fails, it does not have to pay back the promised deposit rate. This implies an incentive problem in risk taking. Thus, moral hazard problem becomes more stringent as competition in the deposit market increases. We find that the positive relationship between deposit market competition and risk taking, established in the extant literature on prudential regulation, is observable under this market structure.¹

¹See the following section for a review of the existing literature.

Next, we analyze the effects of deposit market competition on depositor surplus. We show that in some situations greater competition measured by the degree of substitutability of the deposit services may be detrimental to the welfare of the depositor. When there are many banks in the financial sector, i.e., the market is already very competitive, the depositor surplus reaches its maximum when deposit services are sufficiently but imperfectly substitutes.

Boyd and De Nicoló (2005) arrive at a striking result by introducing a loan market in the banking sector. They show that in a loan market when the entrepreneurs/borrowers of the banks choose the riskiness of the projects, the well-established positive relationship between competition and risk taking does not remain valid anymore. These authors consider an economy where banks compete in homogeneous deposit and loan services, and hence the number of banks nmeasures the degree of competition in moth markets. Later, Dam et. al. (2014) show in a model of spatial banking competition that when the banks compete in differentiated loan services in addition to differentiated deposit services, equilibrium risk decreases with the degree of competition of the loan market, but is independent of the characteristics of the deposit market. In this section we intend to explore this issue. We emphasize that in Boyd and De Nicoló (2005) an increased number of banks does not tell us whether the deposit or the loan market results in to be more competitive. Therefore, we try to separate the two markets from each other, and analyze the characteristics of which of the two markets are relevant for the equilibrium risk behavior of banks. We show that when a loan market is incorporated into the model, the equilibrium level of risk does not depend on the characteristics of the deposit market since the banks do not have any control over the choice of risk by their borrowers. This result conforms to the view that when the banks act as intermediaries between the depositors and the entrepreneurs, increased competition in the deposit market not necessarily results in greater probability of bank failure.

2 Related Literature

Banks have among their functions the provision of financial services as transaction and payment system, insurance and the coordination between lenders and borrowers. In the last type where banks act as financial intermediaries it is natural to find moral hazard problems due to informational asymmetries that tend to exist between the depositors and the entrepreneurs. Moreover, depositors do not have direct access to investment opportunities since individual depositors are small entities. Thus, intermediaries help overcome such frictions in the financial markets. The agency problem in the banking sector leads to excessive risk taking, which implies large probabilities of bank failure, which in turn is associated with large social cost (see e.g. Matutes and Vives, 2000). Often the banking sector is characterized by having strong negative externalities, for example, if a big bank fails this may cause the instability of the entire banking system.

For the reasons described before, among many other, the banking is a regulated market. Regulators use different mechanism to maintain the competition and the risk at an "optimal" level. The use of a minimum capital requirement and deposit rate ceiling to curb banks' incentive for risk taking has been analyzed in Hellmann, Murdock and Stiglitz (2000), and Repullo (2004). Another policy instrument is the use of deposit insurance in order to enhance depositors' confidence and prevent systemic financial crises (Diamond and Dybvig, 1983).

A negative relationship between competition and bank risk taking is a well established result in the literature. For example, Matutes and Vives (2000) show, in a model of spatial competition, that it is clear that there are trade-offs between competition and the potential failure rates. Some authors argue that when banks face high competition in the deposit market they become more prone to gambling (e.g. Keeley, 1990; Hellmann et al., 2000; and Repullo 2004). Allen and Gale (2000) conclude that, when banks compete à la Cournot, the optimal risk of failure reaches a maximum when the number of banks in the deposit market is large. These authors show that, when the depositors are insured as the number of banks grows, banks have incentives to take high risk. A typical approach to the analysis of this relationship between competition and risk taking is the valuation of the charter value of the banks. Under intense competition, the banks take high risk in order to earn high profits. On the there hand, market power enhances the charter values of the banks and makes them more prudent (e.g. Besanko and Thakor, 1993; Boot and Greenbaum, 1993). In the same sense, Boyd and De Nicoló (2005) use a model with homogeneous banks and show the trade off between competition in the deposit market and bank risk taking is robust.

Nonetheless, competition in the loan market reduces the rates that borrowers have to pay for loans, Thus, firms earn higher profits and become more prudent, which establishes a positive correlation between loan market competition and risk taking (e.g. Caminal and Matutes, 2002). Boyd and De Nicoló (2005) suggest that the usual trade off between competition and bank risk taking can be reversed if a loan market is considered. They argue that when the deposit market become more concentrated, banks use their market power to become more profitable. As banks face low competition in the loan market they earn more rents by charging higher loan rate to borrowers. This effect implies that borrowers adjust their investment policies in favor of more risk. Dam et. al. (2014) show that, although the relationship between market power and bank risk taking can be reversed by incorporating the loan market in the model, the equilibrium level of risk in the economy does not depend on the degree of competition in the deposit market but on that of the loan market. We find a result that reinforces the latter view that markets as deposit and loan markets are in general independent.

3 The model

In this section we consider a model of deposit market competition where there are two classes of agents: a representative depositor and $n \ge 2$ banks. The economy lasts for 3 dates. At t = 0, banks simultaneously raise deposits by offering deposit rates $\mathbf{r} = (r_1, \ldots, r_n)$. The supply of deposits with bank *i* is thus given by $D_i(r_i, r_{-i})$, where $r_{-i} = (r_1, \ldots, r_{i-1}, r_{i+1}, \ldots, r_n)$ with $r_i > 1$ for all *i*. Deposit services offered by banks is differentiated with the degree of substitutability $\delta \in (0, 1)$. We assume that the depositor has the following quadratic utility function:

$$V = \sum_{i=1}^{n} r_i D_i - U(D_1, \dots, D_n),$$

where

$$U(D_1,...,D_n) = \sum_{i=1}^n D_i + \frac{1}{2} \sum_{i=1}^n D_i^2 + \delta \sum_{i=1}^n \sum_{j \neq i} D_i D_j.$$

The above utility function gives rise to the following inverse supply function of deposits:

$$r_i = 1 + D_i + \delta D_{-i},$$

where $D_{-i} = \sum_{i \neq j} D_j$ is the aggregate supply of deposits of the rival banks. Therefore, the supply of deposits with bank *i* is given by:

$$D(r_i, r_{-i}) = \frac{-(1-\delta) + [1+\delta(n-2)]r_i - \delta r_{-i}}{\Delta(\delta, n)},$$
(1)

where $\Delta(\delta, n) = (1 - \delta)[1 + \delta(n - 1)]$. Note that the degree of deposit market competition is measured by two parameters: the degree of substitutability δ and the number of banks in the

industry *n*. In the limiting cases, banks are local monopolies when $\delta = 0$, and there is perfect competition when $\delta = 1.^2$ On the other hand, n = 1 corresponds to a monopolistic banking sector, and $n \to \infty$ corresponds to perfect competition.

At date 1, banks simultaneously choose their investment strategy. Each bank *i* has access to a continuum of constant return-to-scale risky technologies indexed by $\theta_i \in [0, 1]$. In particular, given an investment I_i , the risky technology θ_i yields:

$$f(I_i, \theta_i) = \begin{cases} (\alpha + \theta_i)I_i & \text{with probability } 1 - \theta_i, \\ 0 & \text{with probability } \theta_i. \end{cases}$$

We assume that $\alpha \in [1, 3]$ so that that return to a safe project, i.e., $\theta_i = 0$ is neither too low nor too high, i.e., $\alpha \ge 1$, makes a safe project viable, and $\alpha \le 3$ does not kill incentives for banks to invest in the risky projects. Note that higher values of θ_i imply greater per unit return, but at the same time, higher probability of failure, and hence a riskier project. The choice of risk by each bank cannot be observed by the depositor, and hence banks face a potential moral hazard problem. Deposit contracts are subject to the banks' limited liability meaning that in the case the project of a bank *i* fails it is not obliged to pay back the depositor. As a consequence, banks may opt for riskier project since it earns higher income with a positive probability. Such incentive problems of the banks will be crucial for the analysis of the baseline model when they compete in the deposit market and choose their own investment strategies. The deposits are fully insured by the central banking authority or by some private insurer, e.g. the Federal Deposit Insurance Corporation (FDIC) who charges a flat per unit insurance premium which

 $^{^{2}}$ In general, for the depositor the most important thing to consider is the deposit rate offered by a bank. However, there exists other features as the distance between banks, the availability of ATM or the simple fact to make a queue standing or sitting in a comfortable armchair.

is normalized to zero. Hence, the depositor gets back the promised deposit rate r_i from bank *i* even if its project fails.³ Finally at t = 3, the project returns are realized and the depositor is paid off.

4 The Deposit Market Equilibrium

4.1 Effect of Competition

We analyze a symmetric subgame perfect Nash equilibrium (SPNE) of the deposit market game in which all banks choose the same levels of risk θ and the same deposit rates r.⁴ At t = 1, given the supply of deposits $D_i(r_i, r_{-i})$, each bank *i* solves

$$\max_{\theta_i} [1-\theta_i](\alpha+\theta_i-r_i)D_i(r_i,r).$$

The above maximization problem yields

$$\theta_i^* = \theta(r_i) \equiv \frac{1 - \alpha + r_i}{2} \quad \text{for} \quad i = 1, \dots, n.$$
(2)

Thus, at t = 0 bank *i* chooses r_i to solve

$$\max_{r_i} \left[1 - \theta(r_i)\right] \left[(\alpha + \theta(r_i) - r_i]D_i(r_i, r) = \left(\frac{\alpha + 1 - r_i}{2}\right)^2 D_i(r_i, r),$$

³There is no loss of generality in assuming a linear probability function instead of the probability of failure given by $\pi(\theta_i)$ where $\pi'(\theta) < 0$ and $\pi''(\theta) \le 0$ for all $\theta \in [0, \bar{\theta}]$ with $\pi(0) = 1$ and $\pi(\bar{\theta}) = 0$.

⁴We are only interested in the SPNE since all banks are symmetric and compete à la Bertrand, that is, banks compete through deposit rates. If a bank sets a deposit rate lower than the other banks' deposit rates, it will lose its share of the market. On the another hand, if a bank sets a deposit rate higher than the other banks' deposit rates, the other banks will increase their deposit rates, in order to recover their share of market, until the point in which all banks set the same deposit rate, which is given by the symmetric equilibrium.

where *r* is the deposit rate offered by each of the rival banks in a symmetric equilibrium. The first-order condition of the above maximization problem yields the best reply function $r_i(r)$ of bank *i* which is given by:

$$\frac{1}{2}[\alpha + 1 - r_i(r)][1 + \delta(n-2)] = -(1-\delta) + [1 + \delta(n-2)]r_i(r) - \delta r.$$

Notice that $r'_i(r) > 0$, i.e., the deposit rates offered by a pair of banks are strategic complements. Putting $r_i = r = r(\delta, n)$, we get

$$r(\delta, n) = \frac{(1+\alpha)[1+\delta(n-2)]+2(1-\delta)}{3+\delta(n-4)}.$$
(3)

The optimal risk level is thus given by:

$$\theta(\delta, n) = \frac{1-\alpha+r(\delta, n)}{2}.$$

The following proposition analyzes the behavior of the equilibrium deposit rate and risk taking with respect to the degree of deposit market competition.

Proposition 1 Let $r(\delta, n)$, $\theta(\delta, n)$ and $m(\delta, n) := \alpha + \theta(\delta, n) - r(\delta, n)$ be the deposit rate, risk level and intermediation margin, respectively in a symmetric SPNE.

(a) For a given number n of banks in the economy, the equilibrium deposit rate and the equilibrium risk level are monotonically increasing in the degree of substitutability δ.
 Moreover, the equilibrium intermediation margin is monotonically decreasing in δ with lim_{δ→1} m(δ, n) = 0.

(b) For a given value of δ, the equilibrium deposit rate, and the equilibrium risk level are monotonically increasing in the number of banks n. Moreover, the equilibrium intermediation margin is monotonically decreasing in n with lim_{n→∞} m(δ, n) = 0.

The results of the above proposition are fairly intuitive. Recall that we measure the degree of competition either by the degree of substitutability or by the number of banks. When competition increases, in either case the depositor finds it easier to switch between banks. Thus, in order to attract higher deposits, each bank offers a higher rate. As the intermediation or profit margin of each bank is lower with greater degree of competition, banks tend to take more risk in the expectation of maintaining a positive intermediation margin. This is the "gambling in resurrection" effect of the deposit market competition. Boyd and Di Nicoló (2005) obtain a similar conclusion in a model of homogeneous deposit competition, i.e., $\delta = 1$. Dam et. al. (2014), on the other hand, obtain that risk taking increases with the degree of substitutability for a fixed number of banks in the economy. The above result can be seen as a generalization of the results obtained in the above two papers in the sense that we vary both the degree of substitutability and the number of banks to analyze their effects on risk taking.

4.2 The Depositor Surplus

It is often argued by the prudential regulators of banks that more competition is detrimental to the stability of the banking system in an economy. Therefore, several regulatory measures such as minimum capital requirements, deposit rate ceiling, risk-based deposit insurance, etc. have been adopted by the regulation authorities in order to prevent bank failure. In the present paper we have thus far abstracted from these issues. Now, we analyze the impact of increased competition on the welfare of the representative depositor. Note that the equilibrium depositor surplus is given by:

$$V(\delta, n) = nDr((\delta, n))[r(\delta, n) - 1] - \frac{n}{2}(D(r(\delta, n))^2[1 + \delta(n - 1)]] = \frac{n(r(\delta, n) - 1)^2}{2[1 + \delta(n - 1)]}.$$
 (4)

Differentiating the above expression with respect to *n* and δ we get

Proposition 2 Let $V(\delta, n)$ be the equilibrium depositor surplus which is given by equation (4).

- (a) The equilibrium depositor surplus is monotonically increasing in the number of banks for any $\delta \in (0, 1)$.
- (b) If $n \le 5$, then the equilibrium depositor surplus is monotonically increasing in the degree of substitutability.
- (c) If n > 5, then the equilibrium depositor surplus is non-monotone and concave in the degree of substitutability, reaching its maximum at $\hat{\delta} \in (0, 1)$ where $\hat{\delta}$ is the positive root of the following quadratic equation in δ :

$$-[(n-2)(n-4)]\delta^2 + 6\delta + 1 = 0$$

with $\lim_{n\to\infty} \hat{\delta} = 0$.

Since the deposits are fully insured, the depositor is not forced to share the loss incurred by the banks in case some of the projects fail. Therefore, higher deposit rates increase the welfare of the depositor. Thus, as expected, the presence of more banks in the economy causes higher surplus for the depositor. The effect of an increase in the degree of substitutability on the depositor

surplus is not that straightforward, neither very intuitive. When there are few banks ($n \le 5$) in the economy, greater the degree of substitutability, higher is the surplus of the depositor.⁵ This result suggests that when there is a large number of banks in the economy, restricting the product variety, i.e., decreasing δ below 1 may result in greater depositor surplus.⁶

5 Effect of Loan Contracts on Risk Taking

Boyd and De Nicoló (2005) show that when the loan market is considered, the well-established positive relationship between competition and risk taking is reversed. Dam et al. (2014) establish that the risk level does not depend on the grade of competition of the deposit market. We show that the two markets have to be analyzed separately, and analyze the characteristics for the equilibrium risk behavior of banks.

We add two more dates to the initial timeline described in Section 2. After each bank *i* has its deposits D_i in date 1, in the following date a fraction q_i of it is lent to an entrepreneur at a loan rate ρ_i . Then, the entrepreneur invests the amount lent in a risky project. The remaining $(1-q_i)D_i$ is invested in the money market which earns the risk-free interest factor $r_f \ge 1$. At date 3, when the loan obligations are paid off, the bank pays back r_i to the depositor. Both the borrowers and the banks are protected by limited liability, i.e., if the project of the borrower of bank *i* fails neither the bank nor the depositor is repaid.

Without loss of generality, by borrower or entrepreneur *i* we refer to the borrower of bank

⁵The threshold value 5 of *n* is a mere coincidence because of the assumptions on the fundamentals. Under weaker assumptions such as a general probability of failure function $\pi(\cdot)$ the above result continues to hold with a threshold \hat{n} possibly different from 5.

⁶Remember that we are working with a quadratic utility function, therefore the ratio in (4) inherits this shape and this is the reason because the depositor surplus has a change from monotone increasing (for $n \le 5$) to nonmonotone (for $n \le 5$).d

i who is risk neutral. Each identical borrower has access to a continuum of nondecreasingreturns-to-scale technologies indexed by θ_i with stochastic returns

$$f(I_i, \theta_i) = \begin{cases} (\alpha + \theta_i) I_i^{\gamma} & \text{with probability } 1 - \theta_i, \\ 0 & \text{with probability } \theta_i \end{cases}$$

with $\gamma \in (0, 1]$, where $I_i = q_i D_i$ is the amount invested in each of the risky projects.⁷ Further, the entrepreneurs do not have direct access to money market, and hence invests the entire loan obtain in the project, i.e., $I_i = q_i D_i$.

5.1 The Optimal Loan Contracts

A loan contract is thus given by (ρ_i, q_i) where ρ_i is the loan rate and q_i is the fraction of deposit D_i to be lent. We drop the subscript *i* for the time being unless it creates any confusion. An optimal loan contract (ρ, q) thus solves the following maximization problem:

$$\max_{\{\rho,q\}} [1-\theta]\rho qD + r_f(1-q)D$$

subject to $\theta = \operatorname{argmax}_x[1-x][(\alpha+x)(qD)^{\gamma} - \rho qD].$ (IC)

The incentive compatibility constraint (IC) implies

$$\theta = rac{1}{2} [1 + lpha +
ho(qD)^{1-\gamma}] \equiv heta(
ho, q).$$

⁷Under $\gamma = 1$ as in the model of deposit market in Section 2 would trivially imply our subsequent results in this section.

Therefore, substituting $\theta = \theta(\rho, q)$ the bank's problem reduces to:

$$\max_{\{\boldsymbol{\rho},q\}} [1-\boldsymbol{\theta}(\boldsymbol{\rho},q)]\boldsymbol{\rho}q\boldsymbol{D} + r_f(1-q)\boldsymbol{D}.$$

The following proposition characterizes the optimal risk level, loan rate and loan amount.

Proposition 3 Let θ_i^* , ρ_i^* and $q_i^*D_i$ be the optimal risk level, loan rate and loan amount in the lender-borrower relationship i. The optimal values are given by:

$$egin{aligned} & heta_i^* = rac{3-lpha}{4}, \ & heta_i^* = rac{4r_f}{\gamma(1+lpha)}, \ & heta_i^* D = \left[rac{\gamma(1+lpha)^2}{8r_f}
ight]^rac{1}{1-\gamma}. \end{aligned}$$

When α , the per unit returns to prudent behavior increase, the entrepreneur takes lower risk. The optimal loan rate is increasing in the risk-free rate as r_f is the opportunity cost of lending. It is decreasing in both α and γ since an increase in either of these parameters means a more productive project which implies greater bargaining power of the borrower. Therefore, the bank needs to sacrifice more at the margin by charging lower loan rate.

Note that the loan contracts are independent across borrower-lender relationships, and they do not depend on the characteristics of the deposit market, namely the degree of substitutability via the equilibrium deposit rates. Therefore, the terms of the loan contracts such as the loan rates and the amount lent, and the optimal risk level are constant across banks, i.e., $\theta_i^* = \theta^*$, $\rho_i^* = \rho^*$ and $q_i^* = q^*$ for all i = 1, ..., n.

5.2 The Deposit Contracts

At date 1, each bank *i* solves the following maximization problem:

$$\max_{r_i \in [1, \rho^*]} [1 - \theta^*] [\rho^* - r_i] q^* D_i(r_i, r) + r_f (1 - q^*) D_i(r_i, r)$$

$$\equiv Q + \left[\frac{[1 + \delta(n-2)]r_f}{\Delta(\delta, n)} - A \right] r_i,$$

where

$$A \equiv (1 - \theta^*)q^*D_i,$$
$$Q \equiv A\rho^* - r_f q^*D_i - \frac{[1 + \delta(n-2)][(1-\delta) + \delta r]r_f}{\Delta(\delta, n)}.$$

Therefore, the optimal deposit rate offered by bank *i* is given by:

$$r_i^* = \begin{cases} \rho^* & \text{if } \frac{r_f}{A} \ge \frac{\Delta(\delta, n)}{1 + \delta(n-2)}, \\ 1 & \text{otherwise.} \end{cases}$$
(5)

Therefore,

Proposition 4 All the banks offer the same deposit rate in equilibrium, $r_i^* = r^*$ for all i = 1, ..., n, which is given by equation (5). Moreover,

(a) For a given degree of substitutability δ , the equilibrium deposit rate r^* is increasing in the number of banks in the sense that there exits a unique \hat{n} such that $r^* = 1$ for $n < \hat{n}$ and $r^* = \rho^*$ for $n \ge \hat{n}$. (a) For a given number of banks n, the equilibrium deposit rate r^* is increasing in δ in the sense that there exits a unique $\hat{\delta}$ such that $r^* = 1$ for $\delta < \hat{\delta}$ and $r^* = \rho^*$ for $\delta \ge \hat{\delta}$.

The behavior of the deposit rate remains the same as in the model in Section 3 without loan contracts. When banks face perfect competition banks offer a high deposit rate in order to capture deposits. On the another hand, when banks enjoy enough power market offer a low deposit rate, that is, they extract all the depositor surplus. Finally, from Proposition 3 it follows that

Corollary 1 The equilibrium risk level θ^* does not depend on the degree of deposit market competition, being measured either by δ or by n.

When the borrowers of the banks choose the level of risk of their projects, it is obvious that this choice would depend only on the loan market conditions. Since the project returns are independent across borrowers, the terms of the loan contracts no not depend even on how many banks are there in the economy. This is a very particular situation which arises because of the particular assumptions we have made in the present model. But even under a more general situation such as Dam et. al. (2014), the terms of the loan contracts depend on the fundamentals of the loan market, and not on those of the deposit market.

Our model thus implies that in view of the stability of a banking industry, i.e., lower probability of bank failure, little intervention is necessary in the deposit market such as restraining entry of new banks so as to allow the banks to exercise market power and behave prudently. An intervention in the loan market is however necessary if the central banking authority aims at lowering risk taking.

6 Conclusions

Boyd and De Nicoló (2005) consider a base model with only the deposit market. They study a financial market in which banks offer homogeneous goods and are Cournot competitors. Banks have no initial resources however they have access to a set of risky technologies. Banks choose an optimum deposit level and optimum risk level, they have complete control over the risk. The deposit rate is function of the total deposit and it does not depend on the risk level. They show that with a higher number of banks, the economy becomes more competitive and this in turn encourages banks to be more risky. As the competition increases, the deposit level of each bank increases in order to become more profitable. Since banks have to pay a higher deposit rate to the depositors, they will take more risk to obtain higher returns from the projects. Then, these authors add a loan market to the economy and argue that it is more realistic. They consider many entrepreneurs who have access to risky projects. They need borrow from banks to implement their projects. In this case, banks have no direct control over risk, but the entrepreneurs. They show that the last relationship is reversed. That is, more competition through a higher number of banks in the economy implies that the entrepreneurs behave prudently. Intuitively, more competition results in lower loan rate for the entrepreneurs, who in turn become less risky because they borrow to less cost.

Dam et al. (2014) analyze a model of locational competition à la Salop for the loan market. They consider a continuum of identical entrepreneurs uniformly distributed on the unit circle. The entrepreneurs own two projects risky and safe and require of banks as financial intermediaries. Borrowers choose in which to take loan and then simultaneously decide whether to invest in the prudent or the gambling project. If the per unit transportation cost for the borrowers is high, because less competition, then banks set a high loan rate due to their increased market power. Then, the entrepreneurs choose the gambling strategy in order to obtain higher returns. Thus, the authors how the negative relationship between competition and risk taking bank. Additionally, they emphasize that the both deposit and loan should be analyzed separately and therefore competition in the deposit market does not affect the risk level in the economy.

We consider a financial market with differentiated banks competing for deposits and establish that there exists a positive relationship between competition and risk taking by the banks. The degree of competition in the deposit market is induced either by an increase in the number of banks or an increase in the degree of substitutability. The fact that banks face an intense competition implies that in order to collect more deposits banks have to offer higher deposit rate, this in turn causes that banks become more risky in an attempt to increase their intermediation margin. We find that the deposit rate and risk level are increasing with the degree of competition. An additional finding is that when the number of banks in the market is large enough, less than perfect substitutability maximizes the depositor surplus.

In summary, Boyd and De Nicoló (2005) argue that this relationship between market power and risk taking can be reversed if a loan market is considered on the top of the deposit market. They find that when the loan market is considered a positive association between competition (in deposit market) and risk taking arises. However, Dam et. al. (2014) argue that the deposit market and the loan market work independently, and find that in fact there exists a positive relationship between competition and risk taking, but the competition in the loan market is the only determining factor for the equilibrium risk level. In this paper, we find a similar conclusion. When we consider the loan market in which the entrepreneurs choose the risk level, and the bank only set the deposit and loan rates, the risk level is independent of the degree of competition in the deposit market (Proposition 5). The result that the deposit rate is increasing with respect to the number of banks in the market and the degree of substitutability remains the same when we include the loan market.

One of the crucial assumptions in the current paper is that the project returns are independent across borrowers which to a large extent drives that result that the equilibrium risk level is constant with respect to even the degree of loan market competition measured in terms of the number of banks. A future research agenda would incorporate that the project returns are correlated with the conjecture that the equilibrium risk level may have a monotone relation with respect to the intensity of competition in the loan market depending upon the nature of such correlation. Another direction of future research is to allow an individual bank to finance more than one project. In this case, it would even be more natural to assume that the projects are correlated.

Another interesting issue is to know which parameter, number of banks or grade of differentiation, has a higher impact on the banks risk taking. In the same way, we consider that both parameters are independent however will be interesting to analyze the behavior of the market when an increase in the number of banks changes the grade of substitutability of banks. It is natural to think that when more banks compete in the market, the variety of goods increases and with this the grade of substitutability.

Appendix 1

Proof of Proposition 1: We take the partial derivate of (3) with respect to δ , we get

$$\frac{\partial r(\delta,n)}{\partial \delta} = \frac{2\alpha(n-1)}{[3+\delta(3n-4)]^2} > 0.$$

Then,

$$\frac{\partial \theta(\delta, n)}{\partial \delta} = \frac{1}{2} \frac{\partial r(\delta, n)}{\partial \delta} > 0.$$

Now, differentiating (3) with respect to n we get

$$\frac{\partial r(\delta,n)}{\partial n} = \frac{2\alpha\delta(1-\delta)}{[3+\delta(3n-4)]^2} > 0.$$

Then

$$\frac{\partial \theta(\delta,n)}{\partial n} = \frac{1}{2} \frac{\partial r(\delta,n)}{\partial n} > 0.$$

Now, notice that

$$\lim_{\delta \to 1} r(\delta, n) = \frac{(1+\alpha)(n-1)}{n-1} = 1 + \alpha \qquad \text{and}$$

$$\lim_{n\to\infty}r(\delta,n)=\frac{(1+\alpha)\delta}{\delta}=1+\alpha.$$

Thus,

$$\begin{split} \lim_{\delta \to 1} \theta(\delta, n) &= \lim_{\delta \to 1} \frac{1 - \alpha + r(\delta, n)}{2} = 1 & \text{and} \\ \lim_{n \to \infty} \theta(\delta, n) &= \frac{1 - \alpha + r(\delta, n)}{2} = 1. \end{split}$$

Finally, it is easy to see that

$$\lim_{\delta \to 1} m(\delta, n) = 0 \quad \text{and}$$
$$\lim_{n \to \infty} m(\delta, n) = 0,$$

and therefore the Proposition 1 is proved.

Proof of Proposition 2: To find $\hat{\delta}$ we need solve the program

$$\max_{\delta} \qquad \frac{n(r(\delta,n)-1)^2}{2[1+\delta(n-1)]} \ s.t. - \delta \leq 0, \qquad \delta \leq 1.$$

The Lagrangean is given by

$$\mathscr{L} = V(\delta, n) + \lambda_1 \delta - \lambda_2 (\delta - 1).$$

The Karush-Kuhn-Tucker (KKT) conditions are given by:

$$V_{\delta}(\delta, n) + \lambda_1 - \lambda_2 = 0, \lambda_1 \delta = 0, -\lambda_2(\delta - 1) = 0, \lambda_1, \lambda_2 \ge 0.$$

Two cases will be considered. The first implies that $\hat{\delta} = 1$ when $n \le 5$. The second implies that the problem has a internal solution at $\delta \in [0, 1)$ and δ is the positive root of

$$-[(n-2)(n-4)]\delta^2 + 6\delta + 1 = 0.$$

Proof of Proposition 3:

$$\max_{\{\boldsymbol{\rho},q\}} [1-\boldsymbol{\theta}(\boldsymbol{\rho},q)]\boldsymbol{\rho}q\boldsymbol{D} + r_f(1-q)\boldsymbol{D},$$

we get the first order conditions:

$$p:[1-\theta(\rho,q)] = \rho \frac{\partial \theta}{\partial \rho},\tag{6}$$

$$q: [1 - \theta(\rho, q)]\rho = r_f + \rho q \frac{\partial \theta}{\partial q}.$$
(7)

From (6) and (7) we get

$$ho^*=rac{4r_f}{\gamma(1+lpha)},$$
 $ho(qD)^{1-\gamma}=rac{1+lpha}{2}.$

And finally,

$$\theta_i^* = \frac{3-\alpha}{4},$$
$$q_i^* D = \left[\frac{\gamma(1+\alpha)^2}{8r_f}\right]^{\frac{1}{1-\gamma}}.$$

Proof of Proposition 4: See that

$$\frac{\Delta(\delta,n)}{1+\delta(n-2)} = \frac{(1-\delta)[1+\delta(n-1)]}{1+\delta(n-2)}$$

has derivate strictly negative since

$$rac{\partial rac{\Delta(\delta,n)}{1+\delta(n-2)}}{\partial n}=-rac{\delta(1-\delta)(2-\delta)}{[1+\delta(n-2)]^2}<0.$$

So, if there exists one point of interception with the line given by $\frac{r_f}{A}$ this must be unique.

References

Allen F, Douglas G (2000) "Comparing Financial Systems". MIT press

Besanko D, Thakor A (1993) "Relationship banking, deposit insurance and bank portfolio". In Mayer C, Vives X eds. *Capital markets and Financial intermediation*. *Cambrige University press* 292-318

Boot A, Greenbaum (1993) "Bank regulation, reputation and rents: theory and policy implications". In Mayer C, Vives X eds. *Capital markets and Financial intermediation. Cambrige University press* 262-285

Boyd J, De Nicoló G (2005) "The theory of bank risk taking and competition revisited". *J Finance* 60:1329–1343

Caminal R, Matutes (2002) "Market power and banking failures". *Int J of Industrial Organization* 20 9 1341-1361

Dam K, Escrihuela-Villar M, Sánchez-Pagés S (2014) "On the relationship between market power and bank risk taking". *Springer-Verlag Wien*

Diamond D, Dybvig P (1983) "Banks runs. Deposit insurance, and liquidity". *J Polit Econ* 91:401-419

Hellmann T, Murdock C, Stiglitz JE (2000) "Liberalization, moral hazard in banking, and prudential regulation: are capital requirements enough?" *The American Economic Review*

Keely M (1990) "Deposit insurance. Risk, and market power in banking". *Am Econ Rev* 80:1183-1200

Matutes C, Vives X (2000) "Imperfect competition, risk taking, and regulation in banking". *European Economic Review* 44:1-34

Repullo R (2004) "Capital requirements. Market power, and risk-taking in banking". *J Fin Intermediat* 13:156-182

Vives X (2010) "Competition and stability in banking". Working paper IESE Business School