Optimal Tax Rules, Pigou Taxation, and the "Double Dividend"

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Abstract: In this paper optimal tax formulae are computed when consumption of a commodity produces pollution. Then a test of the double dividend hypothesis based on the shadow prices of those formulae is proposed.

Resumen: En este artículo se calculan las fórmulas fiscales óptimas cuando se consume un bien que contamina. Luego se propone una prueba de la hipótesis del doble dividendo, una base en los precios sobre de dichas fórmulas.

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1. Introduction

Since Pigou the main objective of tax policies when considering externalities has been the correction of their negative effects on optimality. The type of remedy proposed was the direct taxation of the externality causing activity. For general externalities differential taxation would be needed requiring information about the damage and the offender. This type of taxation is generally not feasible because of the high cost of its implementation and because of informational constraints of several types, some of them causing incentive constraints. These reasons make the economic literature look for a more appropriate second best framework.

Diamond (1973) is aware of these problems and uses only uniform taxes that do not need information about the differences in the pollution activity pursued by individuals. Green and Sheshinski (1976) extend the approach of Diamond (1973) by introducing a commodity related to the externality creating good. By taxation of the related good additional control of the externality is provided. They show cases in which it is better to tax or subsidythe related good than to tax directly the externality creating good. Sometimes the optimal policy involves a subsidy of a complement of the externality creating good. These results are improved further by Balcer (1980) and Wijkander (1985).

Another line of research relating taxes and externalities begins with Sandmo (1975) and Sandmo (1976). He considers the problem of optimal collection of a given amount of tax revenue with one representative consumer and when consumption of a commodity creates externalities. He focusses on the optimal tax on the externality causing good. In the second paper he explores the problem of three commodities when one of them is related to the externality causing good. The substitution effects are very important to the determination of optimal taxes and also to the effect on the externality.

Closely related are Bovenberg and Goulder (1996), and Bovenberg and van der Ploeg (1994). An optimal tax on externality creating commodities will have two parts. The first one corrects for the externality. The other component will be a kind of Ramsey term for optimal taxes. Also related are Bovenberg and de Mooij (1994) and Cremer, Gahvari, and Ladoux (1998). The latter also discusses conditions for the validity of the (called since Sandmo) *additivity property* where the presence of pollution only alters the tax formula of the externality creating good. We will also review this property in this paper.

We will set the optimal tax problem of the government in which it will be very important the way commodities relate to the externality creating good and also how they relate to other commodities, even to those not related to the externality creating good. The government will also have other important objectives related to income distribution, efficiency of taxation, and tax collection. We will follow the methodology of Atkinson and Stiglitz (1976) to find optimal tax formulae in economies with many agents. We extend their model to cover the case in which consumption of some commodities causes externalities.

We analyze then the *Double Dividend* proposition. This proposition claims that a tax on externality creating goods reduces externalities and also allows the reduction of distortions by using its revenue to reduce, in a revenue neutral way, taxes on other commodities. We will set a methodology inspired on Ahmad and Stern (1991) together with our optimal tax problem to review and analyze this proposition. The relevant literature on the double dividend is summarized by Goulder (1995). (See also Bovenberg and van der Ploeg, 1994, and Bovernberg and de Mooij, 1997.)

The paper is organized as follows: Section 2 presents the model. Section 3 computes the optimal tax formulae. Section 4 relates optimal tax formulae with those arising from Pigouvian taxation. Section 5 presents our test for the double dividend hipothesis. Section 6 concludes the paper with some final remarks.

2. The Model, Notation, and Main Assumptions

Let H be the finite set of consumers in the economy indexed by h. They consume two types of goods, L tradable in the market and pollution which is not tradable.

Let $p \in \Re^L$ be the vector of producer prices and $t \in \Re^L$ be the vector of commodity taxes. Then $q = p + t \in \Re^L$ is the vector of consumer prices. For each $h \in H$, let m^h be the unearned income of consumer h. For each $h \in H$, let $\chi^h \in \Re^L$ be h's consumption of marketed goods. Let z be the amount of pollution in the economy.

We assume that only the consumption of the commodity indexed by 1 causes pollution and that pollution is produced depending on a given technology $z = f\left(\Sigma_h x_1^h\right)$. Thus we refer here to what Meade (1952) calls *atmosphere externalities.* Meaning those types that have a negative impact on the public. They depend on aggregate consumption of a

certain commodity provided consumers disregard the effect of their actions on the level of externalities.

We will assume that *f* is a twice differentiable, convex, and increasing function.

Each consumer $h \in H$ has a preference order defined over consumption and pollution levels that is represented by a continuous twice differentiable, strictly-quasi-concave utility function $U^h(x^h, z)$. Each consumer $h \in H$ solves a problem of utility maximization subject to a budget constraint. For each consumer $h \in H$ the indirect utility function is denoted by $v^h(q, m^h, z)$.

3. Optimal Tax Rules

The problem considered by the government is the maximization of a social welfare function of the type of Bergson-Samuelson

$$W(v^1(q, m^1, z), v^2(q, m^2, z),..., v^H(q, m^H, z))$$

subject to obtaining a given tax revenue *R*. The government budget constraint is $\Sigma_i \Sigma_h t_i x_i^h \geq R$. Let λ be the shadow price associated with this constraint. An additional constraint faced by the government is the technology of generation of pollution.

Before showing the first order conditions of the government problem, we compute the derivative of the welfare function with respect to t_k for $k \in L$,

$$\frac{dW}{dt_k} = \Sigma_h \frac{dW}{dv^h}\left(\frac{dv^h}{dt_k} + \frac{dv^h}{dz}\frac{dz}{dt_k}\right) = \Sigma_h \frac{dW}{dv^h}\left(-\alpha^h x_k^h + w^h f'\Sigma_h \frac{dx_1^h}{dt_k}\right) \quad (1)$$

where α^h is the marginal utility of income of *h* for $h \in H$, $w^h = dv^h/dz$, and Roy's Identity was used. For $h \in H$ let $\beta^h = (dW/dv^h)\alpha^h$ be the direct social value of a unit of money given to *h*. We substitute the Slutsky term for a change in the demand of the first commodity upon changes in the price of commodity $k \left(dx_1^h/dt_k = S_{k1}^h - x_k^h dx_1^h/dm^h\right)$ and add up for *h* in the last term. Then from expresion (1) we obtain

$$\frac{dW}{dt_k} = -\Sigma_h \beta^h x_k^h + W_z f'\Sigma_h\left(S_{k1}^h - x_k^h \frac{dx_1^h}{dm^h}\right) \quad (2)$$

where $W_z = \Sigma_h (dW/dv^h) w^h$ is the change in social welfare due to a change in the pollution level.

The first order conditions of the government problem are

$$\Sigma_h \beta^h x_k^h - W_z f' \Sigma_h \left(S_{k1}^h - x_k^h \frac{dx_1^h}{dm^h} \right) =$$

$$\lambda \left[\Sigma_h x_k^h + \Sigma_h \Sigma_i \left(t_i \frac{dx_i^h}{dt_k} + \frac{dx_i^h}{dz} \frac{dz}{dt_k} \right) \right] \tag{3}$$

Let $T_z = \Sigma_h \Sigma_i t_i \frac{dx_i^h}{dz}$ be the change in tax revenue due to a unitary change in the pollution level.

In (3) we substitute the Slutsky equation for changes in the demand of commodity *i* due to changes in t_k, and for $\frac{dz}{dt_k}$. Then the following expression is obtained

$$\Sigma_h \beta^h x_k^h - W_z f' \Sigma_h \left(S_{k1}^h - x_k^h \frac{dx_1^h}{dm^h} \right) = \lambda \left[\Sigma_h x_k^h + \Sigma_h \Sigma_i t_i \left(S_{ki}^h - x_k^h \frac{dx_i^h}{dm^h} \right) \right]$$

$$+ T_z f' \Sigma_h \left(S_{k1}^h - x_k^h \frac{dx_1^h}{dm^h} \right)$$

The optimal tax formula for $k \in L$ is

$$\Sigma_h \left[\frac{\beta^h}{\lambda} + \left(\frac{W_z}{\lambda} + T_z \right) f' \frac{dx_1^h}{dm^h} + \Sigma_i t_i \frac{dx_i^h}{dm^h} - 1 \right] x_k^h =$$

$$\Sigma_h \left[\Sigma_i t_i S_{ki}^h + \left(\frac{W_z}{\lambda} + T_z \right) f' S_{k1}^h \right] \tag{4}$$

We also define

$$b_o^h = \frac{\beta^h}{\lambda} + \Sigma_i t_i \frac{dx_i^h}{dm^h}$$

as the effect in social welfare (excluding the effect on the externality) of a monetary unit given to consumer *h* valued at shadow prices of public revenue. This term is formed by: the direct effect, and the indi-

rect change in tax revenue induced by the income effect of the change in the tax. This term coincides with the one in Atkinson and Stiglitz (1976).

We also define

$$b_z^h = \left(\frac{W_z}{\lambda} + T_z \right) f' \frac{dx_1^h}{dm^h}$$

as the effect in social welfare of a monetary unit given to consumer *h* because of the implied change in emissions. This term has two parts: (1) The direct effect on welfare due to induced changes in pollution valued in terms of shadow prices of public revenue; and (2) the indirect effect on tax revenue induced by the change in pollution.

If we substitute these expressions in (4), we obtain for $k \in L$

$$\Sigma_h \left(b_o^h + b_z^h - 1 \right) x_k^h = \Sigma_h \left[\Sigma_i t_i S_{ki}^h + \frac{W_z}{\lambda} f' S_{k1}^h \right] \qquad (5)$$

equivalently

$$\left[\Sigma_h \frac{\left(b_o^h + b_z^h \right)}{H} \frac{x_k^h}{\overline{x}_k} - 1 \right] \overline{x}_k H = \Sigma_h \left[\Sigma_i t_i S_{ki}^h + \frac{W_z}{\lambda} f' S_{k1}^h \right] \qquad (6)$$

for all $k \in L$. These are the first order conditions that have to satisfy an optimal tax structure. These formulae are similar to those obtained by Atkinson and Stiglitz (1976). The differences are that they include b_z^h, meaning the welfare effect of the change in pollution due to giving a unit of money to agent *h*. Also on the right hand side appears the distortion on demand of the externality causing commodity.

These equations introduce criteria to design optimal taxes. The effects on consumption distribution had to be taken into account and consumption deviations should be smaller when commodities are mostly consumed by consumers with greater b_os. If the social welfare function gives more weight to poorer people (β is greater for the poorer) and the taxed commodity is mostly consumed by them, then the tax rate should be lower. Therefore even though to have clean air is important, this is only one of the multiple objectives that policy-makers can have. In some countries income distribution and extreme poverty could be more important objectives.

At the same time we would have to compare the amount of distortions in consumption (a good proxy is the first term of the right hand side of the equation) with the effect on the pollution creating commod-

ity. The first term of the right hand side of the equation is directly related with the harbergerian measure of the deadweight loss. Here this is a function of the effect on consumption of the taxed commodity and also of the effect on the commodities related with that one by the terms of the Slutsky matrix. Therefore when we tax the first commodity we have to consider the deviation of consumption caused on other commodities.

The formulae also consider the effect of consumption distribution between individuals through the b_zs. It is important to identify who are the greater marginal polluters. If, as would be reasonable to assume (if we are considering a public bad), W_z and T_z are both negative, then distortions on goods consumed mainly by consumers with large b_z must be larger. This is because taxes on those commodities would lead to larger reductions in social disutility due to pollution.

When we compute the optimal tax formulae, it is also considered that by changing taxes on any commodity there are substitution effects and income effects. This means that if the first commodity is a normal good then with any other tax we can change its consumption. This effect could be reinforced by the substitution effect when the taxed commodity is complementary of the first commodity or when these commodities are substitutive and we are talking of a subsidy. The relation between commodities related to the externality causing good, given by the corresponding Slutsky term in order to design their taxes, is also important.

This reasoning implies that taxes on goods different than the externality creating ones will be different than they would be in the absence of pollution. This would mean that the *additivity property* does not generally hold in this model. Trivially, a sufficient condition for the *additivity property* to hold is that the utility functions are separable in pollution. So summarizing:

PROPOSITION 1: *Unless the utility functions are separable in pollution, the additivity property will not generally be satisfied.*

4. Pigou Taxes versus Optimal Taxes

Another important remark is that the optimal tax of the first commodity is not necessarily a Pigou tax. The formula includes distributive considerations, of efficiency in taxation, of tax revenue, and of the

social evaluation of changing the level of pollution. It also includes the fact that we can change the level of pollution by changing tax rates on goods different than the first commodity. By changing taxes on the first commodity we can also change the consumption levels of other commodities changing welfare.

A Pigou tax in the first commodity would have the value $t_1 = -W_z f'$. We are going to show that under certain conditions this would be the value obtained from the optimal tax problem. The first of these conditions is that all consumers share the same utility function and income to avoid distributional problems. Then the social welfare function could be the indirect utility function of a representative consumer. We also need to assume that the externality causing commodity is not related to other commodities with the substitution terms and that there are no income effects. This is obtained when externalities are separable in the utility function.

PROPOSITION 2: *Assume that all consumers share the same utility function that is separable in pollution. Assume also that the marginal value of public funds coincides with the marginal utility of income. Then the optimal tax on the externality creating commodity is a Pigou tax.*

PROOF: With these assumptions the derivative of the social welfare function with respect to t_1 is

$$\frac{dW}{dt_1} := \frac{dv}{dt_1} = \frac{dv}{dt_1} + \frac{dv}{dz}\frac{dz}{dt_1}$$

Applying Roy's identity and substituting $W_z = dv/dz$ and $f' dx_1/dt_1 = dz/dt_1$, we obtain the following

$$\frac{dW}{dt_1} = -\alpha x_1 + W_z f' \frac{dx_1}{dt_1}$$

The first order conditions of the optimal taxation problem are then

$$\alpha x_1 - W_z f' \frac{dx_1}{dt_1} = \lambda x_1 + t_1 \frac{dx_1}{dt_1}$$

so under the additional condition of the marginal value of private funds (α) being equal to the marginal value of public funds (λ) (condition

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that means that the government is looking for Pareto optimum allocations), $t_1 = - W_z f'$, as we wanted to show. ɣ

The conditions for equivalence between optimal taxes on externality causing goods and classical Pigou taxes are very restrictive. There is no reason for that equivalence as soon as the government has distributive objectives, has to collect revenue for any other reason (so shadow price of public revenue is different than shadow prices of private income), or if the rest of goods are not separable from the externality causing good.

5. The Double Dividend

The double dividend proposition claims that a tax on an externality causing good could reduce the externalities and also could give revenue allowing for a reduction (in a revenue neutral way) on other distortionary taxes. The different forms of the proposition and their criticism have been surveyed by Goulder (1994). In this section we present a different way of presenting and appraising the proposition based on the tax model that we have presented.

Following Ahmad and Stern (1991) the marginal revenue cost of changing a tax rate on a given commodity is given by the marginal change in welfare due to the tax divided by the marginal change in revenue due to the tax. In our model, for a tax on the externality creating commodity, the marginal revenue cost is given by

$$\lambda_1 := \frac{\Sigma_h \beta^h x_1^h - W_z f' \Sigma_h \dfrac{dx_1^h}{dt_1}}{\Sigma_h x_1^h + \Sigma_h \Sigma_i t_i \dfrac{dx_i^h}{dt_1} + T_z f' \Sigma_h \dfrac{dx_1^h}{dt_1}}$$

so it has two parts. The first is the welfare loss due to taxation per unit of public revenue. The second is the welfare gain because the reduction of externalities per unit of revenue. For a tax on a different commodity $k \in L$, the marginal revenue cost is given by

$$\lambda_k := \frac{\Sigma_h \beta^h x_k^h - W_z f' \Sigma_h \dfrac{dx_1^h}{dt_k}}{\Sigma_h x_k^h + \Sigma_h \Sigma_i t_i \dfrac{dx_i^h}{dt_k} + T_z f' \Sigma_h \dfrac{dx_1^h}{dt_1}}$$

Goulder (1995) defines the strong form of the double dividend proposition as: "The revenue neutral substitution of the environmental tax for a typical or representative distortionary tax involves a zero or negative gross cost." In our model of optimal taxation, this would be equivalent to $\lambda_1 \le \lambda_k$ holding for every k.

The intermediate form of the double dividend proposition instead is defined as: "It is possible to find a distortionary tax such that the revenue neutral substitution of the environmental tax for this tax involves a zero or negative gross cost." In our model, what is required is $\lambda_1 \le \lambda_k$ to hold for at least one $k \ne 1$.

Testing the double dividend proposition in any of its forms requires the computation of the λ for the different taxes. The data needed consists of the parameters of a demand system and estimations of the reduction in pollution and also of the willingness to pay for that reduction.

6. Final Remarks

In this paper we have shown conditions for the validity of the *additivity property* of optimal taxes in presence of externalities and for the equivalence of Pigou taxes and optimal taxes. We also presented a simple form based on our optimal tax rules to test the double dividend proposition. This test could be done with household expenditure data available for México, complemented with computations of the willingnesses to pay for clean environment.

Obviously the model has important limitations that are shared with other tax models of the type of Atkinson and Stiglitz (1976). The production prices are assumed constant so perfect competition and constant returns to scale (or a small international economy) are assumed. Another important implicit assumption is that our model is static. Environmental issues have a clear dynamic component.

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