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SONIA DI GIANNATALE AND ITZA T. Q. CURIEL-CABRAL

Compromises and Incentives

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Abstract

We analyze a situation where a Principal does not necessarily have all the bargaining power while negotiating a contract with an Agent by studying a dynamic multi-objective moral hazard model with hidden action. We .nd that the structure of the optimal contracts change along the Pareto Frontier, and that compromise solutions implement higher Agent.s e¤ort levels when compared to contracts located at the Pareto Frontier.s extremes. Our numerical results indicate that in compromise solutions, compared to the contracts located at the Pareto Frontier.s extremes, the Agent exerts higher e¤ort levels, the Agent.s future compensation schedules show higher spread, and the Agent.s salaries become more directly related to productivity outcomes as time goes on. When the coe¢ cient of relative risk aversion increases, compromise solutions tend to become closer to the Agent.s most advan-tageous contract. Improvements in the .rm.s productivity environment bene.t, in relative terms, the Agent more than the Principal when compromise solutions are implemented. *Keywords: Asymmetric information, Principal-Agent Model, Incentives, Pareto Frontier, Compromise Solutions, Multi-Objective Problems, Evolutionary Algorithms.*

JEL Classi.cation Numbers: C63, C78, D61, D82, D86, L14.

Resumen

Analizamos una situación donde un Principal no necesariamente tiene todo el poder de negociación a la hora de negociar un contrato con un Agente. Para ello estudiamos un modelo dinámico multi-objetivo de riesgo moral con acciones ocultas. Encontramos que la estructura de los contratos óptimos cambia a lo largo de la Frontera de Pareto del modelo y que las soluciones compromiso implementan niveles de esfuerzo gerencial más altos que los contratos situados en ambos extremos de la Frontera de Pareto. Nuestros resultados numéricos indican que en las soluciones compromiso, en comparación con aquéllas situadas a los extremos de la Frontera de Pareto, el Agente despliega un mayor nivel de esfuerzo, la compensación futura del Agente muestra mayor variabilidad y la compensación presente del Agente se vuelve más directamente relacionada con los resultados de productividad conforme avanza el tiempo. Cuando aumenta el coeficiente relativo de aversión al riesgo del Agente, las soluciones compromiso se aproximan al contrato más ventajoso para el Agente. Mejoras en el ambiente productivo de la firma proporcionan *mayores beneficios, en general, al Agente cuando las soluciones compromiso son implementadas.*

Palabras Clave:

Información Asimétrica, Modelos de Agente-Principal, Incentivos, Frontera de Pareto, Soluciones Compromiso, Problemas Multi-Objetivo, Algoritmos Evolutivos.

Números de clasificación JEL: C63, C78, D61, D82, D86, L14.

Compromises and Incentives

by

Itza T. Q. Curiel-Cabral, Sonia Di Giannatale*

Abstract

We analyze a situation where a Principal does not necessarily have all the bargaining power while negotiating a contract with an Agent by studying a dynamic multi-objective moral hazard model with hidden action. We find that the structure of the optimal contracts change along the Pareto Frontier, and that compromise solutions implement higher Agent's effort levels when compared to contracts located at the Pareto Frontier's extremes. Our numerical results indicate that in compromise solutions, compared to the contracts located at the Pareto Frontier's extremes, the Agent exerts higher effort levels, the Agent's future compensation schedules show higher spread, and the Agent's salaries become more directly related to productivity outcomes as time goes on. When the coefficient of relative risk aversion increases, compromise solutions tend to become closer to the Agent's most advantageous contract. Improvements in the firm's productivity environment benefit, in relative terms, the Agent more than the Principal when compromise solutions are implemented.

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1 Introduction

The level and composition of CEO pay have been greatly discussed both in the academic and in the popular literature. Recently, the debate has focused on the role that incentive pay might have played in excessive risk-taking that caused the 2007-2009 recession.

Agency models provide a formal environment to analyze the design of incentive-based compensation in the presence of asymmetric information between the Principal and the Agent, Holmstrom (1982). However, Jensen and Murphy (1990) questioned the formal agency models' relevance because they found empirical evidence that CEO pay did not include incentive-based compensation as extensively as those models would prescribe. Wang (1997) showed that the dynamic Principal-Agent model could rationalize the empirical findings of Jensen and Murphy for certain parameter specifications, providing support to formal agency models. Later, Murphy (1999) pointed out that even though there was heterogeneity in CEO pay practices among U.S. industries, several stylized facts could be identified. In particular, the level of CEO pay increased in the period 1992-1996, and this increase was mostly due to increases in stock option grants to executives.

In order to explain how managerial talent might account for the differences in the level of CEO pay, Tervio (2008) proposed an assignment model between managers and firms and demonstrated that differences in managerial abilities might cause high levels of CEO pay in a competitive equilibrium. However, the results of the model's empirical application indicated that the variation of CEO pay was mainly explained by firm heterogeneity, while differences in managerial ability played a significantly smaller role on this variation.

In the same line, Gabaix and Landier (2008) used an assignment model between managers and firms, and obtained that the direction and magnitude of the change in the size of all bigger firms was equal to the direction and magnitude of the change in their CEO's compensation, given those firms' higher willingness to pay for managerial talent. They called this theory on managerial pay the "size of stakes" view. Using data from 1970-2003, the authors found that their empirical results were consistent with this view. In a recent paper, Gabaix *et al.* (2013) find that managerial compensation in the bigger firms follows closely the average firm value during the years 2004-2011, that include the great recession of 2007-2009. They conclude that this result empirically supports the "size of stakes" view.

On the other hand, Gayle and Miller (2009) propose an empirical strategy to examine the reasons of the moral hazard cost's increase in the U.S. since 1944. They find that the leading explanation for this increase is exogenous growth in firm size, while they find no reason to attribute it to variation in managers' preferences because they find evidence of stable CEO preferences towards risk aversion. So, from the aforementioned articles, it seems that the relationship between firm size and the size and composition of CEO compensation should carefully looked at.

Moreover, an explanation for the behavior of CEO pay that has emerged in the literature is that CEOs have acquired a higher ability to extract rents from shareholders; Shivdasani and Yermack (1999), and Murphy and Sandino (2010). Pitchford (1998) explores the possibility that assuming that the Principal has all the bargaining power when negotiating a contract with the Agent is not innocuous, and proposed a bargaining model between the Principal and the Agent, assuming that both are risk neutral and a limited liability constraint for the Agent. Interestingly, the result is that when the distribution of bargaining power between the Principal and the Agent changes, the contracts and the Agent's effort also change.

In this article, we seek to further study the structure of incentive pay and its relationship with firm size in situations where the Principal does not have all the bargaining power while negotiating a contract with the risk-averse Agent. With this objective in mind, we propose a dynamic Principal-Agent model viewed as a multi-objective optimization problem. Many situations in the real world that involve optimizing conflicting objectives between two or more parts can be thought as multi-objective optimization problems. Most of the time it is impossible to find a unique solution to such problems. Thus, multi-objective optimization problems are characterized by a set of alternative and equivalent solutions because of the lack of information about the relevance of one objective with respect to the others. The set of optimal solutions is possibly of infinite dimensions by definition, and it is called the Pareto Optimal Frontier.

The structure of the dynamic Principal-Agent problem makes it possible to envision it as a multi-objective optimization problem, given the conflict of interest between the Principal and the Agent. The advantage of using this approach is that we can consider several incentive-based arrangements between the Principal and the Agent in which their utilities have several levels of priority.

On the other hand, evolutionary algorithms are heuristic methods of search, Goldberg (1989), based on the evolutionary analogy of the "survival of the fittest." This analogy is inspired on the modern evolutionary synthesis, where natural selection can be seen as a learning process in which the fittest individuals survive in a defined environment after a long time. Evolutionary algorithms are often used to provide numerical solutions to multi-objective optimization problems. We numerically approximate the Optimal Pareto Frontier that emerges from our model by using a recently proposed multi-objective evolutionary algorithm named RankMOEA, Herrera-Ortiz *et al.* (2011).

The remaining of this paper is organized as follows: in section 2 the model is presented. In Section 3 we explain the evolutionary algorithm approach used to numerically approximate the model's Pareto Frontier. The numerical results are discussed in section 4. Finally, we offer our concluding remarks.

2 The Model

In this section, we develop a multi-objective dynamic Principal-Agent model based on the standard repeated moral hazard model of Spear and Srivastava (1987).

We assume that time is discrete and that it goes on until infinity: $t = 0, 1, 2, ...$ There are two individuals: a risk neutral Principal and a risk averse Agent, who are both discounted expected utility maximizers with a common discount rate $\beta \in (0, 1)$. Suppose that the Agent has a continuous period utility function represented by: $v(w_t, a_t)$, which is assumed to be bounded, strictly increasing, and strictly concave with respect to w_t; and strictly decreasing and convex with respect to a_t. The variable $w_t \geq 0$ is the Agent's salary or present compensation at the end of every period. The variable a_t is the Agent's effort choice made at the beginning of every period, drawn from a compact set $A = [\underline{a}, \overline{a}]$, and it is unobservable to the Principal. We also assume that v is either additively or multiplicatively separable in its two arguments, w_t and a_t.

Every period a realization of the output y_t, drawn from the finite set Y, is observed by the Principal and the Agent. The stochastic relationship between the output realization and the Agent's effort choice is given by the time-invariant and i.i.d. distribution $F(y_t; a_t) > 0$, $\forall y_t \in Y$ and $\forall a_t \in A$. We assume that this distribution has a density $f(y_t; a_t)$.

The timing of the Principal-Agent relationship is the following: At $t = 1$ the Agent decides on a_1, output y_1 is drawn from the distribution $F(y_1; a_1)$, and the agent receives a compensation w_1. As pointed out above, the Principal observes y_1 but not a_1, so w_1 only depends on y_1. Assuming that the Principal and the Agent employ history-dependent pure strategies, at $t = 2$ the Agent decides on $a_2 = a(y_1)$, output y_2 is drawn from the distribution $F(y_2; a_2)$, and the agent receives a compensation $w_2 = w(y_1, w_1 = w(y_1), y_2)$. Hence, $w_2 = w(y_1, y_2)$. Therefore, at any time t there is a history of output realizations $h_t = \{y_s\}_{s=1}^{t}$, such that $y_s \in Y$. The Principal's decision is $w(h_t)$, and the Agent's decision is $a(h_{t-1})$, because this last decision has to be made before y_t has been realized.

The payoffs of the Principal and the Agent, given h_t, are determined as follows. Let $\pi(h_{t+\tau}; h_t, a)$ be the probability distribution of $h_{t+\tau}$ conditional on h_t and a. This distribution is recursively expressed in the following way:

$$d\pi(h_{t+\tau}; h_t, a) = f(y_{t+\tau}; a(h_{t+\tau-1}))d\pi(h_{t+\tau-1}; h_t, a)$$

where $d\pi(h_{t+1}; h_t, a) = f(y_{t+1}; a(h_t))$ defines a stationary Markov chain.

The payoffs of the Principal and the Agent, respectively, derived from their decisions in the subgame starting from h_t are defined as follows:

$$U(h_t, w, a) = \sum_{\tau=1}^{\infty} \beta^{\tau-1} \int_Y (y_{t+\tau} - w(h_{t+\tau}))d\pi(h_{t+\tau}; h_t, a)$$

$$V(h_t, w, a) = \sum_{\tau=1}^{\infty} \beta^{\tau-1} \int_Y v(w(h_{t+\tau}), a(h_{t+\tau-1}))d\pi(h_{t+\tau}; h_t, a).$$

Likewise, $U(w, a)$ and $V(w, a)$ are the Principal's and the Agent's payoffs, respectively, from the beginning of $t = 1$.

Given a sequence $w = \{w(h_t)\}$, the sequence $a = \{a(h_{t-1})\}$ is incentive compatible at h_t if:

$$V(h_t, w, a) \geq V(h_t, w, \overline{a}) = \sum_{\tau=1}^{\infty} \beta^{\tau-1} \int_Y v(w(h_{t+\tau}), \overline{a}(h_{t+\tau-1}))d\overline{\pi}(h_{t+\tau}; h_t, \overline{a}),$$

for any other sequence $\overline{a} = \{\overline{a}(h_{t-1})\}$, and $\overline{\pi}$ is the distribution in the future histories induced by w and $\overline{a}$.

A contract is defined by a history dependent Agent's effort recommendation $a_t(h_{t-1})$, and a history dependent Agent's compensation plan $w_t(h_t)$, that is $\sigma_t = (w_t(h_t), a_t(h_{t-1}))$. We say that a contract σ_t is feasible if:

$$a_t(h_{t-1}) \in A, \qquad \forall t \geq 1, \qquad \forall h_{t-1} \in Y_{t-1}, \tag{1}$$

$$0 \leq w_t(h_t) \leq y_t, \quad \forall t \geq 1, \qquad \forall h_t \in Y_t. \tag{2}$$

Condition (2) requires that the Agent's salary be non-negative and not greater than the current ouput.

Here we intend to solve a problem in which two conflicting objective functions are maximized: the *ex-ante* Principal's discounted expected utility, and the *ex-ante*

Agent's discounted expected utility. Given the multi-objective nature of this optimization problem, the expected solution is not a unique contract but a unique series of contracts that satisfy Pareto optimality. Each element of this Pareto optimal contract series is associated to some level of priority given to each of the objective functions to be maximized. Let $\gamma \in [0,1]$ be the priority assigned to the *ex-ante* Principal's discounted expected utility, and $(1-\gamma)$ the priority assigned to the *ex-ante* Agent's discounted expected utility. Now, a contract $\sigma^\gamma = (w_t^\gamma(h_t), a_t^\gamma(h_{t-1}))$ is Pareto optimal if there is no other feasible and incentive compatible contract φ^γ such that $(U(h_t, \varphi^\gamma), V(h^t, \varphi^\gamma)) \succeq (U(h_t, \sigma^\gamma), V(h^t, \sigma^\gamma))$, for all h_t, and for all $\overline{a}^\gamma$. Each Pareto optimal contract σ^γ maximizes both $U(h_t, \sigma^\gamma)$ and $V(h^t, \sigma^\gamma)$ subject to feasibility, and $V(h_t, \sigma^\gamma) \geq V(h_t, w^\gamma, \overline{a}^\gamma)$, for all h_t, and for all $\overline{a}^\gamma$.

Now, we follow Spear and Srivastava (1987) to transform this problem into a static variational one. The continuation profile from time $t+1$ onwards for contract σ^γ at any t, given h_t, is determined by $\sigma^\gamma \mid h_t$. This implies a continuation value from time $t+1$ onwards of $U(\sigma^\gamma \mid h_t)$ for the Principal, and of $V(\sigma^\gamma \mid h_t)$ for the Agent.

A contract σ^γ is temporary incentive compatible if:

$$a_t^\gamma(h_{t-1}) \in \arg\max_{a^\gamma \in A} \int_Y [v_t(w_t^\gamma(h_t)), a^\gamma) + \beta V(\sigma^\gamma \mid h_t)] f(y_t; a^\gamma) dy_t \tag{3}$$

This constraint ensures that there will be no deviations in the optimal path of the Agent's effort decisions, for any γ. Furthermore, in order to ensure the validity of the first order approach to this incentive compatibility constraint, we assume that the Monotone Likelihood Ratio Property and the Convexity of the Conditional Distribution Condition are satisfied, following Rogerson (1985).

Now, let us assume that $\mathcal{U}$ and $\mathcal{V}$ are the spaces where the Principal's and Agent's discounted expected utilities can take values, respectively. Also, assume that $\mathcal{U}$ and $\mathcal{V}$ are both non-empty and compact. Our multi-objective optimization problem, for any $\gamma \in [0,1]$, is:

$$\max_{\sigma^\gamma} \{(U(\sigma^\gamma \mid h^0), V(\sigma^\gamma \mid h^0)) \in (\mathcal{U}, \mathcal{V})\} \;\; s.t. \;\; (1), (2), \text{ and } (3).$$

A solution, when considering all values of $\gamma \in [0,1]$, to this problem is a unique series of Pareto optimal contracts, or Pareto Frontier PF^*.

Define $\mathcal{W} = \{(U(\sigma_t), V(\sigma_t)\}$ as the set that contains all the values of the Principal's and the Agent's discounted expected utilities, respectively, that are feasible and incentive compatible.

Proposition 1 *$\mathcal{W}$ is compact*[1].

Given that a unique series of Pareto optimal contracts exist, we will characterize them in a Bellman equation. Given Proposition 1, we define $(U, V)^*$ as the Pareto optimal values of the Principal's and the Agent's, respectively, discounted expected utilities that belong to $\mathcal{W}$. Now, Γ is an operator that maps from the space of the cartesian product of two spaces of continuous and bounded functions, one for the Principal and one for the Agent, with the $\sup^*$ norm, where $\sup^* = \max(\sup, \sup)$[2], into itself. The function $U : \mathcal{W} \to \mathbb{R}$ is bounded because the Principal's rewards are bounded, and the function $V : \mathcal{W} \to \mathbb{R}$ is also bounded because the Agent is risk averse and his compensations are bounded. Hence, $\forall W \in \mathcal{W}$:

$$\Gamma(U, V)(W) = \max_{w(y,W), \overline{V}(y,W), \overline{U}(y,W)} \{U, V\} \tag{4}$$

$$U = \int_Y [y - w(y, W) + \beta \overline{U}(y, W)] f(y; a^*) dy \tag{5}$$

$$V = \int_Y [v(w(y, W), a^*) + \beta \overline{V}(y, W)] f(y; a^*) dy \tag{6}$$

subject to

$$a^* \in \arg\max_{a(W) \in A} \int_Y [v(w(y, W), a(W)) + \beta \overline{V}(y, W)] f(y; a) dy \tag{7}$$

$$0 \leq w(y, W) \leq y \qquad \forall y \in Y \tag{8}$$

$$(\overline{U}(y, W), \overline{V}(y, W)) \in \mathcal{W} \quad \forall y \in Y \tag{9}$$

where, (7) is the incentive compatibility constraint; (8) indicates the Agent's temporary inability to borrow; and (9) ensures that the Principal's and the Agent's future

[1] Proofs of Propositions 1 and 2 are in Appendix 1.

[2] The operator $\sup^*$ should actually be defined as follows: $\sup^* = \sup\{\sup, \sup\}$. But, given that the functions (U, V) are bounded and continuous, we can define the operator $\sup^*$ as follows: $\sup^* = \max\{\sup, \sup\}$. Also, the operator max of Γ should be understood in the Pareto optimal sense.

utility plans are feasible. We now establish that $(U^*(W), V^*(W))$ is a fixed point of Γ.

Proposition 2 $(U^*(W), V^*(W)) = \Gamma(U^*, V^*)(W)$, $\forall W \in \mathcal{W}$.

The operator Γ satisfies Blackwell's sufficient conditions for a contraction, and the contraction mapping theorem ensures that the fixed point $(U^*(W), V^*(W))$ is unique, $\forall W \in \mathcal{W}$. This means that along the PF^* there exists only one maximal value of Principal's discounted expected utility, given a value of the Agent's discounted expected utility, for all $W \in \mathcal{W}$; and viceversa. Now, PF^* must be non-increasing because otherwise either the Principal or the Agent can achieve a higher level of discounted expected utility and the other individual would be better off, see Spear and Srivastava (1987).

In order to characterize the optimal contracts in the Pareto Frontier, we make an additional assumption, following Spear and Srivastava (1987); *i.e.*, the Agent's static preferences can be represented by an additively separable utility function of the form: $\phi(w) - \psi(a)$; where ϕ is strictly increasing and strictly concave with respect to w for all $0 \leq w \leq y$, and ψ is strictly decreasing and strictly convex with respect to a for all $a \in A$.

Proposition 3 *The optimal contracts that belong to the Pareto Frontier satisfy:*

$$\frac{\gamma}{\phi'(w(y, W)} = (1-\gamma) - \mu(w)\frac{f_a(y;a)}{f(y;a)},$$

and

$$\gamma\left[\int_Y [y - w(y, W) + \beta U(y, W))] f_a(y;a)dy\right]$$
$$+\mu(w)\left[\int_Y [\phi(w(y, W)) + \beta V(y, W)] f_{aa}(y;a)dy - \psi''(a)\right] = 0,$$

where $\gamma \in (0, 1)$, and $\mu(w)$ is the Lagrange multiplier associated with the Temporary Incentive Compatibility Constraint.

Proof. According to Hernandez-Lerma and Romera (2004), our multi-objective dynamic optimization problem admits the following scalarization: $\max\limits_{w(y,W),\overline{V}(y,W),\overline{U}(y,W)} [\gamma U+$

$(1-\gamma)V]$ subject to the specialization of constraints (7), (8), and (9). The results are obtained by differentiating the Lagrangean of this optimization problem. ■

Notice that when $\gamma \to 1$, the structure of the optimal contracts in this model is similar to that of the optimal contract in Spear and Srivastava (1987); and therefore, the interpretation is similar. However, when $\gamma \to 0$, we obtain that: $1 = \mu(w)\frac{f_a(y;a)}{f(y;a)}$, which suggests an almost perfect risk sharing in terms of current compensation. Also, $\mu(w)\left[\int_Y [\phi(w(y,W)) + \beta\overline{V}(y,W)]f_{aa}(y;a)dy - \psi''(a)\right] = 0$. If the incentive compatibility constraint holds, the term $\overline{V}$ affects $\mu(w)$. Consequently, $\mu(w)$ is not a constant. Moreover, in the expression $\frac{1}{\mu(w)} = \frac{f_a(y;a)}{f(y;a)}$, $\frac{f_a(y;a)}{f(y;a)}$ is not equal to a constant, and the resulting contract is not first best. Hence, for these types of contracts, current compensation must have a direct relationship with the output realization but it is not a constant through all the output realizations.

In the next section we propose a methodology to numerically approximate the Pareto Frontier derived from this model, and to look at the structure of several types of contracts that emerge in different areas of the Pareto Frontier.

3 Numerical Approach

3.1 Functional Forms and Parameter Values

We assume that the Principal's temporary utility function is: $u(y, w(y,W) = y - w(y,W)$; while that of the Agent is: $v(a(W), w(y,W)) = \frac{(w(y,W))^{1-h}}{1-h} - a(W)$, where $1 > h > 0$. The Agent's temporary utility function is of the CRRA type with respect to current compensation, and the coefficient of relative risk aversion is h, where higher degrees of relative risk aversion are observed with higher values of h. We assume that the Agent's feasible effort choices are continuous and that they belong to the compact set $A = [0; \overline{a}]$.

Also, we suppose that there are two levels of output: low (L) or high (H), described by the set $Y = \{y_L, y_H\}$. The probability function that formalizes the stochastic relationship between effort and output is:

$$\begin{aligned} f(y_L; a) &= \exp(-a) \\ f(y_H; a) &= 1 - \exp(-a), \end{aligned}$$

and these probabilities capture the idea that the higher the Agent's effort level choice is, the greater the likelihood of the realization of the high output level.

The parameter values we use for our bechmark numerical exercise are the following: $h = \frac{1}{2}$, $\beta = 0.96$, $Y = \{y_L = 2, y_H = 4\}$, $A = [0; \overline{a}]$, where $\overline{a}$ is a sufficiently high number.

Further numerical exercises are variations of this benchmark case, as follows: $h = \{\frac{1}{4}, \frac{3}{4}\}$. Also, we analyze the cases with several output sets: $Y = \{y_L = 3, y_H = 4\}$, $Y = \{y_L = 2, y_H = 3\}$, $Y = \{y_L = 2, y_H = 5\}$, and $Y = \{y_L = 4, y_H = 8\}$.

3.2 Optimal Contracts

In order to gain insight on the structure of the optimal contracts that emerge from our multi-objective Principal-Agent model, we specialize it using the functional forms provided above.

First, we analyze the static version of our multi-objective optimization problem, which can be expressed in the following way:

$$\max_{(w_L, w_H)} \gamma[\exp(-a^*)(y_L - w_L) + (1 - \exp(-a^*))(y_H - w_H)]$$
$$+(1-\gamma)[\exp(-a^*)(\frac{w_L^{1-h}}{1-h}) + (1 - \exp(-a^*))(\frac{w_H^{1-h}}{1-h}) - a^*]$$

subject to

$$a^* \in \arg\max_{a \in A}[\exp(-a)(\frac{w_L^{1-h}}{1-h}) + (1 - \exp(-a))(\frac{w_H^{1-h}}{1-h}) - a]$$

$$0 \leq w_i \leq y_i, \qquad i = L, H.$$

From the incentive compatibility constraint, we obtain that the optimal effort choice is $a^* = \ln(\upsilon_H - \upsilon_L)$, where $\upsilon_i = \frac{w_i^{1-h}}{1-h}$, for $i = L, H$. This result indicates that a higher Agent's effort follows a higher spread in the Agent's utility from compensation. Now, let $\chi = \upsilon_H - \upsilon_L$, and $e(\upsilon_i) = [(1-h)\upsilon_i]^{\frac{1}{1-h}}$, for $i = L, H$; then, our static optimization problem can be expressed as follows:

$$\max_{\chi \geq 1} \gamma[\frac{1}{\chi}(y_L - e(\upsilon_L)) + (1 - \frac{1}{\chi})(y_H - e(\upsilon_H))]$$

$$+(1-\gamma)[\frac{1}{\chi}(\upsilon_L) + (1 - \frac{1}{\chi})(\upsilon_H) - \ln(\chi)]$$

subject to

$$0 \leq w_i \leq y_i, \qquad i = L, H.$$

The first order condition of this static optimization problem, given an interior solution, is the following:

$$\begin{aligned} \gamma\frac{1}{\chi^2}(y_H - y_L) &= \gamma\frac{1}{\chi^2}[e(\upsilon_H) - e(\upsilon_L)] - \gamma\frac{1}{\chi}[e'(\upsilon_H) - e'(\upsilon_L)] + \gamma e'(\upsilon_H) \\ &\quad -(1-\gamma)(\upsilon'_H - \frac{1}{\chi}) \end{aligned}$$

Notice that if $\gamma = 1$, this result is equal to:

$$\frac{1}{\chi^2}(y_H - y_L) = \frac{1}{\chi^2}[e(\upsilon_H) - e(\upsilon_L)] - \frac{1}{\chi}[e'(\upsilon_H) - e'(\upsilon_L)] + e'(\upsilon_H),$$

and it is equal to the expression obtained by Clementi *et al.* (2010) when they analyze the standard static agency model. The interpretation offered for this result is that the expected marginal revenue gain from an Agent's higher effort should be equal to the raise in the marginal cost of compensating the Agent from exerting a higher effort.

If $\gamma = 0$, the above expression reduces to:

$$\upsilon'_H = \frac{1}{\chi},$$

and this result defines a partial differential equation. Its solution can be characterized by $\upsilon_H = \ln(\chi) + k$, where k is a constant. That is, $\upsilon_H = a^* + k$. Morevoer, if $a^* = 0$, we have that $\upsilon_H = 0$; and this defines a boundary condition that implies that $k = 0$. Hence, $\upsilon_H(\gamma = 0) = a^*$, a result that can be interpreted as a constant compensation prescription in the case of a high productivity shock for this particular case ($\gamma = 0$).

Finally, if $\gamma = \frac{1}{2}$, the first order condition can be written as follows:

$$\frac{1}{\chi^2}(y_H - y_L) = \frac{1}{\chi^2}[e(\upsilon_H) - e(\upsilon_L)] - \frac{1}{\chi}[e'(\upsilon_H) - e'(\upsilon_L)] + e'(\upsilon_H) - (\upsilon'_H - \frac{1}{\chi}),$$

and this implies that the marginal cost of compensating the Agent from exerting a higher effort in this case is lower than in the case where $\gamma = 1$ if $\upsilon'_H - \frac{1}{\chi} > 0$. Given the result we obtained for the case $\gamma = 0$, we can say that $\upsilon'_H(\gamma = \frac{1}{2}) - \frac{1}{\chi} > 0$ if $\upsilon_H(\gamma = \frac{1}{2}) = \upsilon_H(\gamma = 0) + \xi$, for $\xi > 0$. Hence, we expect that solutions that are located around the middle section of the numerically approximated Pareto Frontier offer higher Agent's compensation for the high productivity shock, which is more likely to occur if the implemented Agent's effort is higher than in the two other cases.

On the other hand, a higher difference between the productivity shocks should increase the marginal gain of implementing a higher Agent's effort, $\forall\gamma$.

Now, the dynamic version of our multi-objective optimization problem is as follows:

$$\begin{aligned}
&\max_{w(y,W),V(y,W),U(y,W)} \gamma[\exp(-a^*)(y_L - w(y_L,W) + \beta U(y_L,W)) \\
&+(1-\exp(-a^*))(y_H - w(y_H,W) + \beta U(y_H,W))] \\
&+(1-\gamma)[\exp(-a^*)(\frac{w(y_L,W)^{1-h}}{1-h} + \beta V(y_L,W) \\
&+(1-\exp(-a^*))(\frac{w(y_H,W)^{1-h}}{1-h} + \beta V(y_H,W)) - a^*]
\end{aligned}$$

subject to

$$\begin{aligned}
a^* \in\ & \arg\max_{a(W)\in A}[\exp(-a(W))(\frac{w(y_L,W)^{1-h}}{1-h} + \beta V(y_L,W)) \\
& +(1-\exp(-a(W)))(\frac{w(y_H,W)^{1-h}}{1-h} + \beta V(y_H,W))) - a(W)]
\end{aligned}$$

$$0 \le w(y_i,W) \le y_i \qquad i = H,L$$

$$(U(y_i,W),V(y_i,W)) \in \mathcal{W} \qquad i = H,L.$$

From the incentive compatibility constraint, we obtain that the optimal effort is $a^* = \ln(\upsilon_H - \upsilon_L + \beta(\overline{V}_H - \overline{V}_L))$; where $\upsilon_i = \frac{w(y_i,W)^{1-h}}{1-h}$, and $\overline{V}_i = V(y_i,W)$, for $i = L,H$. This result is similar to that of the static case, only that here the spread of the Agent's promised discounted expected utilities is also included in the determination of the Agent's optimal effort level. Given this similarity, we expect that, in the dynamic case, higher effort levels from the Agent are to be implemented in the solutions that are located in the intermediate section of the numerically approximated Pareto Frontier. Next, we explain the strategy we use to approximate the Pareto Frontier.

3.3 The Computational Algorithm

Multi-Objective Evolutionary Algorithms (MOEAs) constitute a reliable methodology to achieve the two ideal goals of MO: attaining a good convergence to the Optimal Pareto Frontier, and maintaining the distribution of the Pareto Frontier approximation as diverse as possible.

Evolutionary Algorithms (EAs) are stochastic methods of search often applied to optimization, Goldberg (1989). EAs have shown to be a promising approach to deal with MOPs; however, they usually do not guarantee the identification of optimal trade-offs, only that they will find good assessments, *i.e.*, the set of solutions ($Pareto\ Frontier\ Approximation - PF^*_{known}$) whose objective vectors are not too far from the optimal objective vectors. In recent years, several MOEAs have been proposed, but most of them are unable to deal with incommensurable objectives. In this article, we use a recently proposed MOEA, named RankMOEA because of some advantages observed in numerical approximations of the solution of a dynamic model similar to the one proposed here, Herrera *et al.* (2011). The details of our computational algorithm are described in Appendix 2.

In Figure 1 we can observe the evolution of our model's Pareto Frontier as the number of generations increases. Notice that both the spread and proximity of the points of the Frontier improve as more generations are considered, and that the difference between the Frontier when 100,000 generations have been considered and when 500,000 generations have been considered is negligible.

4 Results

4.1 Conceptual Framework

With the objective of studying the structure of the contracts that emerge at different points of the numerically approximated Pareto Frontier, we introduce some concepts that allow us to identify specific contractual arrangements that are located the intermediate region of the Pareto Frontier. Namely, we identify some contracts that can be characterized using compromise solutions in multiobjective optimization, Yu and Leitmann (1974).

First, the utopia point of our dynamic model is defined as (U^*, V^*), where $U^* = \sup U$ and $V^* = \sup V$ for all feasible and incentive compatible strategies a^*, $w(y, W)$, and $V(y, W)$.

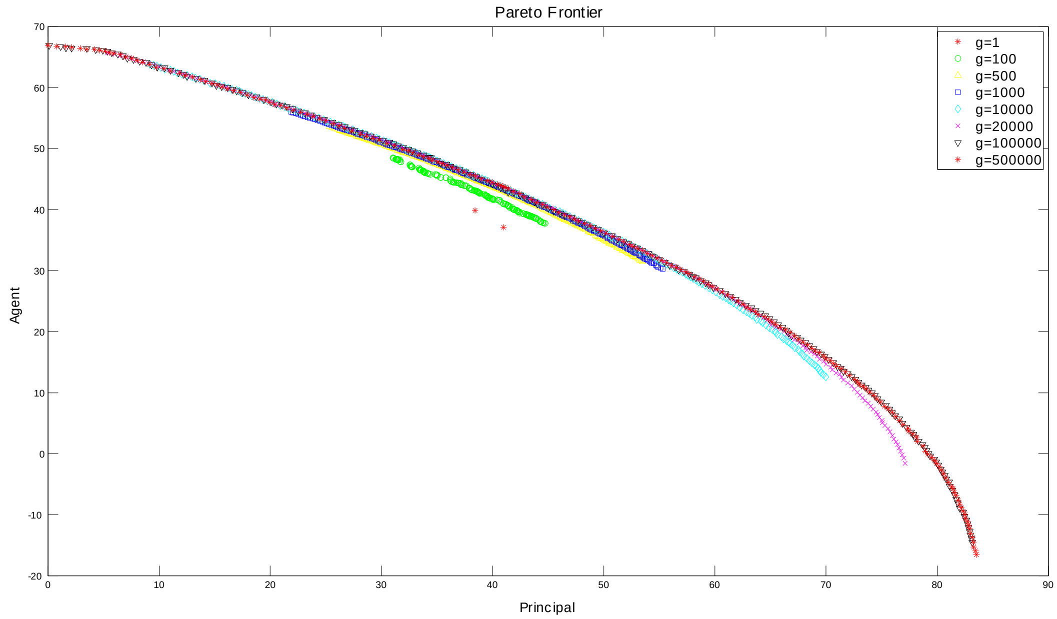

Figure 1: Approximating the Pareto Frontier.

Now, we define a regret function:

$$R_p(U,V) = [\sum_j ((U^*,V^*) - (U_j,V_j))^p]^{\frac{1}{p}}, \qquad p \geq 1,$$

where j is an indicator of of the pairs (U,V) that are Pareto Optimal.

The pair (U^p, V^p) is the compromise solution with parameter $p \geq 1$ if an only if (U^p, V^p) minimizes $R_p(U,V)$. Notice that if $p = 2$, the regret is associated with the Euclidean norm (also known as Salukvadze's solution, see Yu and Leitmann, 1974). If $p = \infty$ the regret corresponds to a minimax criterion: $\underset{(U,V)}{min}\ \underset{j}{max}(U^*,V^*) - (U_j,V_j)$.

These compromise solutions satisfy feasibility, least group regret, no dictatorship, Pareto optimality, uniqueness, symmetry or principle of equity, independence of irrelevant alternatives, continuity, monotonicity, and monotonicity of the group utilities and the individual regrets, Yu and Leitmann (1974).

In the discussion of our results, we will identify, in the numerically approximated Pareto Frontier, both compromise solutions: the Euclidean and minimax compromise solutions, namely (U^2,V^2) and (U^∞,V^∞).

4.2 The Benchmark Case

First, we discuss the benchmark case, in which the difference $(y_H - y_L) = 2$ and $\frac{y_H}{y_L} = 2$. In Figure 2, we show the results of the contract that gives priority to the Principal's expected discounted utility, $(\gamma \to 1)$. That is, the contract that is located in the right extreme of the Pareto Frontier depicted in the upper-left panel. The Pareto Frontier is decreasing and strictly concave, as expected from other related articles (Spear and Srivastava, 1987; and Wang, 1997). The Agent's effort schedule is decreasing and concave, meaning that the probability of the high productivity shock decreases more rapidly as the contractual lifespan approaches its maximal value of $p = 70$. Both the Agent´s expected discounted utilities for the low (L) and high (H) productivity levels are increasing and become closer (meaning that the spread between the two diminishes) as $p \to 70$. It is interesting to notice that given that this contract is the most advantageous for the Principal, the incentives in future utility that the Principal offers to the Agent are very punitive (negative utilities); and that only when $p \to 70$ and if there is a high productivity shock the Principal offers him a positive expected discounted utility. Negative Agent's discounted expected utilities are admissible under our optimization program, because they are a result of the Agent exerting feasible but high effort levels, and being paid positive but low salaries. The behavior of the Agent's current compensation schedules for the low and high productivity levels indicates that this incentive tool is used by the Principal when $p \to 70$, since the spread between the two is almost zero for many periods and becomes larger as $p \to 70$. So, in the initial periods of the Principal-Agent relationship, the incentive tool that the Principal favours is the future utility because it is cheaper in terms of the Principal's utility; while in the final periods of this relationship the Principal uses the Agent's current utility or salary as the preferred incentive tool.

In Figure 3, we show the results of the most advantageous contract for the Agent, $(\gamma \to 0)$; that is, the contract that is located in the left extreme of the same Pareto Frontier depicted in previous graph. The Agent's effort schedule has the same shape as in Figure 2; however, in the initial periods the Agent chooses higher effort levels and in the later periods the Agent chooses lower effort levels compared to those in Figure 2. The schedule of the Agent´s expected discounted utilities for the low (L) productivity shocks is increasing and starts at higher levels of utility (still negative) than that of Figure 2, and has positive values since $p = 25$. The schedule of the Agent´s discounted expected utilities for the high (H) productivity shocks is decreasing, unlike that corresponding schedule in Figure 2, and positive-valued. The spread between those two schedules lowers as $p \to 70$, with a similar interpretation as in Figure 2. The behavior of the Agent's current compensation schedules for the low and high

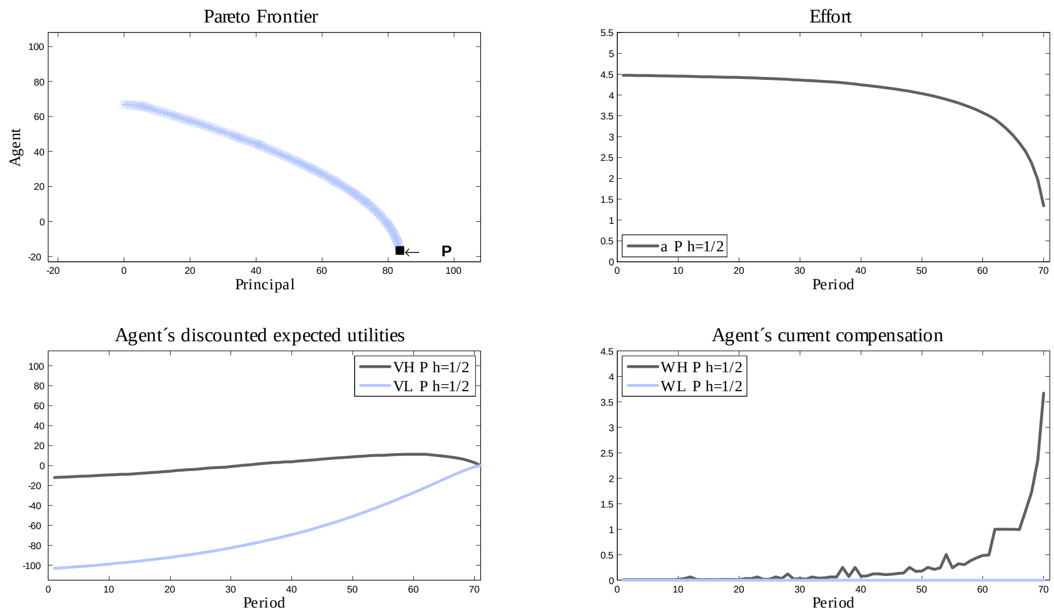

Figure 2: Principal's Most Advantageous Contract: Benchmark Case.

productivity levels indicates the positive relationship between the Agent's salary and the output realization, as pointed out when discussing Proposition 3. This means that the Agent is assuming less risk inherent to the productive activity with respect to what is observed in the previous figure, and only faces compensation variability through future pay. Moreover, the Agent enjoys higher levels of future pay than in the previous contract.

In Figure 4, we show the solutions of minimizing regret functions with respect to the utopia point using both the Euclidean norm (or Salukvadze's solution, solution S from now on), and the infinite norm (or minimax solution, solution M from now on). Notice that solutions S and M are located around the middle of the Pareto Frontier; but solution S is closer to the Principal's most advantageous contract, while solution M is closer to the Agent's most advantageous contract. The Agent's effort schedule has the same shape as in the previous two figures; however, in both solutions S and M, the Agent's effort is higher than in the Principal's and Agent's most advantageous contracts.

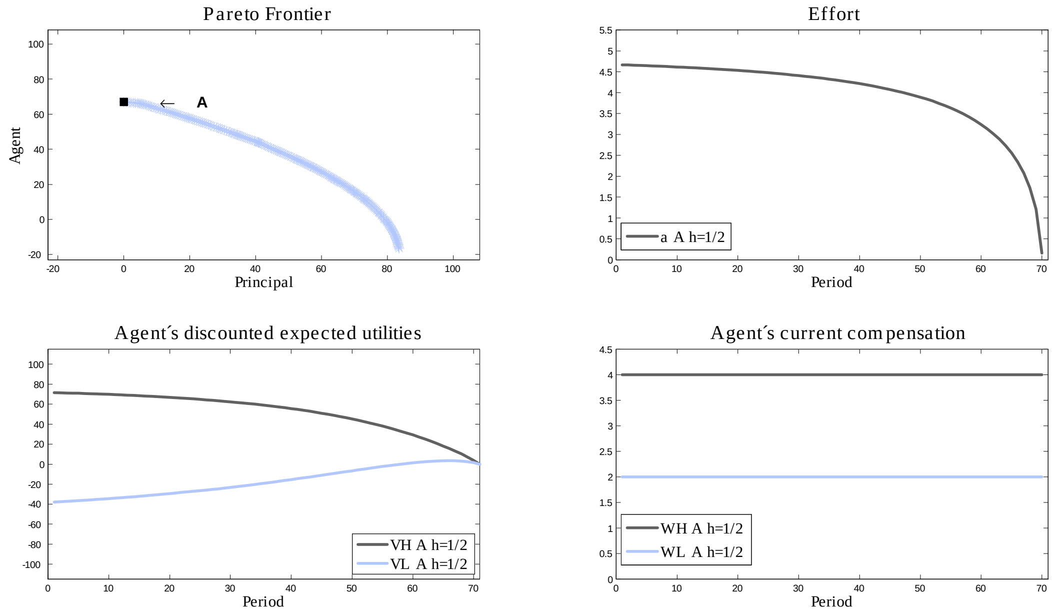

Figure 3: Agent's Most Advantageous Contract: Benchmark Case.

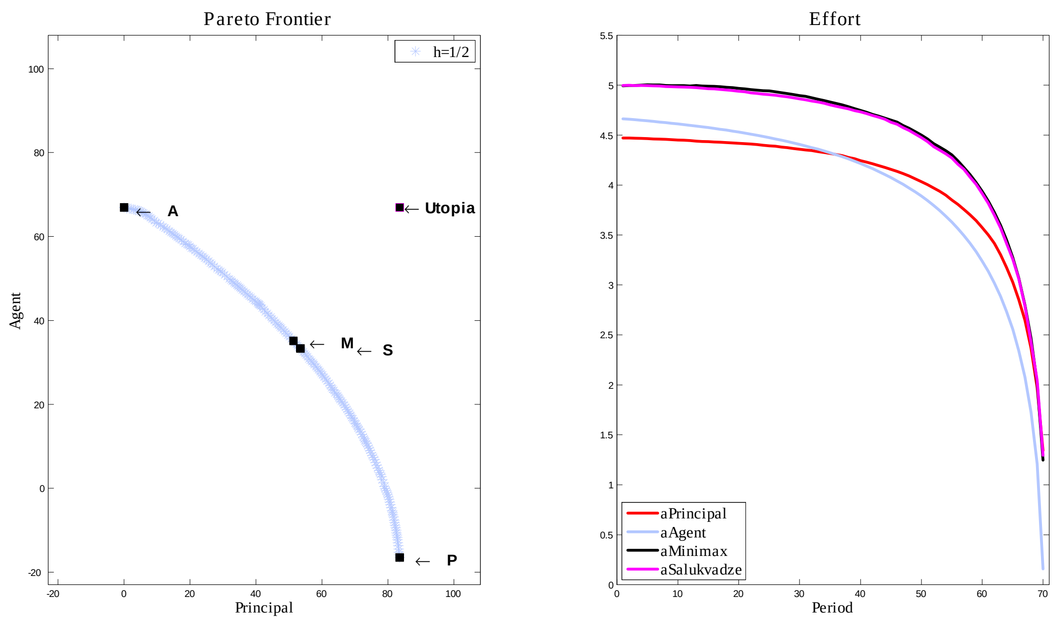

Figure 4: Solutions S and M: Benchmark Case.

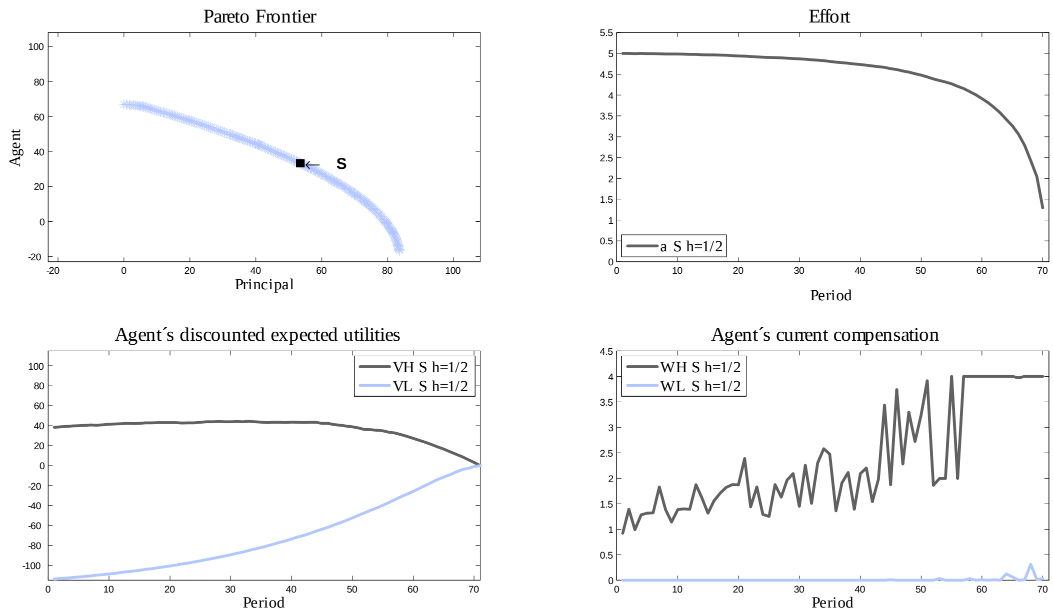

Figure 5: Solution S: Benchmark Case.

In Figure 5, we show the details of solution S. The schedule of the Agent´s expected discounted utilities for the low (L) productivity shocks is increasing and starts at lower levels of utility (negative) than that of Figure 2. The schedule of the Agent´s discounted expected utilities for the high (H) productivity shocks is non-increasing. It is always positive-valued. The spread between those two schedules significantly lowers as $p \to 70$, and it is higher than in the two previous contracts in the first periods. The behavior of the Agent's current compensation schedules for the low and high productivity levels is similar to that in Figure 2; however, the spread between the two is higher and with richer dynamics than that observed in that figure. This incentive tool is actively used by the Principal since the first periods, but it appears to be more intensively used in later periods. So, in this contract the Agent is assuming more risk inherent to the production process and exerting more effort in comparison to the Agent´s most advantageous contract. We must also add the observation that this contract has some aspects that are similar to some observations made while analyzing Figure 3. From $p = 55$ on, current compensation schedules show a behavior that is similar to that observed in Figure 3. However, in this contract the Agent is exerting more effort than in the Agent's most advantageous contract.

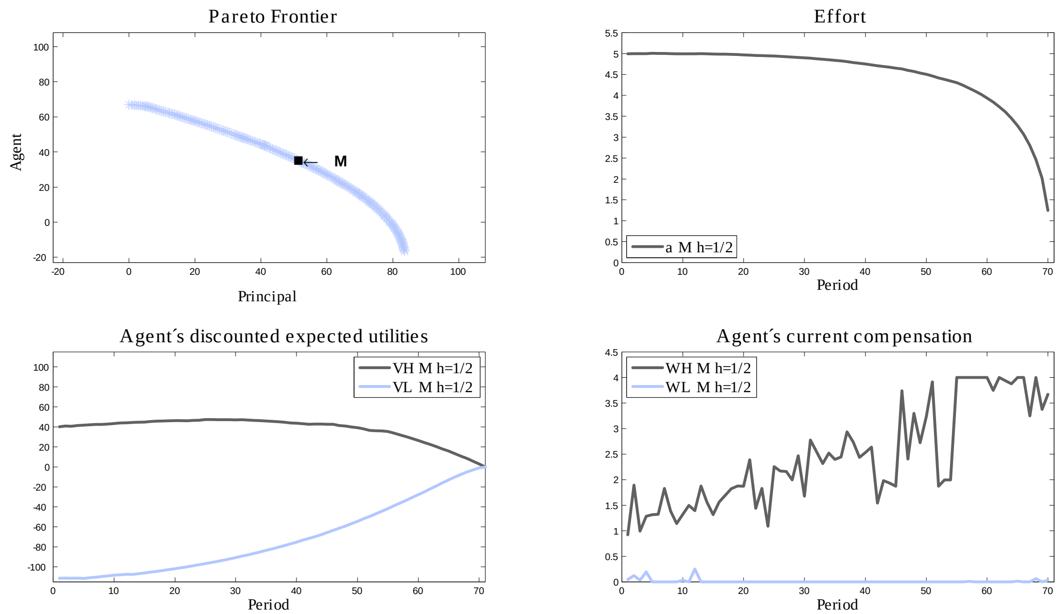

Figure 6: Solution M: Benchmark Case.

In Figure 6, we show the details of solution M. Given the proximity of this solution to solution S, both contracts look similar. The main difference is that the behavior of the current compensation schedules in solution M starts looking similar to the Agent's most advantageous contract at a slightly earlier period than observed in solution S.

The explanation of the result that in both solutions S and M, the Agent's effort is higher than in the Principal's and Agent's most advantageous contracts lies in the spread of both the current and promised discounted expect utilities of the Agent, as pointed out in the subsection about optimal contracts. To show this, we now present in Figure 7 the behavior of the spreads $\beta(\overline{V}_H - \overline{V}_L)$ in the first panel, and $(w_H - w_L)$ in the second panel.

Also, in Table 1, we show the sums of the aforementioned total spreads for each solution:

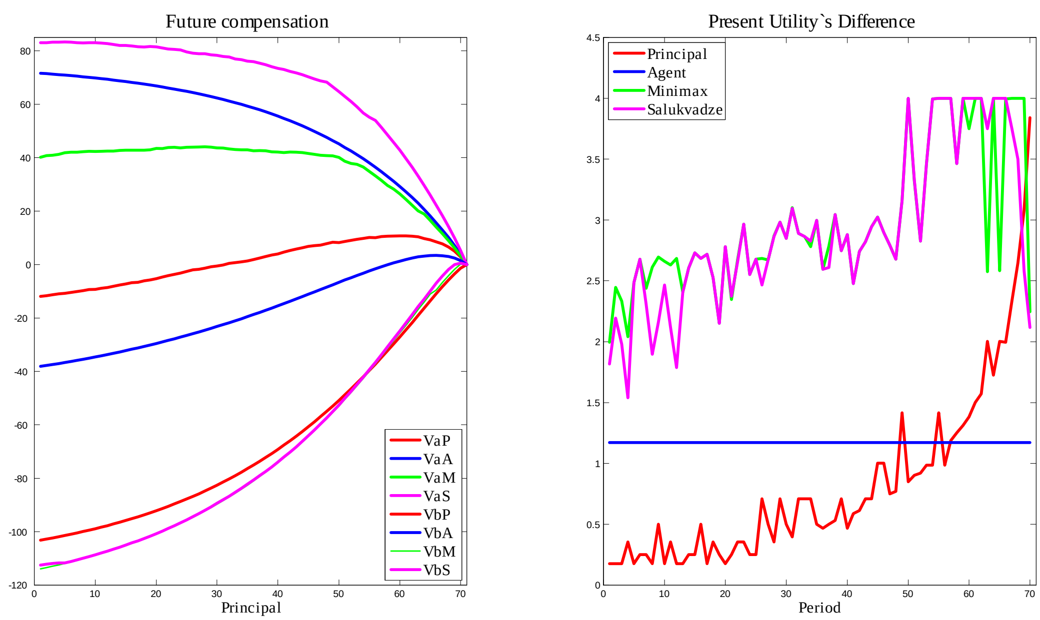

Figure 7: Compensation Spreads: Benchmark Case.

Table 1

Solution	$w_H - w_L$	$\beta(\overline{V}_H - \overline{V}_L)$
Principal	164.49	7156.35
Agent	63	4647.81
S	322.15	9760.87
M	325.97	10113.36

The results showed in Figure 7 and Table 1 confirm that higher spreads in the current and promised discounted expected utilities of the Agent, observed in solutions S and M, result in higher optimal effort levels from the Agent. Also, this occurs in solutions S and M because in those solutions it is marginally cheaper to implement those higher effort levels from the Agent, as can be seen in the sub-section about optimal contracts.

4.3 Risk Aversion

In this sub-section, we explore the effects of risk aversion on the model's numerical results. First, we consider $h = \frac{1}{4}$, which is a value that reflects an Agent's lower level of risk aversion compared with the benchmark case.

In Figure 8 we show the results of the contract that gives priority to the Principal's expected discounted utility. The Pareto Frontier is still decreasing and strictly concave. The Agent's effort schedule, the Agent´s expected discounted utilities for the low (L) and high (H) productivity levels, and the Agent's current compensation schedules for the low and high productivity levels are very similar to those in Figure 2. Hence, the incentive provision mechanism works similarly; but given that the Agent is less risk-averse, his utility entitlement is lower.

In Figure 9 we show the results of the most advantageous contract for the Agent for $h = \frac{1}{4}$. The Agent's effort schedule, the Agent´s expected discounted utilities for the low (L) and high (H) productivity levels, and the Agent's current compensation schedules for the low and high productivity levels are very similar to those in Figure 3. Hence, the incentive provision mechanism works similarly; but, again, given that the value of h is lower, his utility entitlement is lower.

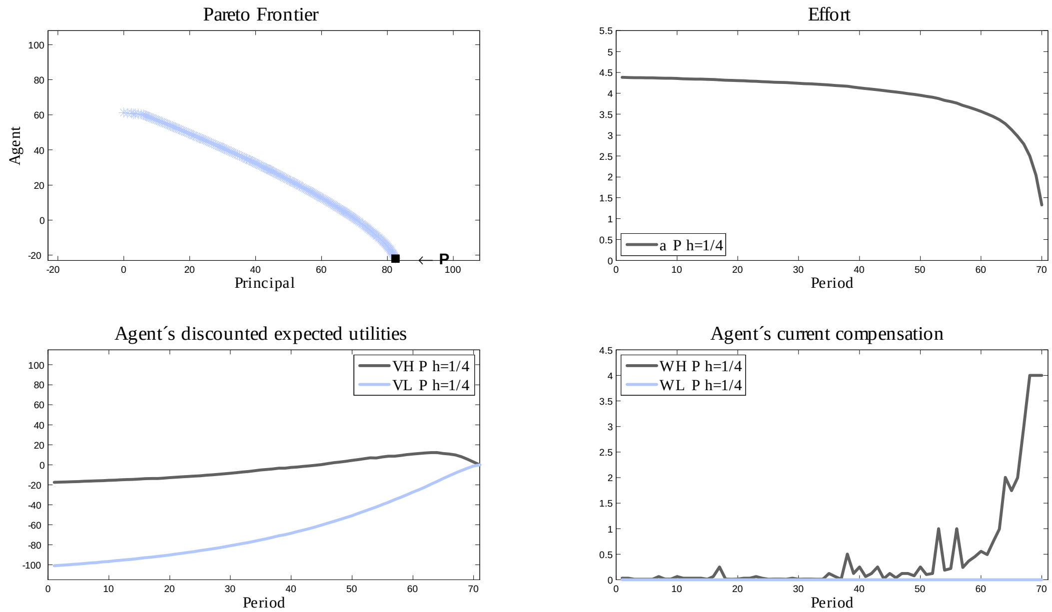

Figure 8: Principal's Most Advantageous Contract: $h = \frac{1}{4}$.

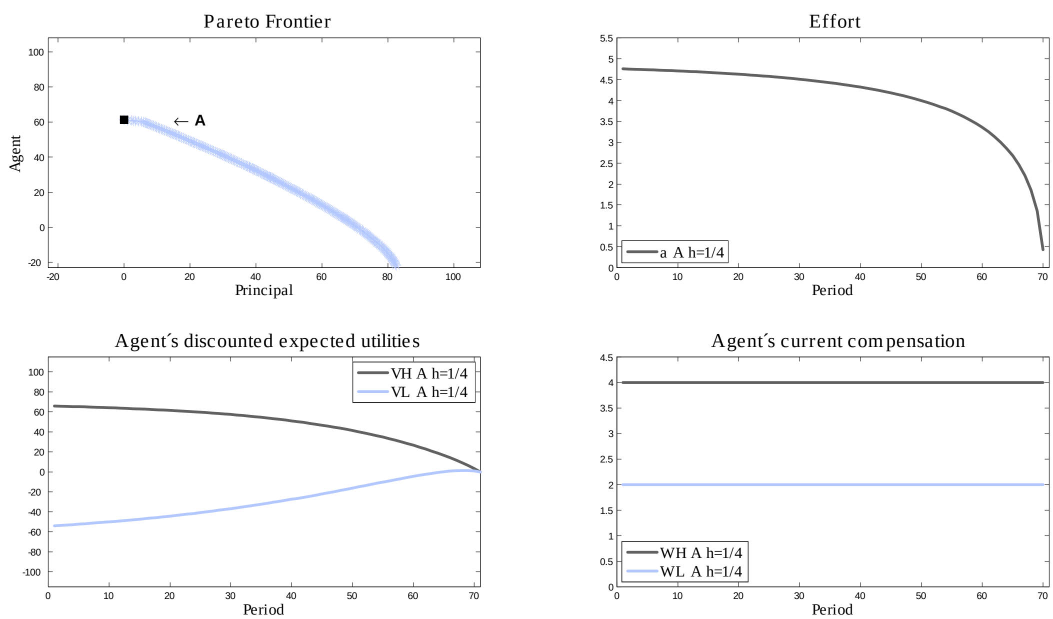

Figure 9: Agent's Most Advantageous Contract: $h = \frac{1}{4}$.

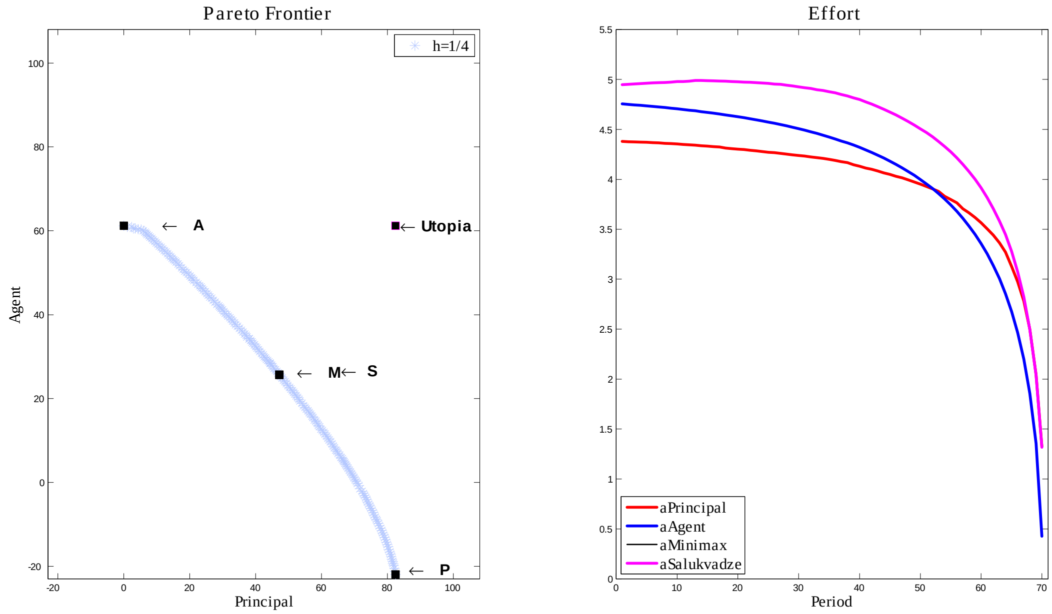

Figure 10: Solutions S and M: $h = \frac{1}{4}$.

In Figure 10 we show solutions S and M for $h = \frac{1}{4}$. Notice that those solutions are located around the middle of the Pareto Frontier, and both solutions are equal. Notice that, in contrast, solutions S and M showed in Figure 4 were close but not equal. The Agent's effort schedule has the same shape as in the previous two figures; however, in both solutions S and M, the Agent's effort is higher than in the Principal's and Agent's most advantageous contracts.

In Figure 11, we show the details of solution S for $h = \frac{1}{4}$. The results are similar to those reported for Figure 6; however the current compensation schedules begin at an earlier period to behave like those in the Agent's most advantageous contract with respect to the results observed in that figure. This result can be attributed to the lower level of the Agent's relative risk aversion parameter.

In Figure 12, we show the details of solution M for $h = \frac{1}{4}$. We can corroborate that this solution is equal to solution S in all aspects.

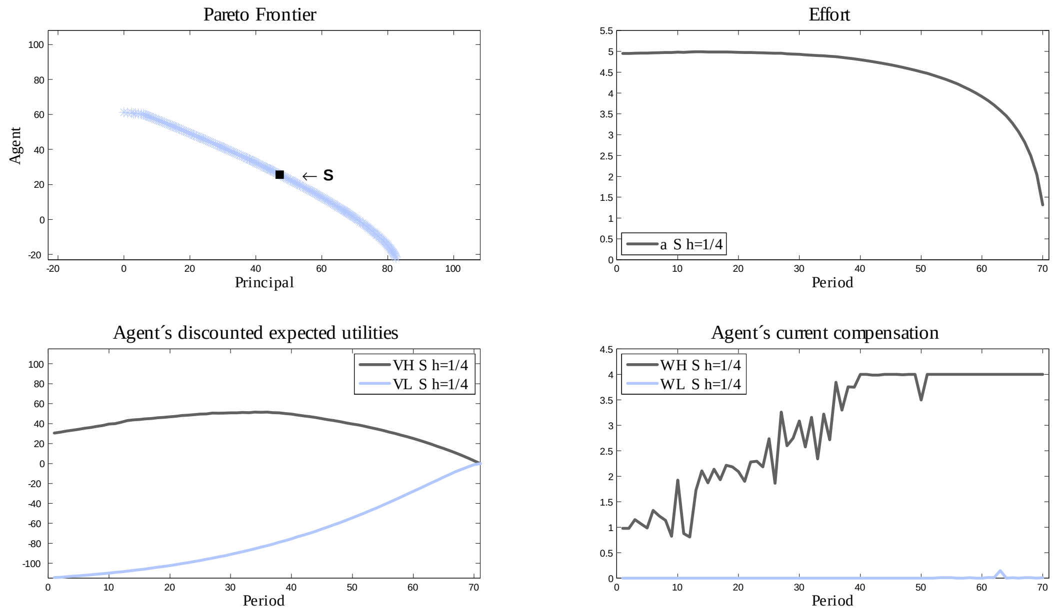

Figure 11: Solution S: $h = \frac{1}{4}$.

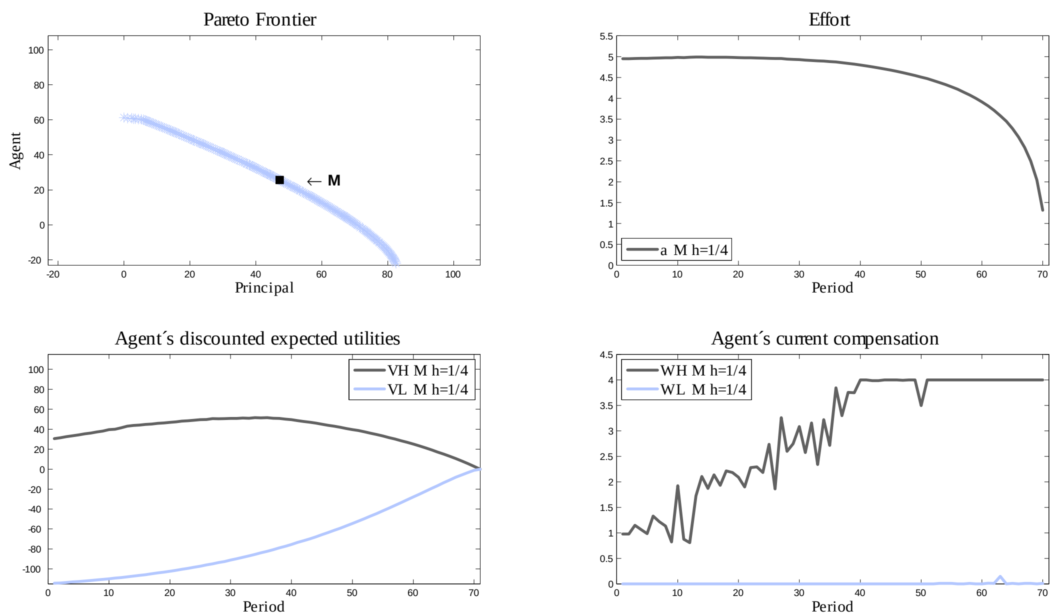

Figure 12: Solution M: $h = \frac{1}{4}$.

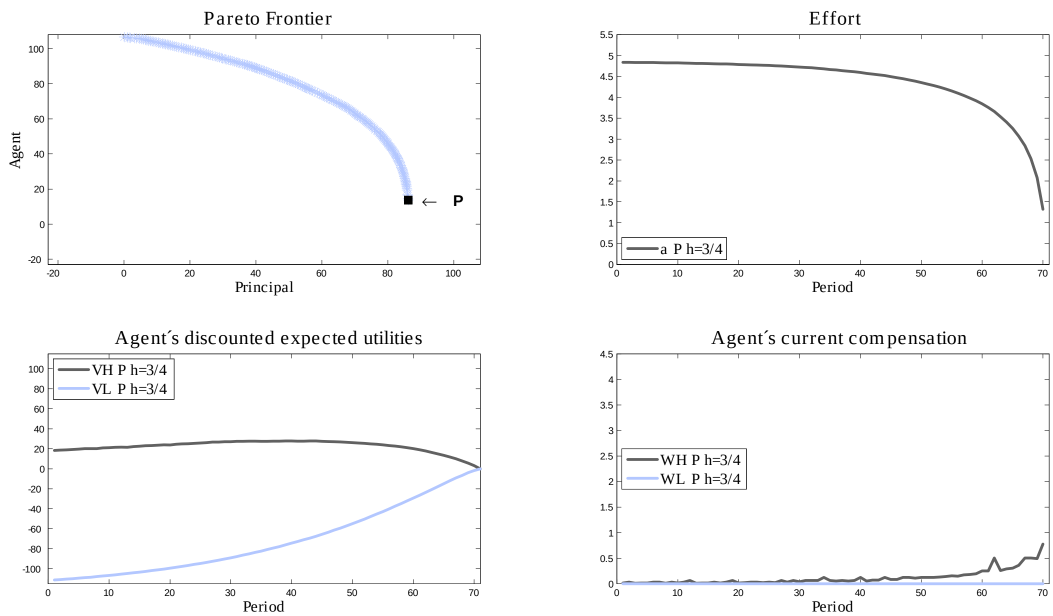

Figure 13: Principal's Most Advantageous Contract: $h = \frac{3}{4}$.

Secondly, we present our results for $h = \frac{3}{4}$. In Figure 13, we show the results of the contract that gives priority to the Principal's discounted expected utility. The Pareto Frontier is decreasing and concave, and the Agent achieves a higher maximal value in discounted expected utility with respect to Figures 2 and 8. The behavior of the Agent´s expected discounted utilities for the low (L) and high (H) productivity levels is similar to those in the aforementioned figures. Again, the Agent's current compensation schedules for the low and high productivity levels are very similar to those in Figures 2 and 8, but the spread in this case is lower here than in those figures. Hence, the incentive provision mechanism works similarly; but given that the Agent is more risk-averse, his utility entitlement is higher, and compensations show a lower level of variability.

In Figure 14, we show the results of the most advantageous contract for the Agent for $h = \frac{3}{4}$. The Agent's effort schedule is similar to that in Figure 3 and 9, but the Agent chooses higher effort levels in most periods with respect to what is observed in those figures. The behavior of the Agent´s expected discounted utilities for the low (L) and high (H) productivity levels, and the Agent's current compensation schedules for the low and high productivity levels are very similar to those of Figure 3 and 9;

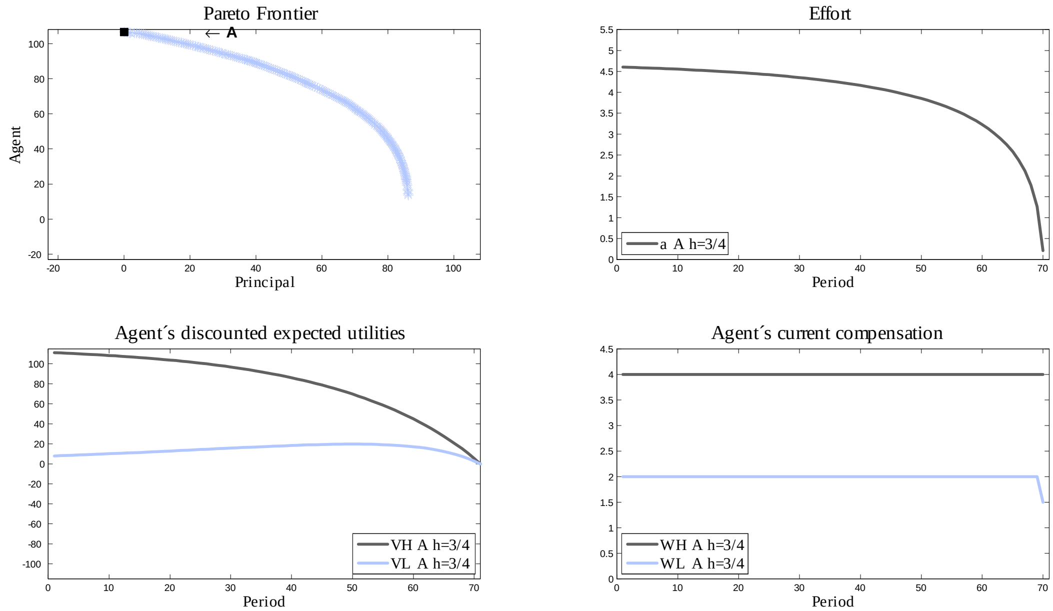

Figure 14: Agent's Most Advantageous Contract: $h = \frac{3}{4}$.

however, the Agent's promised utilities are higher. Hence, the incentive provision mechanism works similarly; but given that the Agent is more risk-averse, his utility entitlement is higher.

In Figure 15, we show the solutions S and M's results for $h = \frac{3}{4}$. Notice that, differently with respect to Figures 4 and 10, solutions S and M are located closer to the Agent's most advantageous contract but a little more distant between them as what it is observed in Figure 4. The Agent's effort schedule has the same shape as in the previous figures; however, in solution S and, in particular, solution M, the Agent's effort is higher than in the Principal's and Agent's most advantageous contracts.

In Figure 16, we show the details of solution S for $h = \frac{3}{4}$. The results are similar to those reported for Figure 5 and 11; however, the current compensation schedules at a much later period to behave like those in the Agent's most advantageous contract with respect to the results observed in those Figures.

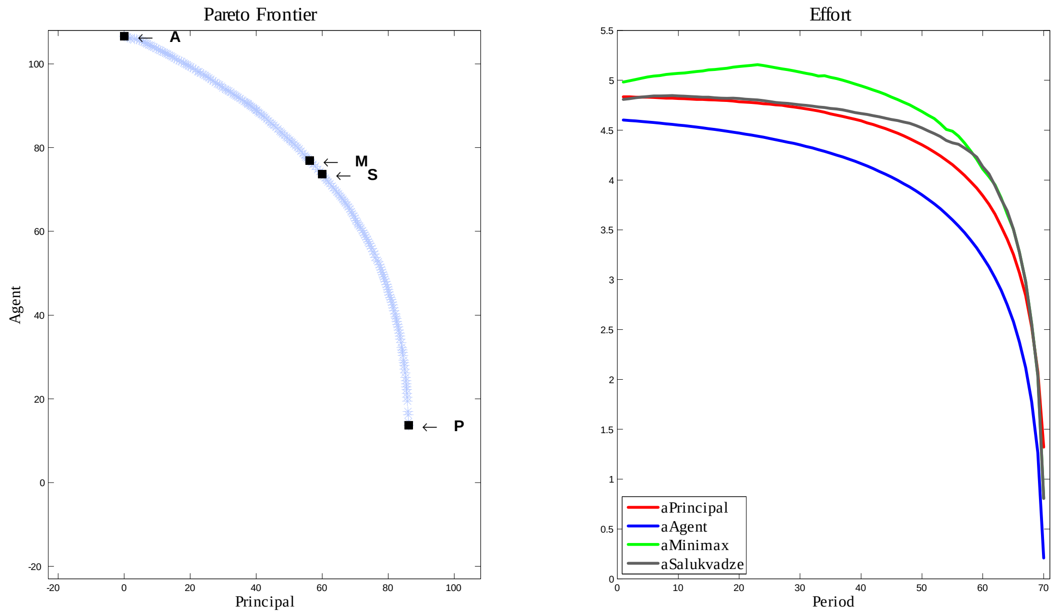

Figure 15: Solutions S and M: $h = \frac{3}{4}$.

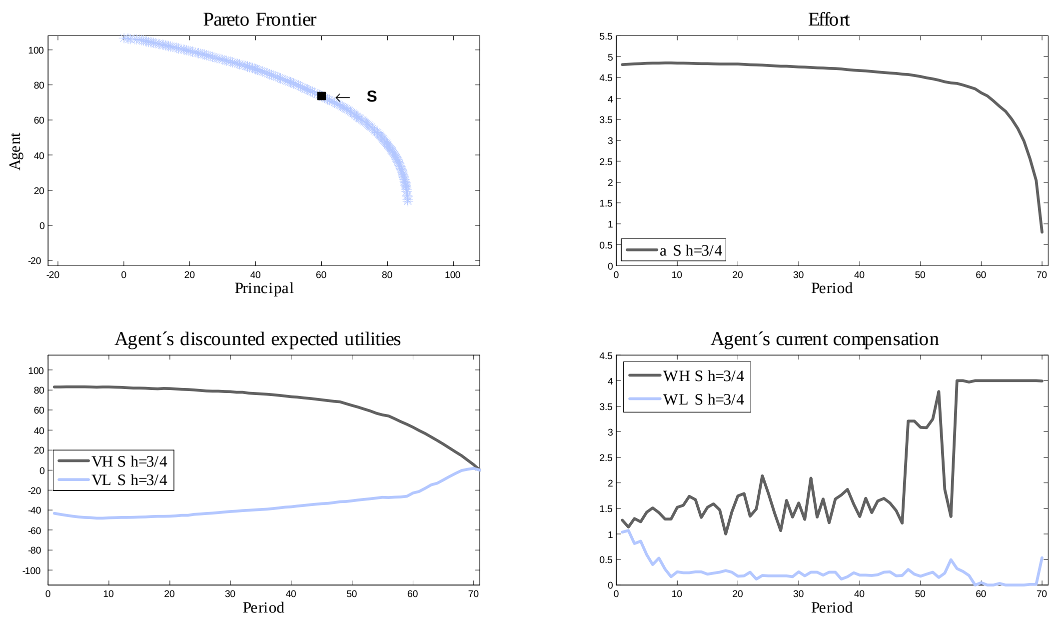

Figure 16: Solution S: $h = \frac{3}{4}$.

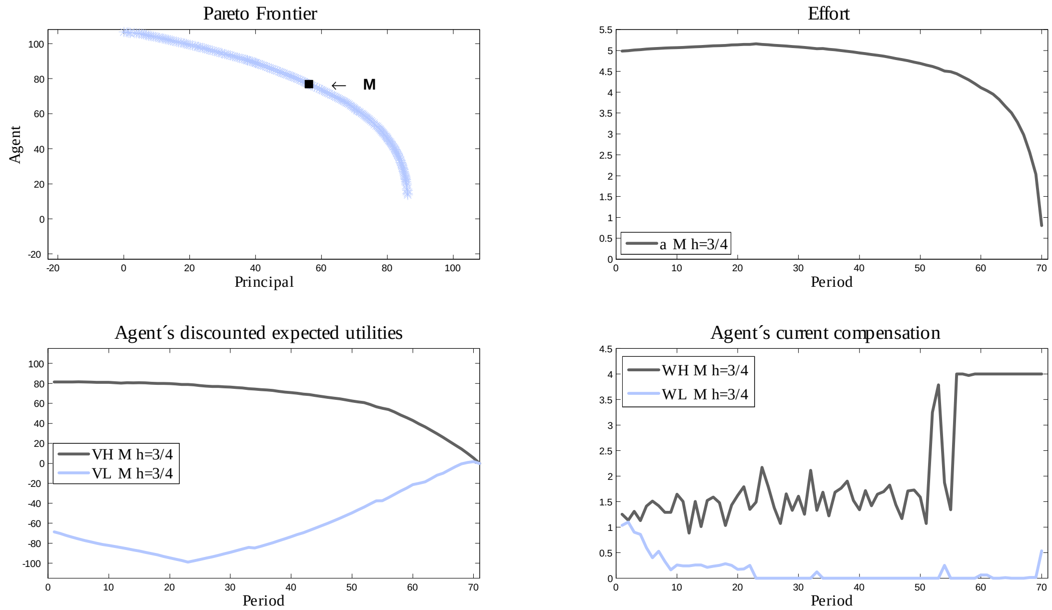

Figure 17: Solution M: $h = \frac{3}{4}$.

In Figure 17, we show the details of solution M for $h = \frac{3}{4}$. Given the relative proximity of this solution to solution S, both contracts look similar. The main difference is that the behavior of the future and current compensation schedules in solution M showed higher spread for the high and low productivity shocks than what is is observed in solution S, and hence, this explains why this solution implements a higher Agent's effort.

In summary, we can conclude that as the Agent's parameter of relative risk aversion increases, the compromise solutions become closer to the Agent's most advantageous contract, and are most distant between them. Solution M implements a higher effort than solution S because it is characterized by a higher spread in future and present compensation schedules for the high and low productivity shocks.

4.4 Productivity Shocks

In this subsection, we present our numerical results from changing the possible output sets. The interesting point of this numerical exercise is to see how changing the productivity shocks affects the incentive schemes. As we saw in the sub-section about optimal contracts, in the static version of our model, higher effort levels from the Agent increase the probability of occurrence of the high productivity shock, and hence increase its marginal revenue gain, given by $\gamma[\frac{1}{\chi^2}(y_H - y_L)]$. Notice, that a higher difference $(y_H - y_L)$ also implies a gain in the marginal revenue derived from implementing higher Agent's effort levels.

In order to study how the difference $(y_H - y_L)$ affects the incentive schemes, we consider the following sets of productivity shocks:

- $Y = \{y_L = 3, y_H = 4\}$: In this case, the low productivity shock is higher than that of the benchmark case. The difference $(y_H - y_L) = 1$ is lower than in the benchmark case, which means that the expected marginal gain from implementing a higher effort level from the Agent is lower than in the benchmark case. Also, the ratio $\frac{y_H}{y_L} = 1.33$ is lower than that of the benchmark case; however, y_L is higher than that of the benchmark case while y_H is equal to that of the benchmark case. So, we consider this as a better productive scenario than that of the benchmark case.

- $Y = \{y_L = 2, y_H = 3\}$: In this case the high productivity shock is lower than in the benchmark case. The difference $(y_H - y_L) = 1$ is lower than in the benchmark case, which means that the expected marginal gain from implementing a higher effort level from the Agent is lower than in the benchmark case. Also, the ratio $\frac{y_H}{y_L} = 1.5$ is lower than that of the benchmark case; however, y_L is equal to that of the benchmark case while y_H is lower than that of the benchmark case. So, we consider this as a worse productive scenario than that of the benchmark case.

- $Y = \{y_L = 2, y_H = 5\}$: In this case the high productivity shock is higher than in the benchmark case. The difference $(y_H - y_L) = 3$ is higher than in the benchmark case, which means that the expected marginal gain from implementing a higher effort level from the Agent is higher than in the benchmark case. Also, the ratio $\frac{y_H}{y_L} = 2.5$ is higher than that of the benchmark case. This is considered as a better productive scenario than that of the benchmark case.

- $Y = \{y_L = 4, y_H = 8\}$: In this case both high and low productivity shocks are higher than in the benchmark case. The difference $(y_H - y_L) = 4$ is higher than in the benchmark and the previous cases, which means that the expected marginal gain from implementing a higher effort level from the Agent is higher than in the both cases. Also, the ratio $\frac{y_H}{y_L} = 2$ is equal to that of the benchmark case. This is considered as a better productive scenario than that of the benchmark case.

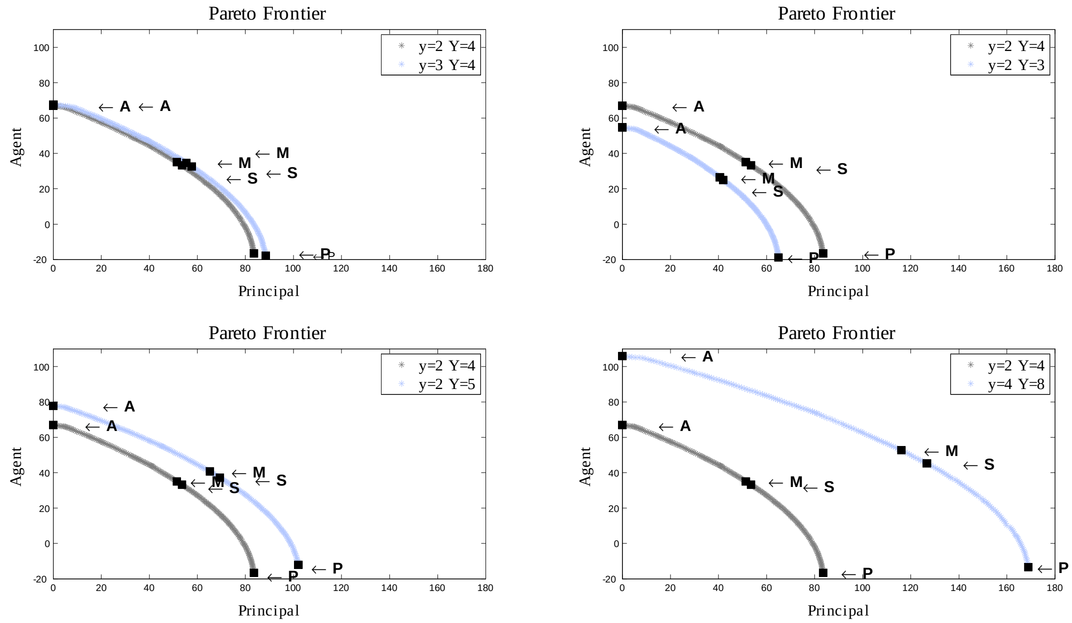

Figure 18: Effect of Changes in Productivity Shocks.

Given that the internal structures of the different solutions that we analyze is similar to those presented in the previous section, in this section we will only show how the Pareto Frontier and the position of the compromise solutions change with the different sets of productivity shocks.

In Figure 18 we present the results from the several sets of productivity shocks mentioned above. We observe that better productive scenarios: (i) move the Pareto Frontier away from the origin with respect to the benchmark case, and (ii) cause an increase in the distance between solutions S and M. It is not clear from this figure how the Principal and the Agent share the gains in the different productive scenarios.

Now, we try to assess how gains/losses derived from changes in productivity shocks are shared between the Principal and the Agent. In order to do that, for every pair of productivity shocks considered above, we measure the distance from the Principal's maximal utility in his most advantageous contract and his utility in the Agent's most advantageous contract, $\Delta PP = |U_P - U_A|$. We also measure the distance from the Principal's utility in every compromise solution, S and M, and his utility in the Agent's most advantageous contract, $\Delta PS = |U_S - U_A|$ and $\Delta PM = |U_M - U_A|$,

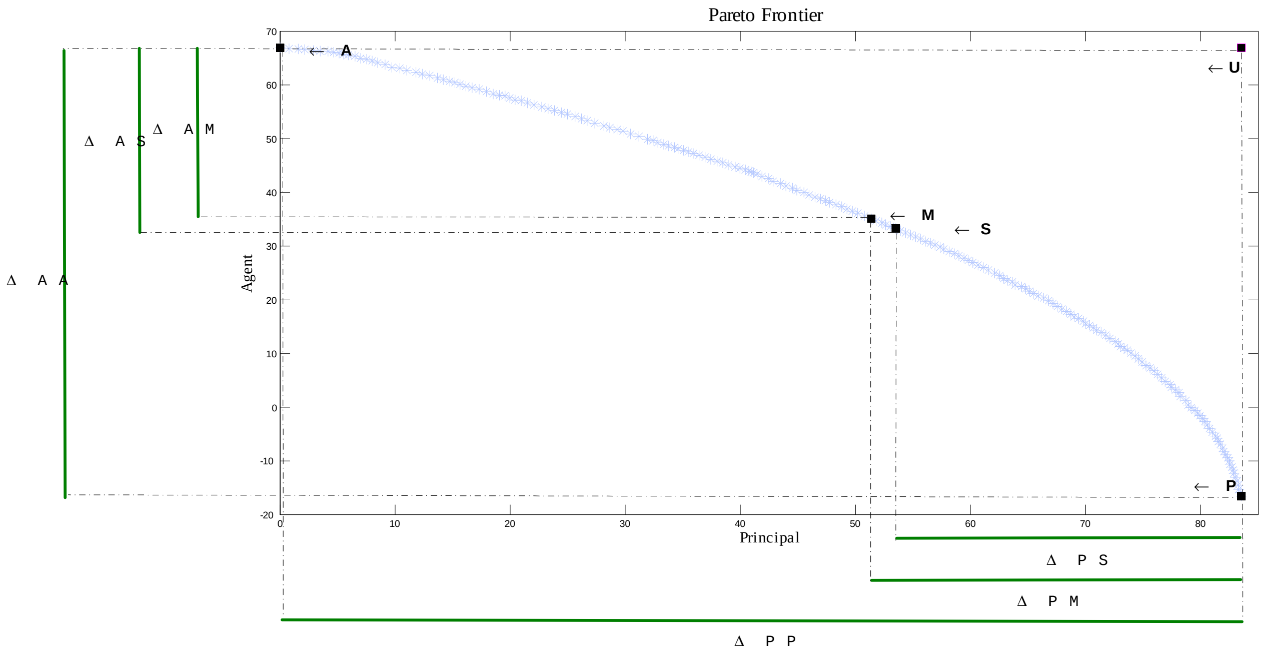

Figure 19: Measuring ΔAA, ΔAS, ΔAM, ΔPP, ΔPS, ΔPM.

respectively. For the case of the Agent, for every pair of productivity shocks considered above, we measure the distance from the Agent's maximal utility in his most advantageous contract and his utility in the Principal's most advantageous contract, $\Delta AA = |V_A - V_P|$. We also measure the distance from the Agent's utility in every compromise solution, S and M, and his utility in the Principal's most advantageous contract, $\Delta AS = |V_S - V_P|$ and $\Delta AM = |V_M - V_P|$, respectively. We show the aforementioned measures on the Pareto Frontier in Figure 19, and the numerical values of those measures in Table 2.

Table 2

	$\{2,4\}$	$\{3,4\}$	$\{2,3\}$	$\{2,5\}$	$\{4,8\}$
ΔPP	83.53029382	88.37666718	64.90066374	102.0904546	168.8769348
ΔPS	32.16146335	33.0887887	22.95938222	36.92543354	52.77272069
ΔPM	30.02356478	30.83816655	24.32125004	32.74530361	42.14593325
ΔAA	83.45872312	85.37478526	73.48293183	89.83868206	119.3997101
ΔAS	31.818884	32.88335668	29.63184069	37.09803278	53.28522985
ΔAM	33.62999177	34.91300547	28.13260074	40.57661896	60.73148422

Then, we construct the following relative measures to determine who gains relatively more when moving from the structure of productivity shocks of the benchmark case to any other set of productivity shocks, where i is the subindex for any new set of productivity shocks, as follows: $i = 1$ means that $Y = \{3,4\}$, $i = 2$ means that $Y = \{2,3\}$, $i = 3$ means that $Y = \{2,5\}$, and $i = 4$ means that $Y = \{4,8\}$; B is the subindex of the productivity shocks of the benchmark case, $Y = \{2,4\}$; and R stands for "relative":

$$\begin{aligned}
\Delta PPR_i &= \frac{\Delta PP_i - \Delta PP_B}{\Delta PP_B} = \frac{\Delta PP_i}{\Delta PP_B} - 1 \\
\Delta PSR_i &= \frac{\Delta PS_i - \Delta PS_B}{\Delta PS_B} = \frac{\Delta PS_i}{\Delta PS_B} - 1 \\
\Delta PMR_i &= \frac{\Delta PM_i - \Delta PM_B}{\Delta PM_B} = \frac{\Delta PM_i}{\Delta PM_B} - 1 \\
\Delta AAR_i &= \frac{\Delta AA_i - \Delta AA_B}{\Delta AA_B} = \frac{\Delta AA_i}{\Delta AA_B} - 1 \\
\Delta ASR_i &= \frac{\Delta AS_i - \Delta AS_B}{\Delta AS_B} = \frac{\Delta AS_i}{\Delta AS_B} - 1 \\
\Delta AMR_i &= \frac{\Delta AM_i - \Delta AM_B}{\Delta AM_B} = \frac{\Delta AM_i}{\Delta AM_B} - 1
\end{aligned}$$

The numerical values we obtain are reported in Table 3, and must be understood as results from moving from the benchmark case $Y = \{2,4\}$ to the cases reported in Table 3.

Table 3

	$\{3,4\}$	$\{2,3\}$	$\{2,5\}$	$\{4,8\}$
ΔPP	0.05801935	-0.223028428	0.222196762	1.021744771
ΔPS	0.028833432	-0.286121344	0.148126661	0.640868144
ΔPM	0.02713208	-0.189927971	0.09065342	0.403761797
ΔAA	0.022958201	-0.119529642	0.076444483	0.430643864
ΔAS	0.033454117	-0.06873413	0.165912443	0.674641696
ΔAM	0.038150878	-0.163466915	0.206560479	0.805872706

From these results, we conclude that: (i) Moving from $Y = \{2,4\}$ to $Y = \{3,4\}$, to $Y = \{2,5\}$, and to $Y = \{4,8\}$ favors the Principal because he obtains a higher relative gain in utility when his most advantageous contract is implemented than the Agent when his most advantageous contract is implemented; while it favors the Agent when implementing any of the compromise solutions. (ii) Moving from $Y = \{2,4\}$ to $Y = \{2,3\}$ favors the Agent overall.

From these results we can conclude that an improvement in the productive scenario causes the Principal to obtain a higher relative gain in utility when his most advantageous contract is implemented with respecto to the Agent when his most advantageous contract is implemented. Also, when any of the compromise solutions is implemented in a better productive scenario, the Agent benefits relatively more from the change. On the other hand, when the productive scenario worsens, the Agent benefits in all of the contracts.

5 Conclusions

In this paper we formulate an infinitely repeated Principal-Agent relationship as a multi-objective optimization problem, and provide a numerical algorithm to approximate this model's Pareto Frontier. We obtain an analytical result that says that the structure of the contract where priority is given to the Principal's discounted expected utility is similar to that of the optimal contract in Spear and Srivastava (1987); while the structure of the contract where priority is given to the Agent is such that there is always a direct relationship between the Agent's salary and the output realization. Even though this contract does not provide the Agent with complete insurance from the uncertainty inherent to the production process, it ensures the Agent a positive relationship between his salary and the output realization.

We also find a result that says that, in the static version of our model, compromise solutions pay the Agent higher compensation levels in the event of the high productivity shock, which is more likely if the Agent chooses a high effort level.

Our numerical results indicate that the compromise solutions are characterized by a higher Agent's effort levels compared to those of both Principal's and the Agent's most advantageous contracts. Also, in these solutions the incentive-provision mechanisms have feature of the other two types of contracts at distinct points of the contractual relationship. In the earlier periods of the relationship, the compromise contracts resemble the Principal's most advantageous contract so that the Agent has to bear some of the risk associated with the production process. However, in the later periods of the contractual relationship, the compromise contracts relieve the Agent from most of the productive risk, and his present compensation is directly related to the productivity shock realizations. located at the middle of the numerically approximated Pareto Frontier.

On the other hand, the more risk averse the Agent becomes, the higher his effort levels and his promised discounted expected utility values. That is, as the Agent's coefficient of relative risk aversion increases, the probability of the high productivity shock and the spread in the present and future compensation schedules for the high and low productivity shocks increase. This causes that the Principal-Agent relationship generates more value to the Agent. Morever, the compromise solutions tend to be located at points in the Pareto Frontier that are closer to the Agent's most advantageous contract and more distant between them along the Pareto Frontier than what is observed in the benchmark case.

When the structure of productivity shocks is such that the Agent's effort yields relatively higher production levels, *i.e.*, the ratio $\frac{y_H}{y_L}$ is either equal or higher than that of the benchmark case, the Principal obtains a higher relative gain in utility when his most advantageous contract is implemented than the Agent when his most advantageous contract is implemented. Also, when the any of the compromise solutions is implemented, then the Agent benefits relatively more from the change in the productivity shocks. On the other hand, when the structure of the productivity shocks is such that the Agent's effort yields relatively lower production levels, *i.e.*, the ratio $\frac{y_H}{y_L}$ is lower than that of the benchmark case, the Agent might benefit in all or some of the contracts. So, this might constitute an incentive-based explanation for the premium paid to managers of larger (or more productive) firms.

As observed in our numerical results, the compromise solutions that we analyze here are characterized by contracts that: (i) make the Agent exert higher effort levels, (ii) offer the Agent future compensation that differentiates more between high and low productivity shocks, and (iii) pay the Agent salaries that become more directly related to productivity outcomes as time goes on. Also, when the coefficient of relative risk aversion increases, compromise solutions tend to become closer to the Agent's most advantageous contract, that is the Agent tend to bear less than the Principal the risk inherent to the productive process. Moreover, improvements in the firm's productivity shock structure benefit, in relative terms, the Agent more than the Principal. So, if we consider the possibility that compromise solutions provide a good characterization of the actual contractual arrangements between Principals and Agents, then our framework might help explain the empirical finding that the increase in the level of CEO pay is caused by an exogenous growth of firm size and not by changes in the CEOs' preferences for risk aversion (Gabaix and Landier , 2008; Gayle and Miller, 2009; and Gabaix *et al.*, 2013), and the fact that the higher levels of CEO pay are characterized by a higher component of stock option grants (Murphy, 1999).

Finally, the analysis of the relationship between incentive provision and the dynamics of firm growth, like in the model of Clementi *et al.* (2010), in our framework might be an interesting line of future research because it would allow us to explicitly investigate how firms grow while simultaneously dealing with the cost of moral hazard.

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6 Appendix 1: Proofs

Proof of Proposition 1. Fix (U, V). $\mathcal{W}$ is bounded. We need to prove that $\mathcal{W}$ is also closed. Let $\{U_n, V_n\} \in \mathcal{W}$ such that $\underset{n\longrightarrow\infty}{lim}\{U_n, V_n\} = \{U_\infty, V_\infty\}$. We have to show that $\{U_\infty, V_\infty\} \in \mathcal{W}$, or that there exists a contract σ_∞ that satisfies (1), (2), (3), $U(\sigma_\infty \mid h_0) = U_\infty$ and $V(\sigma_\infty \mid h_0) = V_\infty$. We construct this optimal contract σ_∞. The definition of $\mathcal{W}$ allows us to say that there exists a sequence of contracts $\{\sigma_n\} = \{a_t^n(h_{t-1}), w_t^n(h_t)\}$ that satisfies (1) and (2), $\forall n$. Hence,

$$U_\infty = \underset{n\longrightarrow\infty}{lim}\sum_{t=1}^{\infty}\beta^{t-1}\int\limits_Y (y - w_t^n(h_t))f(y; a_t^n(h_{t-1}))dh_t$$

$$V_\infty = \underset{n\longrightarrow\infty}{lim}\sum_{t=1}^{\infty}\beta^{t-1}\int\limits_Y (v(w_t^n(h_t), a_t^n(h_{t-1}))f(y; a_t^n(h_{t-1}))dh_t,$$

For $t = 1$, $\{a_1^n(h_0), w_1^n(h_1)\}$ is a finite collection of bounded sequences, so there exists a collection of subsequences $\{a_1^{n_q}(h_0), w_1^{n_q}(h_1)\}$ that satisfy:

$$\underset{n_q\longrightarrow\infty}{lim} a_1^{n_q}(h_0) = a_1^\infty(h_0) \quad \text{and} \quad \underset{n_q\longrightarrow\infty}{lim} w_1^{n_q}(h_1) = w_1^\infty(h_1)$$

Also, (U_∞, V_∞) must be equal to (U, V). If $V_\infty < V$ the Agent would not stop bargaining given that the Principal is obtaining U, and if $V_\infty > V$, V_∞ would not belong to $\mathcal{W}$ because it does not belong to PS^*, given that the Principal is getting U. A similar argument justifies that $U_\infty = U$.

We can repeat this procedure for $t = 2, ..., \infty$, and let $\sigma_\infty = \{a_t^\infty(h_{t-1}), w_t^\infty(h_t)\}$. The contract σ_∞ is the object we desire to obtain. ■

Proof of Proposition 2. Fix $W = (U, V)$. First, we show that $\Gamma(U^*, V^*)(W) \leq (U^*(W), V^*(W))$. This is true if $\exists\sigma$ that is feasible and incentive compatible such that $(U(\sigma \mid h^0), V(\sigma \mid h^0)) = \Gamma(U^*, V^*)(W)$. We construct this contract σ by letting $a(W)$, $w(y, W)$, and $(\overline{U}(y, W), \overline{V}(y, W))$ be the solution to $\Gamma(U^*, V^*)(W)$, and:

$$a_1(h^0) = a(W), \text{ and } w_1(h^1) = w(y_1, W), \forall h^1.$$

For a given $y_1 \in Y$, $\exists\sigma_{y_1}$ such that the Principal receives $\overline{U}(y_1, W)$ and the Agent receives $\overline{V}(y_1, W)$. Let

$$\sigma \mid h_1 = \sigma_{y_1}, \forall h_1.$$

Notice that σ_{y_1} belongs to the PS^*, because $(\overline{U}(y_1, W), \overline{V}(y_1, W)) = W^*(\sigma_{y_1} \mid h_1)$. So, there is no other contract φ_{y_1} in the PS^* such that $W^*(\varphi_{y_1} \mid h_1)$ dominates $W^*(\sigma_{y_1} \mid h_1)$; that is, $W^*(\varphi_{y_1} \mid h_1) \prec W^*(\sigma_{y_1} \mid h_1)$. So, σ_{y_1} is the contract we need, and $\Gamma(U^*, V^*)(W) \leq (U^*(W), V^*(W))$. The second part of the proof shows that $(U^*(W), V^*(W)) \leq \Gamma(U^*, V^*)(W)$. Let σ^* be an optimal contract. Hence,

$$U^*(W) = U(\sigma^* \mid h_0) = \int_Y [y_1 - w^*(y_1) + \beta U(\sigma^* \mid h_1)] f(y_1; a^*(h_0)) dy_1,$$

$$V^*(W) = V(\sigma^* \mid h_0) = \int_Y [v(w^*(y_1), a^*(h_0)) + \beta V(\sigma^* \mid h_1)] f(y; a^*(h_0)) dy_1,$$

and

$$(U^*(W), V^*(W)) \leq \Gamma(U^*, V^*)(W)$$

if we set $a(W) = a^*(h_0)$, $w(y, W) = w^*(y_1)$, and $(\overline{U}(y_1, W), \overline{V}(y_1, W)) = (U^*(\sigma^* \mid y_1), V^*(\sigma^* \mid y_1))$, for $y_1 \in Y$; for (7) , (8), and (9) are satisfied. It must be noted that $V^*(W) = V$ and $U^*(W) = U$ because, σ^* must belong to the PS^*, given that $\overline{U}(y_1, W) = U^*(\sigma^* \mid y_1)$ and $\overline{V}(y_1, W) = V^*(\sigma^* \mid y_1)$; and there is no other contract φ^* in the PS^* such that $W(\varphi^*)$ dominates $W(\sigma^*)$; that is, $W(\varphi^*) \prec W(\sigma^*)$. ■

7 Appendix 2: Computational Algorithm

Our computational algorithm is the following:

(i) We set the numerical values of the parameters of the production shocks $Y = \{y_L, y_H\}$, the discount rate β, and the coefficient of relative risk aversion h. We specify that the coding or nature of the genotypes is binary in order to work with the genetic operators whose parameters we specify at this point. The genetic operators are described in step (ix). For details of the coding and the genetic operators' parameters, see Herrera *et al.* (2011).

(ii) We define the two objective functions, namely the Principal's discounted expected utility and the Agent's discounted expected utility, and specify that both of them should be maximized (RankMOEA allows us to choose whether or not every objective function should be maximized).

(iii) Each generation g, $g = 1, ..., G$, where G is a finite number; contains a total of J individuals (solutions), where j is the index of individuals in a generation, $j = 1, ...J$, and J is also a finite number. An individual's chromosome is characterized by 2 substrings of length N, where N is the total number of contractual periods considered in an individual that belongs to generation g. The generation-g individual-j's chromosome has a length of $2N$, and it is defined by the Agent's salary w_p for every y_i, $i = H, L$; and every period $p = 1, ..., N$. Formally:

$$\begin{bmatrix} \overline{w}_{gj1}, \overline{w}_{gj2}, ..., \overline{w}_{gjN}; \\ \underline{w}_{gj1}, \underline{w}_{gj2}, ..., \underline{w}_{gjN} \end{bmatrix}.$$

For our numerical exercise, we set the following parameter values: $G = 500,000$; $J = 200$; and $N = 70$.

(iv) We generate our first generation of J individuals, that is a number of J chromosomes as defined above. This generation is created randomly and we specify that the following restrictions must hold:

$$0 \leq \overline{w}_{1jp} \leq y_H \qquad \forall p = 1, ..., N$$

$$0 \leq \underline{w}_{1jp} \leq y_L \qquad \forall p = 1, ..., N.$$

(v) We define the model's variables, their feasible upper and lower bounds and their level of specification at 10^{-3}. The variables and their bounds are:

Agent's optimal effort level:

$$a_{gjp} = \ln[\frac{(\overline{w}_{gjp})^{1-h}}{1-h} - \frac{(\underline{w}_{gjp})^{1-h}}{1-h} + \beta(\overline{V}_{gjp+1} - \underline{V}_{gjp+1})]$$

Agent's maximal effort level:

$$\overline{a}_{gjp} = \ln[\frac{(y_H)^{1-h}}{1-h} - \frac{(0)^{1-h}}{1-h} + \beta(\overline{V}_{gjp+1} - \underline{V}_{gjp+1})]$$

Agent's minimal effort level:

$$\underline{a}_{gjp} = 0$$

Principal's discounted expected utility:

$$\begin{aligned} E[U_{gjp}] &= \exp(-a_{gjp})[y_L - \underline{w}_{gjp} + \beta\underline{U}_{gjp+1}] + \\ &\quad (1 - \exp(-a_{gjp}))[y_H - \overline{w}_{gjp} + \beta\overline{U}_{gjp+1}] \end{aligned}$$

Maximal Principal's discounted expected utility:

$$\begin{aligned}
E[\overline{U}_{gjp}] &= \exp(-\overline{a}_{gjp})[y_L - \underline{w}_{gjp} + \beta \underline{U}_{gjp+1}] + \\
&\quad (1 - \exp(-\overline{a}_{gjp}))[y_H - \overline{w}_{gjp} + \beta \overline{U}_{gjp+1}]
\end{aligned}$$

Minimal Principal's discounted expected utility:

$$\begin{aligned}
E[\underline{U}_{gjp}] &= \exp(-\underline{a}_{gjp})[y_L - \underline{w}_{gjp} + \beta \underline{U}_{gjp+1}] + \\
&\quad (1 - \exp(-\underline{a}_{gjp}))[y_H - \overline{w}_{gjp} + \beta \overline{U}_{gjp+1}] \\
&= [y_L - \underline{w}_{gjp} + \beta \underline{U}_{gjp+1}]
\end{aligned}$$

Agent's discounted expected utility:

$$\begin{aligned}
E[V_{gjp}] &= \exp(-a_{gjp})[\frac{(\underline{w}_{gjp})^{1-h}}{1-h} - a_{gjp} + \beta \underline{V}_{gjp+1}] + \\
&\quad (1 - \exp(-a_{gjp}))[\frac{(\overline{w}_{gjp})^{1-h}}{1-h} - a_{gjp} + \beta \overline{V}_{gjp+1}]
\end{aligned}$$

Maximal Agent's discounted expected utility:

$$\begin{aligned}
E[\overline{V}_{gjp}] &= \exp(-\overline{a}_{gjp})[\frac{(\underline{w}_{gjp})^{1-h}}{1-h} - \underline{a}_{gjp} + \beta \underline{V}_{gjp+1}] + \\
&\quad (1 - \exp(-\overline{a}_{gjp}))[\frac{(w_{gjp})^{1-h}}{1-h} - \underline{a}_{gjp} + \beta \overline{V}_{gjp+1}] \\
&= \exp(-\overline{a}_{gjp})[\frac{(\underline{w}_{gjp})^{1-h}}{1-h} + \beta \underline{V}_{gjp+1}] + \\
&\quad (1 - \exp(-\overline{a}_{gjp}))[\frac{(w_{gjp})^{1-h}}{1-h} + \beta \overline{V}_{gjp+1}]
\end{aligned}$$

Minimal Agent's discounted expected utility:

$$
\begin{aligned}
E[\underline{V}_{gjp}] &= \exp(-\underline{a}_{gjp})[\frac{(\underline{w}_{gjp})^{1-h}}{1-h} - \overline{a}_{gjp} + \beta\underline{V}_{gjp+1}] + \\
&\quad (1-\exp(-\underline{a}_{gjp}))[\frac{(w_{gjp})^{1-h}}{1-h} - \overline{a}_{gjp} + \beta\overline{V}_{gjp+1}] \\
&= [\frac{(\underline{w}_{gjp})^{1-h}}{1-h} - \overline{a}_{gjp} + \beta\underline{V}_{gjp+1}]
\end{aligned}
$$

Notice that the variables $E[\overline{U}_{gjp}]$, $E[\underline{U}_{gjp}]$, $E[\overline{V}_{gjp}]$, $E[\underline{V}_{gjp}]$ define the two dimensional state space W. We also set a marker of violated restrictions and all the variables start at zero.

(vi) Each chromosome (solution) is evaluated recursively using backward induction, so that in the last period of every solution in each generation all variables will be zero.

(vii) For each g, j, and p, we evaluate whether the following conditions are satisfied:

$$\underline{w}_{gjp} \leq \overline{w}_{gjp}$$

$$a_{gjp} \geq 0.$$

When any of the above is not satisfied, a marker of violated restrictions is activated.

(viii) We create a routine to recursively evaluate, using backward induction, each period p of an individual j belonging to generation g. First we evaluate the values of $[a_{gjp}, \overline{a}_{gjp}, \underline{a}_{gjp}, E[U_{gjp}], E[\overline{U}_{gjp}], E[\underline{U}_{gjp}], E[V_{gjp}], E[\overline{V}_{gjp}], E[\underline{V}_{gjp}]]$ at period N of the individual j belonging to generation g. We publish the values $[E[U_{gjp}], E[V_{gjp}]]$ and count the number of violated restrictions. Then, we move to period $N-1$, and so on until we reach period 1 of individual j belonging to generation g. We do the same for all the individuals belonging to generation g, that is for 200 individuals. Then we plot all the values $[E[U_{gjp}], E[V_{gjp}]]$, for $p = 1, ..., 70$ and $j = 1, ..., 200$, given a generation g, to obtain a Pareto Frontier.

(ix) We create the next generation by using the individuals of the previous generation and the genetic operators of cross over, mutation and selection. Cross over involves generating new individuals from individuals from previous generations and its function is to accelerate the new individuals' searching process by using information from previous generations. Mutation provides the population with diversity by exploring new searching areas through the isolated modification of genetic material, as it happens with living beings. Finally, the selection process chooses the individuals that are fitter to survive to form a new generation without compromising the population's diversity, and considering that the two aforementioned objective functions must me maximized.

(x) The new generation is evaluated in step (vii). One can choose from several stopping conditions, defined either by the proximity of the resulting Pareto Frontier of each successive generation or by considering the total number of generations. We use the second criterion and we consider that at the end of $500,000$ generations the final Pareto Frontier is obtained.