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MARÍA JOSÉ ROA, DULCE SAURA AND FRANCISCO J. VÁZQUEZ

Economic Growth, Labour Market and  
Demographic Patterns

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## Abstract

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*The purpose of this work is to study the dynamic interaction between income growth, demographic variables patterns and labour market characteristics. We develop a general and simple framework of economic growth with unemployment, endogenous population growth and age structure. Our dynamic results show a potential role for state intervention and institutions in order to enhance growth or smooth out its irregular pattern. Unemployment rate and per capita income dynamics fluctuate along cycles of different periods, and they may even have aperiodic paths. The characteristics of labour market institutions are an endogenous source of instability. In particular, as the rigidity of the labour market increases, the possibility of irregular behaviour increases as well. Next, we show how the introduction of endogenous population growth generates a demographic transition that affects the dynamics of unemployment and economic growth rate. Specifically, the entrance of the young population to the labour force could smooth fluctuations caused by labour market rigidities. However, this stabilising factor would disappear by the delayed entrance into the labour market due to the age structure mechanism. The delay, corresponding to the Malthusian cycle, is one source of endogenous oscillations and economic growth. The paper aims to contribute to the literature that studies understand the origin and nature of long run macroeconomic fluctuations in economic growth models.*

*Keywords: economic growth, human capital, unemployment, chaotic dynamics.*

*JEL classification: E10, O40, C61, E24*

## Resumen

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*El objetivo de este trabajo es estudiar la interacción dinámica entre el crecimiento de la renta, el patrón de comportamiento de las variables demográficas y las características del mercado de trabajo. Para ello, se desarrolla un marco teórico general y sencillo de crecimiento económico con desempleo, crecimiento endógeno de la población y estructura de edades. Nuestros resultados dinámicos muestran el papel potencial de la intervención del Estado y las instituciones para promover el crecimiento y suavizar su comportamiento irregular.*

*La tasa de desempleo y la renta per cápita fluctúan con distintas dinámicas periódicas e incluso sendas aperiódicas. Las características de las instituciones del mercado de trabajo son una de las fuentes endógenas de*

*inestabilidad. En particular, cuando las rigideces del mercado de trabajo aumentan, las posibilidades de obtener comportamientos irregulares lo hacen también. A continuación, se muestra cómo la introducción del crecimiento endógeno de la población genera una transición demográfica que afecta a las dinámicas del desempleo y de la tasa de crecimiento económico. Específicamente, la entrada de la población joven al mercado laboral podría suavizar las fluctuaciones causadas por las rigideces del mercado de trabajo. No obstante, este factor estabilizador podría desaparecer debido a la demora en la entrada al mercado laboral de la población joven. Esta demora corresponde al ciclo maltusiano, y es una de las fuentes endógenas de las fluctuaciones y del crecimiento económico. Este documento intenta contribuir a la literatura que estudia el origen y la naturaleza de las fluctuaciones de largo plazo en modelos de crecimiento económico*

*Palabras clave: crecimiento económico, capital humano, desempleo, dinámicas caóticas.*

*Clasificación JEL: E10, O40, C61, E24*

# Economic growth, labour market and demographic patterns

María José Roa<sup>(1)(\*)</sup>, Dulce Saura<sup>(2)</sup> and Francisco J. Vázquez<sup>(3)</sup>

<sup>(1)</sup> The Ohio State University and Universidad Francisco de Vitoria

<sup>(2)</sup> Universidad de Zaragoza

<sup>(3)</sup> Universidad Autónoma de Madrid

## Abstract

The purpose of this work is to study the dynamic interaction between income growth, demographic variables patterns and labour market characteristics. We develop a general and simple framework of economic growth with unemployment, endogenous population growth and age structure. Our dynamic results show a potential role for state intervention and institutions in order to enhance growth or smooth out its irregular pattern. Unemployment rate and per capita income dynamics fluctuate along cycles of different periods, and they may even have aperiodic paths. The characteristics of labour market institutions are an endogenous source of instability. In particular, as the rigidity of the labour market increases, the possibility of irregular behaviour increases as well. Next, we show how the introduction of endogenous population growth generates a demographic transition that affects the dynamics of unemployment and economic growth rate. Specifically, the entrance of the young population to the labour force could smooth fluctuations caused by labour market rigidities. However, this stabilising factor would disappear by the delayed entrance into the labour market due to the age structure mechanism. The delay, corresponding to the Malthusian cycle, is one source of endogenous oscillations and economic growth. The paper aims to contribute to the literature that studies understand the origin and nature of long run macroeconomic fluctuations in economic growth models.

**Keywords:** Economic growth, human capital, unemployment, chaotic dynamics

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(\*) Corresponding author: María José Roa García

Address: División de Economía

Centro de Investigación y Docencia Económicas, CIDE A.C.

Carretera México-Toluca 3655

México D.F., 01210

e-mail: mariajose.roa@cide.edu

# 1 Introduction

The purpose of this work is to study the dynamic interaction between income growth, demographic patterns and labour market characteristics. We aim to contribute to the literature on the origin and nature of long run macroeconomic fluctuations in economic growth models.

Since Malthus (1798) hypothesised that rapid population growth would lead to mass starvation and death, there have been debates over the economic effects of demographic variables. Demographers and economists have argued about the role of population growth rate on macroeconomic performance, maintaining three main positions: population growth restricts (Malthus, 1798), promotes (Boserup, 1981), or is independent of economic growth<sup>1</sup> (Solow, 1956). More recently, discussions about population size have given way to theories suggesting population age structure and health status are key demographic determinants of economic progress (Bloom et al., 2001).

On the other hand, there has been increasing interest in explaining the long-run oscillations of population and economy (Lee, 1974; Feichtinger and Sorger, 1989; Manfredi and Fanti, 2006). In particular, these works investigated the concept of Malthusian cycles, and considered the consequences “...of the lags between the response of fertility to current labour market conditions and the time when the resulting labour births actually enter the labour force,” (Lee, 1997). Although in these works the Malthusian cycle is the result of the endogenous interaction between economic and demographic forces, most of this literature is limited to the demographic part, paying little attention to the macroeconomics.

There are different works of economic growth models in which imperfections in the labour market affects economic fluctuations and in which growth are contemplated (Ito,

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<sup>1</sup> This view is also based on empirical research showing little correlation between the growth rate of income per capita and the rate of population growth (Kelley, 1995).



1978; Chiarella et al., 2000; Manfredi and Fanti, 2006). Although different in their purpose and ideas, these works have in common the way in which the labor market disequilibrium is introduced. In particular, based in the Goodwin model (1967), the labour market characteristics are represented by wage dynamics (Phillips curve). This modelization tries to introduce, in a simple way, the ideas of the New Keynesian Economics literature (Mankiw and Romer, 1991). The labour market disequilibrium is caused by different imperfections which explain wage and price rigidity from the optimizing behaviour of individuals.

The traditional models of economic growth do not take into account the stylized fact of the existence of unemployment; it has been usually assumed that the labour market is always in equilibrium. Furthermore, the economic growth theory has not given much attention to demographic variables until recently, with the works of Becker et al. (1990), Galor and Weil (2000), and Jones (2001).

This paper aims to complement the economic growth literature by combining and extending the above mentioned literature regarding labour market, endogenous population growth and age structure. First, the labour market characteristics are introduced through a non-market real wage clearing modelled by a non-linear Phillips Curve. Then, following the literature of endogenous population growth (Galor and Weil, 2000; Jones, 2001), it is assumed that labour supply is determined through micro-founded fertility choices of individuals. Finally, the population's age structure is embedded through a time delay in the rate of change of the labour supply.

Our dynamic results show a potential role for state intervention and institutions in order to enhance growth or smooth out its irregular pattern. Unemployment rate and per capita income fluctuate along cycles of different periods, and they may even have aperi-

odic paths. The characteristics of labour market institutions are the endogenous source of instability. In particular, as the rigidity of the labour market increases, the possibility of irregular behaviour increases as well. Next, we show how the introduction of endogenous population growth generates a demographic transition that affects the dynamics of unemployment and economic growth rate. Specifically, the entrance of a young population to the labour force could smooth fluctuations caused by labour market rigidities. However, this stabilising factor would disappear by the delayed entrance into the labour market due to the age structure mechanism. The delay, corresponding to the Malthusian cycle, is one source of endogenous oscillations and economic growth.

The remainder of the paper is organized as follows. In section 2 we study the basic growth model with unemployment and exogenous population growth. In Section 3, we expand the basic model by introducing endogenous population growth and the population's age structure. The dynamics of the expanded model is studied in section 4. The conclusions of the paper are summarized in section 5.

## 2 The basic model with exogenous population growth

In this section we present the basic economic growth model with unemployment. We assume a closed economy with final good  $Y_t$  and knowledge  $h_t$ , and the following technologies<sup>2</sup> :

$$Y_t = \mu (\gamma h_t L_t)^\alpha, \quad 0 < \alpha < 1, 0 < \gamma < 1, \mu > 0, \quad (1)$$

$$h_{t+1} = e^{\delta(1-\gamma)} h_t, \quad 0 < \delta < 1, \quad (2)$$

where  $\mu$  and  $\delta$  are productivity parameters of the final and knowledge sector, respectively, and  $\gamma$  and  $(1 - \gamma)$  are the fractions of time that people devote to the production and

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<sup>2</sup> This framework encompasses specific growth models as special cases (Sollow, 1956; Lucas, 1988; Romer, 1990; Aghion and Howitt, 1992).  $h$  could be considered any factor that causes sustained growth: technical progress, knowledge in the tradition of endogenous growth models, or human capital.

knowledge, respectively.  $L_t$  is the employment and labour is supplied inelastically either in producing the final good and/or in producing knowledge; both activities are paid at the same wage  $w_t$ .

It is assumed the total demand for the final good,  $D_{t+1}$ , is equal to the wage income paid at the end of the previous period:

$$D_{t+1} = w_t L_t, \quad (3)$$

which determines the total labour demand of the economy. On the other hand, we consider that the total labour force  $A$  grows at a fixed exogenous rate  $n = \frac{A_{t+1}}{A_t} - 1$ ,  $n > 0$ .

Finally, following the literature of disequilibrium models (Goodwin, 1967; Chiarella et al., 2000; Manfredi and Fanti, 2003) unemployment is included by adding a nonlinear real wage Phillips' curve:

$$\frac{w_{t+1}}{w_t} = \exp(-a_1 + a_2 l_t), \quad a_2 > a_1 > 0, \quad (4)$$

where  $l_t = \frac{L_t}{A_t}$  is the employment rate, and  $a_1, a_2$  are the characteristic parameters governing the labour market (Pohjola, 1981).

From the above assumptions, we show in Roa et al. (2008) that the evolution of the employment rate is governed by the equation:

$$l_{t+1} = r \exp(-s l_t) l_t, \quad (5)$$

where  $r = \frac{1}{1+n} \exp\left(\frac{\alpha\delta(1-\gamma)+a_1}{(1-\alpha)}\right)$ , and  $s = \frac{a_2}{(1-\alpha)}$ . (5) is the Ricker-Moran equation (Moran, 1950; Ricker, 1954), well-known for its ability to generate chaotic behaviour (May and Oster, 1976).

The dynamics of (5) is investigated in Roa et al. (2008) in great detail. Next, we briefly summarize the main results. The solutions of (5) show a cyclic evolution of the employment rate, where economic growth and knowledge accumulation are interacting with labour

market characteristics. Given the standard values of the knowledge and production sectors parameters (Lucas, 1988; Barro and Sala-i-Martin, 1995), the possibility of chaos reveals to be reasonable if  $a_1$  is big enough. This parameter, in combination with  $a_2$ , determines the elasticity of the wage's growth to the employment rate: the model predicts that the more imperfect the labour market<sup>3</sup>, the greater the possibility of instability.

The qualitative conclusions obtained concerning the dynamics of the employment rate apply directly to the growth rate, given by:

$$\frac{y_{t+1}}{y_t} = z \cdot \left( \frac{l_{t+1}}{l_t} \right)^\alpha, \quad z = e^{\delta(1-\gamma)\alpha} (1+n)^{\alpha-1}, \quad (6)$$

where  $y_t = \frac{Y_t}{A_t}$  is the per capita production.

Population growth rate and knowledge accumulation just determine the income growth trend (measured by the parameter  $z$ ). In the same way as traditional models of growth, sustained growth is due to knowledge production or technological change. Considering realistic values of  $n$ , the economy would disappear only if it does not invest in knowledge ( $\gamma = 1$ ), or if it does at an infinitesimal level. This result coincides with the Malthusian idea concerning the stagnation of the economy. As the population rises, in a non-industrialized economy and with limited resources, at some moment the amount of food falls below the subsistence level. So, we get an inverse Malthusian relation between population and economic growth.

The basic model produces oscillations of employment rate and per capita income with too large a size when compared with real world behaviour, which is mainly due to the simple nonlinear modelling of wage dynamics. Then we consider a wage dynamics modelling

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<sup>3</sup> The rigidity of the labour market is explained from different imperfections, such as cost of mobility, informational imperfections, mismatch between the workers seeking jobs and the vacancies available, minimum wage and union wage setting, etc.

that fits better its assumed behaviour (see Figure 1):

$$\frac{w_{t+1}}{w_t} = \exp(-a_1 + a_2 l_t^\sigma + a_3 (1 - l_t)^{-\varepsilon}), \quad a_1, a_2, a_3 > 0, 0 < \sigma < 1, 0 < \varepsilon < 1, \quad (7)$$

and so more realistic intertemporal evolutions of the employment rate and per capita production are generated (see Figure 2).

(Figure 1 about here)

(Figure 2 about here)

### 3 Endogenous population growth and age structure

Next, we extend the basic model introducing both the endogenous population growth and the population's age structure. First, following the standard models of endogenous population (Galor and Weil, 2000; Jones, 2001), it is assumed that labour supply is determined through micro-founded fertility choices of individuals, in which households choose the number of children.

Let us assume that the representative family's preferences are represented by the following standard log-linear utility function:

$$U(c_{t+1}, b_{t+1}) = c_{t+1}^{1-\epsilon} b_{t+1}^\epsilon, \quad 0 < \epsilon < 1, \quad (8)$$

where  $c$  is the per capita consumption and  $\epsilon$  measures the preference for children. At every point of time  $t$  each family choice determines the number of children  $b$ , who will be born in the next period  $t + 1$ .

The cost of childrearing  $E_t$  includes both the cost of raising a child, regardless of quality,  $\theta_1$ , and education cost  $e_t = \theta_2 h_t^\rho$ :

$$E_t = \theta_1 + \theta_2 h_t^\rho, \quad 0 < \rho < 1. \quad (9)$$

$\theta_1$  is considered as a fixed “maintenance” cost (e.g. food, clothes), and the education cost  $e_t$  depends on the medium level of knowledge<sup>4</sup>.

Given these assumptions, each family’s static optimization problem<sup>5</sup> is:

$$\left. \begin{array}{l} \max \quad U(c_{t+1}, b_{t+1}) = c_{t+1}^{1-\epsilon} b_{t+1}^\epsilon \\ \text{s.t.} \quad c_{t+1} + (\theta_1 + \theta_2 h_t^\rho) b_{t+1} = y_t^F \end{array} \right\}, \quad (10)$$

being  $y_t^F = 2y_t$  the representative family’s income<sup>6</sup>. Solving problem (10) we get that the optimal number of children for each member of a representative family at each period  $t$ :

$$b_{t+1} = \frac{\epsilon y_t}{\theta_1 + \theta_2 h_t^\rho}, \quad (11)$$

and  $b$  can be interpreted as the fertility rate. In line with endogenous population growth literature, it depends positively on the per capita income and negatively on the medium level of knowledge. As individuals receive all income from labour supplied,  $y_t = w_t l_t$ , the fertility rate can be written as:

$$b_{t+1} = \frac{\epsilon w_t l_t}{\theta_1 + \theta_2 h_t^\rho}. \quad (12)$$

Because only adult populations can work and reproduce, the labour force  $A$  coincides with the adult population. The total number of births  $B$  in each period is  $B_{t+1} = b_{t+1} A_t$ , and then the evolution of  $A$  is given by:

$$A_{t+1} = A_t(1 - \pi) + B_{t+1-\nu} = A_t(1 - \pi) + b_{t+1-\nu} A_{t-\nu}, \quad (13)$$

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<sup>4</sup> Schultz (1975) claimed that new technology will create a demand for skilled workers to analyze new production processes. So, education cost would depend on knowledge  $h$ .

<sup>5</sup> Some models of endogenous population growth introduce altruism in the parent’s utility function (Becker et al. 1990). This assumption implies both of the dynastic utility functions considered, and solves dynamic optimization problems of the Bellman equation type. By considering some very simple assumptions: “the more standard dynamic optimization problem reduces to the sequence of static problems” (see Jones, 2001, p. 5)

<sup>6</sup> Manfredi and Fanti (2006) suggest that situations of structural long-term unemployment might cause the fertility rate of employed and unemployed individuals to be different.

where  $\pi$  is the constant mortality rate<sup>7</sup> of the labour force. The parameter  $\nu$  represents the time age transition to adulthood; that is, the delay of recruitment into the labour force. This is taken as exogenous. Therefore, the labour supply is related to past fertility  $b_{t+1-\nu}$ , and thus the labour market situation (past levels of the wage,  $w_{t+1-\nu}$ ).

Finally, taking into account the above assumptions, we obtain the endogenous growth rate of  $A$ :

$$n_t = \frac{A_{t+1}}{A_t} - 1 = -\pi + b_{t+1-\nu} \frac{A_{t-\nu}}{A_t} = -\pi + b_{t+1-\nu} \frac{1}{(1+n_{t-1}) \cdots (1+n_{t-\nu})}. \quad (14)$$

The delay  $\nu$  is related with the existence of the Malthusian cycle. This cycle is the consequence of the endogenous interaction between economic and demographics forces: “...the lags between the response of the fertility to current labour market conditions and the time when the resultating births actually enter the labour force” (Lee, 1997, p. 1097). The key of this mechanism is the positive relation between the fertility rate and the level of the wage<sup>8</sup> (equation 12). As wage increases the workers’ number of offsprings increases as well. However, some years must pass before the response of fertility can be felt in the labour market. In the next section, this effect will be studied.

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<sup>7</sup> Blanchard (1985) states that “evidence on mortality rates suggests low and approximately constant probabilities from age 20 to age 40,” after this, mortality rates depend inversely and exponentially on individual age (see “Gomperty’s Law,” Wetterstrand, 1981). For simplicity we set the mortality rate of children to zero. Though this might seem inappropriate for historical epochs characterised by high child mortality, or currently in poor countries, it is, however, easily incorporated into the model by considering a “diminished” fertility rate in order to include mortality of newborns in child mortality.

<sup>8</sup> Dyson and Murphy (1985) have shown that this positive relation, called Malthusian fertility, has been quite common during the demographic transition experienced by both developing and industrialized countries.

## 4 The dynamics of the extended model

The extended economic growth model with endogenous demographic structure is given by the following equations:

$$\frac{l_{t+1}}{l_t} = \frac{1}{1+n_t} \exp\left(\frac{\alpha\delta(1-\gamma)}{(1-\alpha)}\right) \left(\frac{w_{t+1}}{w_t}\right)^{\frac{1}{1-\alpha}}, \quad (15.a)$$

$$\frac{h_{t+1}}{h_t} = \exp(\delta(1-\gamma)), \quad (15.b)$$

$$\frac{w_{t+1}}{w_t} = \exp(-a_1 + a_2 l_t^\sigma + a_3 (1-l_t)^{-\varepsilon}), \quad (15.c)$$

$$\frac{y_{t+1}}{y_t} = \exp(\delta(1-\gamma)\alpha) \left(\frac{1}{1+n_t}\right)^{1-\alpha} \left(\frac{l_{t+1}}{l_t}\right)^\alpha, \quad (15.d)$$

$$n_t = -\pi + b_{t+1-\nu} \frac{1}{(1+n_{t-1}) \cdots (1+n_{t-\nu})}, \quad (15.e)$$

$$\text{and } b_{t+1} = \frac{\epsilon w_t l_t}{\theta_1 + \theta_2 h_t^\rho}. \quad (15.f)$$

Because of the high analytical complexity of the model we limit its study to numerical simulations (by using MATLAB). Compared to standard endogenous population growth models, the complexity of our model is in part due to the delay, which arises as a consequence of taking age structure into account. The large number of simulations carried out suggest that the subsequent properties are robust for acceptable changes in parameter values<sup>9</sup>.

The first feature is the endogenous emergence of a logistic behaviour in the population growth rate describing its historical evolution (see Figure 3, where we have drawn the fertility rate  $b_t$  and below the growth rate  $n_t$  for four different realistic values of the delay,  $\nu = 12, 16, 20, 24$ ).

(Figure 3 about here)

The evolution is characterized by a first stage habitually named Post Malthusian,

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<sup>9</sup> Although a large number of different values has been simulated, the figures shown in this section correspond to the following standard parameter values:  $\alpha = 0.8$ ,  $\delta = 0.1$ ,  $\gamma = 0.7$ ,  $a_1 = 1.7$ ,  $a_2 = 1.5$ ,  $a_3 = 0.16$ ,  $\sigma = 0.01$ ,  $\varepsilon = 0.1$ ,  $\pi = 0.01$ ,  $\epsilon = 0.02$ ,  $\theta_1 = 0.3$ ,  $\theta_2 = 0.01$ ,  $\rho = 0.8$ .



where population growth rate increases and, after a transition phase, a modern growth stage follows where  $n$  decreases and tends to stagnate (Galor and Weil, 2000). For low levels of knowledge development the education cost is small and income growth raises the fertility rate. But for high levels of knowledge development education cost rises inducing parents to have fewer, but more high-quality children. On the other hand, as the economy develops, new technology and production processes will increase the demand of knowledge. It generates a demographic transition, that is, an economy transitions from a high fertility and low knowledge accumulation stage to a low fertility and rapid knowledge accumulation stage<sup>10</sup>.

Besides the logistic behaviour of population growth, Figure 3 shows the behaviour of  $n$  during different phases. In the stage previous to the transition  $n$  oscillates constantly, in the transition stage it shows a very regular behaviour, and in the last stage it oscillates again. This type of dynamic evolution is not exclusive of the population growth rate. In Figure 4 we have drawn the intertemporal evolution of  $l$  that shows the same dynamics behaviour.

(Figure 4 about here)

This dynamics result would be related with the age structure mechanism. In keeping with the Malthusian cycle, Figures 3 and 4 show that as delayed recruitment into the labour force  $\nu$  increases, instability does as well. Therefore, entrance of a young population to the labour force acts as a stabilising factor<sup>11</sup>. A young labour force would smooth fluctuations caused by labour market rigidities because satisfies the needs of the labour market more easily (e.g. training, geographic and labor mobility) than an adult population

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<sup>10</sup> Because human capital is not a family choice, the model is abstracted from the so-called quantity-quality trade-off in the literature of endogenous population growth.

<sup>11</sup> The destabilising role of age structure in economic growth models is a concept which is vastly discussed in Manfredi and Fanti (2003, 2006).

(corresponding to a low population growth and characteristic of developed economies). Hence, the justification for the stability of employment during the demographic transition would be that a young population (resulting from the typical “baby boom” of the post Malthusian stage) joins the labor force at the beginning of this transition.

The qualitative conclusions regarding the fluctuations of the employment rate apply directly to the growth rate (see Figure 5); the greater the delay, the higher the instability and the earlier the emergence of oscillations. The moment of oscillations’ emergence coincides for the three variables: population growth, employment rate and economic growth rate (see Figures 3-5), and it is produced when the logistic transition is finishing.

(Figure 5 about here)

Besides the oscillations, Figure 5 shows that the greater the delay, the higher the rate of economic growth. This is due to, as the economy evolves and knowledge accumulation increases, the demand for high-quality individuals does as well (footnote 4). This would lead to more years and time spent in education (higher delay), and, so, higher economic growth<sup>12</sup>.

Every time the delay increases the economy undergoes a structural change: it moves up to curves of higher delays (see Figure 5), with higher economic growth, but with more instability. This is a relevant result in terms of policies: there would be a conflict regarding policies related with the control of the age of entry into the labour force and years spent in human capital accumulation. On one hand, they would promote economic growth but, on the other hand, would increase instability.

There are no laws in the early stages of development that forbid children’s work; the delay in general is high. On the other hand, neither are there laws that generate labour

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<sup>12</sup> Normally, while developing countries are characterized by both low values of the delay and economic growth, and the opposite is observed in developed countries.

market frictions. The agricultural sector is the dominant and the labour market is closer to being competitive perfectly. Therefore, our model would predict that at low levels of development economies are more stable<sup>13</sup>. This result coincides with the historical evolution on population and economic growth during the Malthusian Regime in Western Europe (Maddison, 2003). In these economies, the growth rate of GDP per capita between 500 and 1500 was zero, and the population growth was nearly zero.

Later, during the Post Malthusian Stage, the volatility of the population growth rate becomes quite large (Maddison, 2003). However, unlike our model, such large movements in population growth do not seem to be present in the rich world today (Lagerlof, 2006). The model attempts to be representative of a developed economy, and some assumptions are not appropriate for explaining the transition from an agricultural economy to an industrial one<sup>14</sup>. Stage-dependent assumptions (e.g. regarding birth rate, endogenous mortality, and labour market) would generate more realistic intertemporal evolutions of the population growth rate.

Finally, it is important to notice that the logistic behaviour and instability emerge solely because of the internal structure of the economy; it is not exogenously generated. Thus, the model might consider itself as a self-organizing system displaying emergent properties.

## 5 Conclusions

This work complements the theoretical results of economic growth models and cycles introducing demographic factors and labour market institutions. We show the added value

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<sup>13</sup> This is an important and controversial theme in the growth and development literature: the relationship between volatility and economic development (Lucas, 1988, Acemoglu and Zilibotti, 1997).

<sup>14</sup> Galor and Weil (2000) develop a model that captures the historical evolution of population, technology and output. The model has recently been calibrated by Lagerlof (2006).

of jointly considering these three strands of literature: sticky prices, neoclassical economic growth and demography. A standard growth model with endogenous population growth and age structure is elaborated, where the economy fluctuations are endogenous and are generated from the interaction between demand and supply labour factors.

First, we show that the higher the rigidity of the labour market, the greater the possibility of instability<sup>15</sup>. Second, when we assume endogenous population growth by means of optimal fertility choices, the model endogenously generates a logistic behaviour in the population growth rate describing its historical evolution. We get that during the demographic transition the dynamics of the employment rate shows a very regular behaviour. This is due to the entrance of a young population (resulting from a “baby boom”), to the labour force smooth fluctuations caused by labour market rigidities. However, in consonance with the Malthusian cycle, the delay of transition to the adulthood (entry into the labour force) appears to be a source of instability, especially during the demographic transition stage. Therefore, policies which improve the flexibility of the labour market and reduce the age of entry into the labour force will decrease instability.

In regards to economic growth, because an increase in the population growth rate leads to a decrease in the growth rate of income, policies which reduce incentives for fertility may positively affect long run economic growth. On the other hand, we get that by increasing the years of human capital accumulation and delaying the age of entrance into the labour force economic growth would be promoted.

In conclusion, the model predicts an interesting result in terms of policies. State and institutions would find a conflict between enhancing growth or reducing its instability

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<sup>15</sup> Acemoglu and Zilibotti (1997) link the degree of financial market incompleteness to uncertainty in the growth process. They get that the more incomplete the financial market, the greater the inability to diversify idiosyncratic risk and, so, the greater the variability in the growth process. Because financial market incompleteness is associated with early stages of development, they get that more developed economies are less unstable.

through policies of controlling the age of entry into the labour force and years spent in human capital accumulation. We find these preliminary results encouraging enough to perform further work, both at the theoretical and empirical level.

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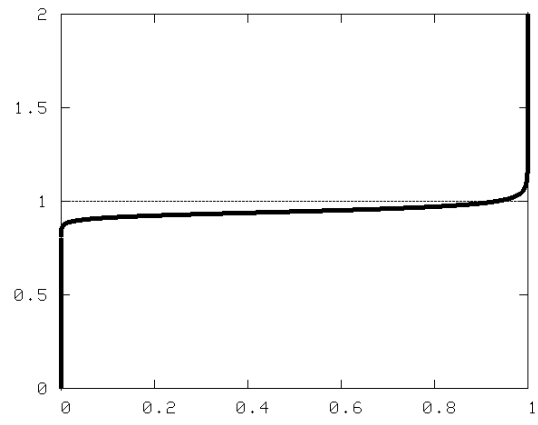


Figure 1. Phillips curve



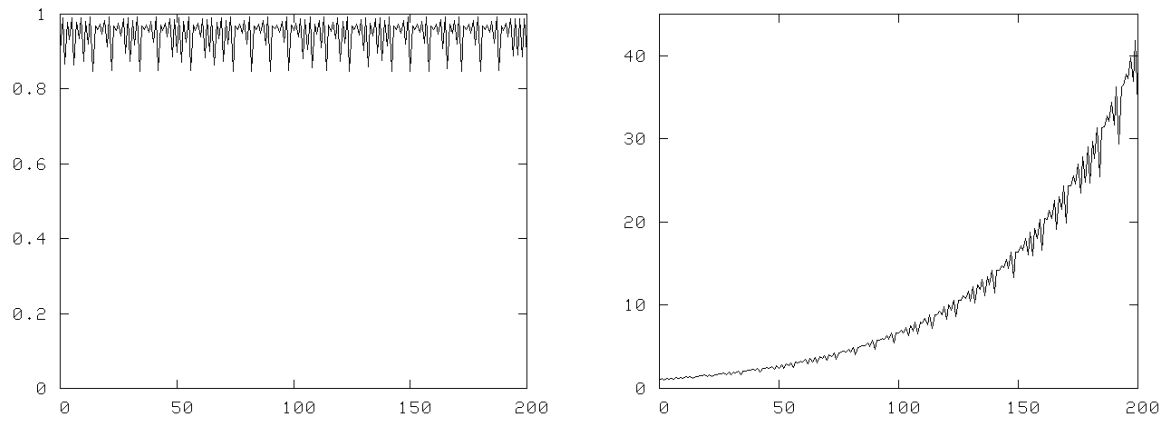


Figure 2. Employment and income evolution

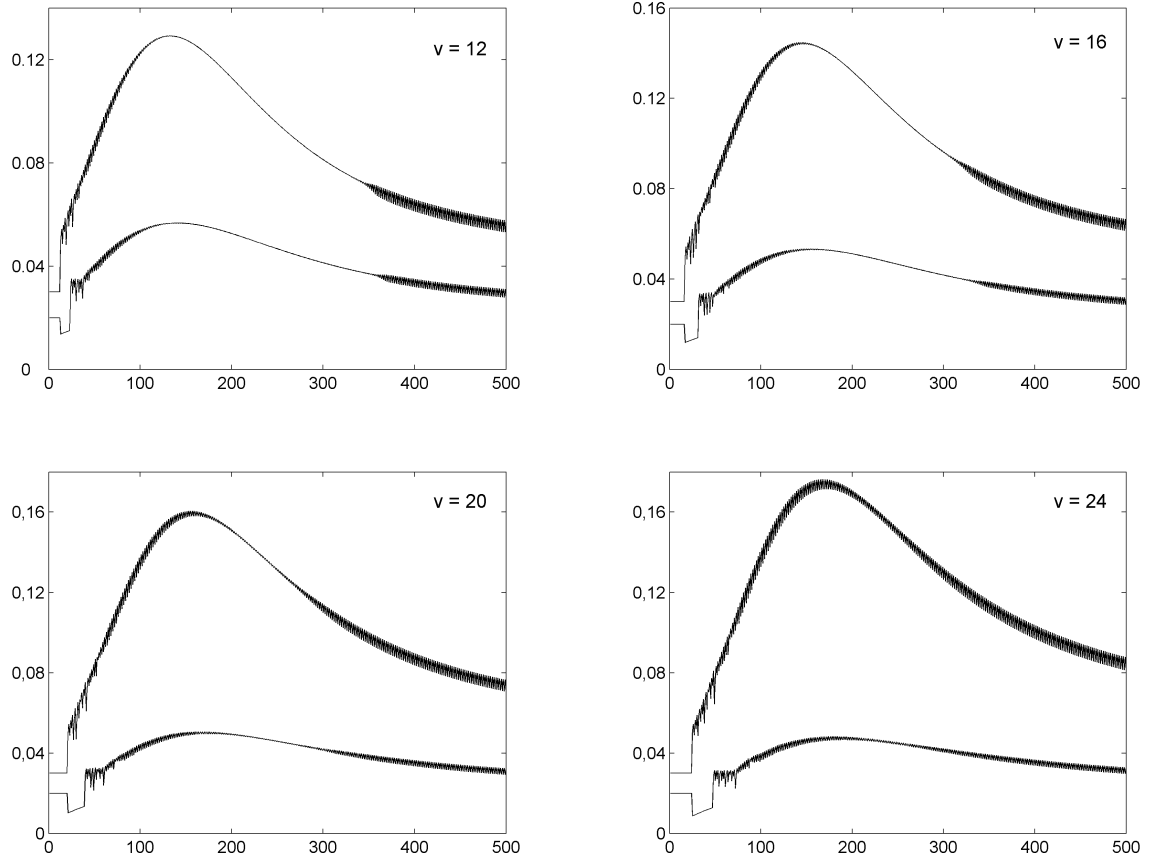


Figure 3. Logistic evolution of the population growth

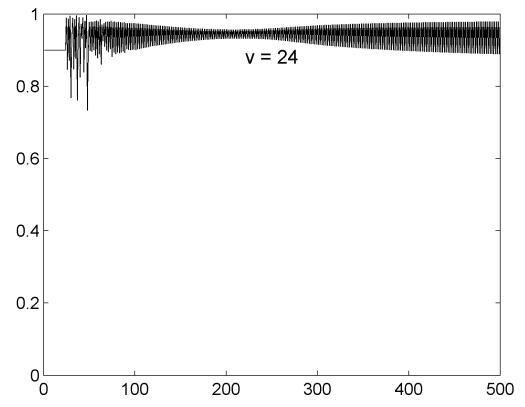
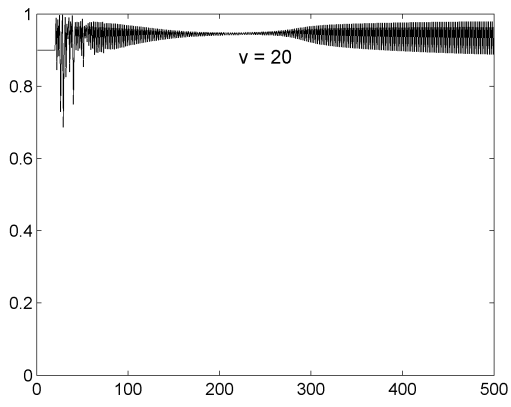
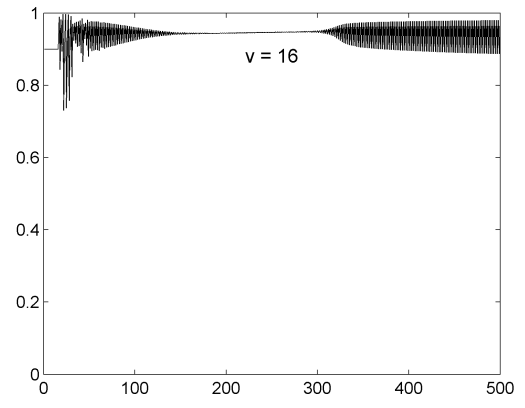
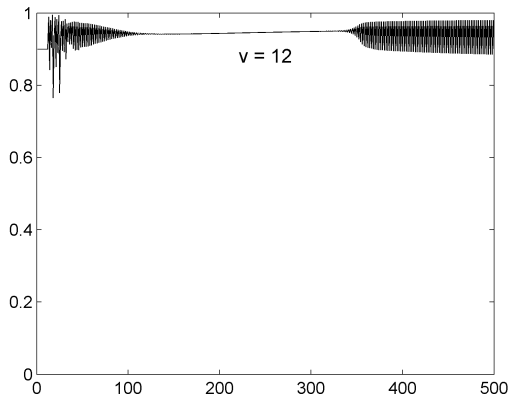


Figure 4. Intertemporal evolution of the employment rate

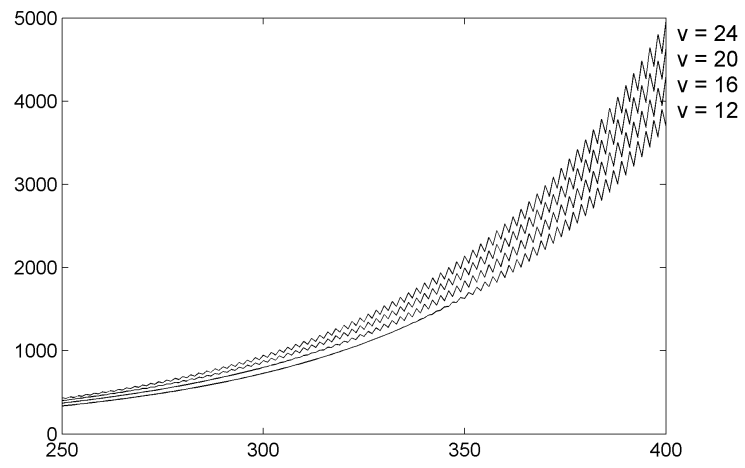


Figure 5. Income growth rate

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