NÚMERO 371

MARÍA JOSÉ ROA, DULCE SAURA, AND FRANCISCO J. VÁZQUEZ

Endogenous Demographic Transition, Unemployment and Complex Dynamics in an Endogenous Growth Model

NOVIEMBRE 2006

CIDE

www.cide.edu

Las colecciones de **Documentos de Trabajo** del **CIDE** representan un medio para difundir los avances de la labor de investigación, y para permitir que los autores reciban comentarios antes de su publicación definitiva. Se agradecerá que los comentarios se hagan llegar directamente al (los) autor(es).

Tel. 5727•9800 exts. 6090, 6091, 6092, 6094
Fax: 5727•9800 ext. 6314
Correo electrónico: publicaciones@cide.edu
www.cide.edu

Producción a cargo del (los) autor(es), por lo que tanto el contenido así como el estilo y la redacción son su responsabilidad.

Abstract

The objective of this research is to elaborate an endogenous growth model which allows us to examine the mutual interaction between economic growth, labour market behaviour and population growth. We show how the introduction of disequilibrium and some nonlinearities give rise to more complex dynamics than the usually obtained. The unemployment rate, per capita income and population growth rate fluctuate along cycles of different periods, and they may even exhibit chaotic paths. In particular, as the level of knowledge sector development and the rigidity of the labour market increase, the possibility of complex dynamics increases as well. Moreover, in line with wage bargaining models, we get the result that the higher workers' bargaining power, the lower both the employment rate and per capita production. When we assume endogenous population growth by means of optimal fertility choices, the model endogenously generates a logistic behaviour in the population growth rate describing its historical evolution. The demographic transition reverses the positive relationship between economic development and population growth, and fertility rates permanently decline. However, besides this structural change, we find that during the transition, the relevant variables evolve regularly; in particular, it seems that labour force fits the labour market needs. As some endogenous population research states, this dynamic result could be related with the population's age structure.

Resumen

El objetivo de la presente investigación es elaborar un modelo de crecimiento endógeno que nos permita examinar la interacción entre el crecimiento económico, la situación del mercado de trabajo y el crecimiento de la población. La introducción de desequilibrios y no linealidades generan comportamientos dinámicos de gran complejidad. La tasa de desempleo, la renta per capita y la tasa de crecimiento de la población fluctúan a lo largo de ciclos de diferentes periodos, e incluso trayectorias caóticas. Introduciendo el crecimiento endógeno de la población mediante un problema de optimización donde los agentes deciden el número de hijos, el modelo genera endógenamente un comportamiento logístico para la tasa de crecimiento de la población, describiendo su evolución histórica. La transición demográfica invierte la relación positiva entre el crecimiento económico y el crecimiento de la población, y la tasa de fertilidad disminuye de forma permanente. Sin embargo, detrás de este cambio estructural, obtenemos que durante la transición las variables del modelo evolucionan de forma regular. En particular, la fuerza de trabajo parece adaptarse a las necesidades del mercado de trabajo. Como afirman algunas investigaciones sobre demografía y crecimiento económico, este comportamiento dinámico podría estar relacionado con la estructura de edad de la población.

1 Introduction

For several decades, economists have discussed the influence of population growth on economic growth and vice versa. Although there are a diversity of approaches, nowadays, there is strong evidence that demographic variables have a significant impact on economic growth. Moreover, as suggested by Bloom and Canning (1999): "causality runs in both directions, from the economy to demography, and from demography to the economy. The interaction is a dynamic process, with each side affecting the other". One of the mechanisms through which demographic variables affect economic growth is the so-called labor market effect: demographic transition affects the age structure of the labor supply with ambiguous effects on growth (Bloom and Canning, 1999; Bloom et al., 2001). This effect depends mainly on the flexibility of the labor market.

The traditional literature of economic growth has considered certain assumptions that hinder the simultaneous analysis of population growth, labor market and economic growth. On the one hand, an exogenous population growth rate is assumed; the population behaviour is determined outside of the model. On the other hand, full employment is assumed; the labor market is always in equilibrium, independently of the economic conditions.

The purpose of this research is to investigate the mutual interaction between economic growth, labor market flexibility and population growth. To do this we elaborate a model of endogenous growth that considers both endogenous population growth and the possibility of labor unemployment.

The analysis of the dynamic behaviour is one of the most important points of our research. We will show how the introduction of disequilibrium and some nonlinearities give rise to more complex dynamics than those usually obtained. This behaviour depends on both the level of knowledge development and the rigidity of the labour market. Besides endogenous cyclical behaviour, the dynamic analysis also points to a positive income growth trend, but we also find periods in which per capita income decreases. Although initially we consider exogenous population growth, later we show how the introduction of endogenous population growth enriches the theoretical results. In particular, the model endogenously generates the demographic transition to a developed economy, and explains both why there is a demographic transition and how it is produced.

Our model is built on three theoretical bases. The first one corresponds to the engine of growth. In the tradition of Lucas (1988), technical progress (interpreted endogenously as knowledge) sustains economic growth in the long term. It is assumed that because of historical, cultural or sociological reasons, the economy decides the allocation of resources between the production of the final good and knowledge. This idea is also found in Goodwin (1967). The second base of the model introduces the possibility of unemployment. The labor market is assumed to be in disequilibrium and the wage dynamic -modeled by a non linear Phillips curve- is determined by this disequilibrium. The third base concerns the study of the mutual interactions between demographic variables and economic growth. Following the standard models of endogenous population growth (Becker et al., 1990; Galor and Weill, 2000), it is assumed that labor supply is determined through micro-founded fertility choices of individuals, in which households choose the number of children via a quantity-quality trade off.

The rest of the paper is organized as follows. In section 2 we detail the hypothesis of our model. In section 3 we analyze the dynamic behaviour of the model's variables with exogenous population growth and different wage dynamics. In Section 4, we introduce endogenous population growth in the model. The conclusions of the paper are summarized in section 5.

2 The model

2.1 Production Functions for the Final Good and Knowledge

We consider a single economy that uses labour L_t and knowledge h_t to produce a final good Y_t. The market for the final good is perfectly competitive. The aggregate production function is:

$$Y_t = F(L_t, h_t) = \mu \left(\gamma h_t L_t\right)^{\alpha} \hat{h}_t^{\beta}, \quad 0 < \alpha < 1, \beta > 0, 0 < \gamma < 1, \mu > 0$$

where γ is the constant exogenous fraction of time that people devote to the production of final goods. μ is the sector productivity. In addition to the effects of an individual's knowledge on his own productivity, we also consider an external effect (Lucas, 1988). Specifically, the average level of knowledge $\hat{h}_t$ also contributes to the productivity of all factors of production. If we assume that all workers are identical, the average level of knowledge $\hat{h}_t$ is just h_t, in which case output production Y can be rewritten as[1]

$$Y_t = F(L_t, h_t) = \mu \left(\gamma L_t\right)^{\alpha} h_t^{\alpha+\beta}$$

Besides producing final goods, the economy also produces knowledge. In the tradition of Lucas, we consider and formalize a general concept of human capital. Human capital accumulation is a "social activity" which survives individuals, so it can be accumulated without limit and guarantees long run economic growth. Consequently, we are not going to distinguish between private human capital of individuals and knowledge of the society as a whole[2] . Therefore h can be interpreted as a composite good made up of knowledge and human capital.

---

[1] For simplicity, we consider constant physical capital (Aghion-Howitt, 1998) which could be included in the parameter μ. We can assume a economy where physical capital accumulation is stable and only by accumulating human capital can the economy obtain growth.

[2] In the literature on economic growth, some authors separate the rival component of knowledge, that is, knowledge embodied in some kind of tangible capital such as conventional physical capital or human capital, from the non rival or intangible component (Romer, 1990)

The knowledge technology is[3]

$$h_{t+1} = e^{\delta(1-\gamma)} h_t, \qquad 0 < \delta < 1 \tag{2.1}$$

where $(1-\gamma)$ is the fraction of time devoted to knowledge accumulation. δ is the productivity of the sector[4] .

The saving and investment decisions are in line with Goodwin (1967). If L_t is used in the production of final goods, that is, if the economy does not invest in knowledge, the final production (potential product) would be $\tilde{Y}_t$:

$$\tilde{Y}_t = \mu L_t^{\alpha} h_t^{\alpha+\beta}$$

So, if all the labour is used in the production of final goods the economy will get more product than if some labour is allocated to the production of knowledge. The diference between the two production levels is:

$$\mu L_t^{\alpha} h_t^{\alpha+\beta} - \mu \left(\gamma L_t\right)^{\alpha} h_t^{\alpha+\beta} = \left(1-\gamma^{\alpha}\right) \tilde{Y}_t$$

This equation shows the oportunity cost of the use of labour inputs in the production of knowledge instead of final goods. This diference is the production or real income value that the economy loses, and can be interpreted as the total savings of the economy. These savings are invested ex-ante in the production of knowledge. Therefore γ^{α} is the fraction of income that individuals devote to consumption and $1-\gamma^{\alpha}$ is the fraction they devote to saving or investing in knowledge.

---

[3] The model can be easy rewritten in terms of a two sector economy, in which there exits a constant proportionality between the population devoted to the productions of final goods and knowledge, and workers are paid at the same wage.

[4] Equation (2.1) is an approximation to Lucas's knowledge technology:

$$h_{t+1} = h_t + h_t \delta \left(1-\gamma\right) \qquad 0 < \delta < 1$$

and we make this assumption because it simplifies the later analysis considerably.

Although the production sector produces knowledge, it is the economy itself that decides to invest in knowledge. On the one hand, workers invest in knowledge in order to improve their skills and get a better job in the future. On the other hand, firms invest in knowledge in order to either improve the productivity of the labour input or to get process and product innovations[5] . Therefore, the fraction of time devoted to each activity, although exogenous, is a decision of the entire economy. These decisions can be interpreted as cultural or social characteristics. For simplicity in the following analysis we assume it is exogenous. However, we should point out that the choice of γ determines the time evolution of the economy. In a specific way, this idea is similar to Lucas (1988, p. 19), where he states that "human capital is a social activity, involving groups of people"[6] .

2.2 Consumers-Workers

Each individual is endowed with one unit of time that can be used either in producing the final good and or in producing knowledge. Labour is supplied inelastically. At the end of each period, individuals receive all the income from labour supplied in these activities, and all labour is paid at the same wage denoted by w_t. We assume that all income is spent on consumption. So, in each period the total demand for the final good is equal to the wage income paid at the end of the previous period:

$$D_{t+1} = C_{t+1} = w_t L_t \tag{2.2}$$

where D_{t+1} is total demand in period $t+1$, C_{t+1} is total consumption in period $t+1$, L_t is total employment and $w_t L_t$ is the total wage income paid at the end of period t.

[5] As was explained earlier, the Lucas concept of knowledge is very general, and it doesn't exclude the possibility of knowledge embodied in the physical capital of the firm. In this case the economy decides to invest in knowledge because by doing so it increases the tangible part of its capital.

[6] In the same line, Jones (2001) relates the exogenous fraction of time devoted to knowledge accumulation with the role of institutional development, which promotes the production of knowledge with the creation, definition and enforcement of property rights. Through numerical simulations, he shows that the improvement in property rights in the 20th century played a critical role in the timing of Industrial Revolution.

On the other hand, initially we assume that the population N grows at the fixed exogenous rate n. Moreover, we assume, consistently with the empirical evidence[7] , that the total population is a constant fraction of the labour force A, $N = \lambda A$ ($\lambda > 1$). Thus

$$\frac{A_{t+1}}{A_t} = \frac{N_{t+1}}{N_t} = 1 + n, \quad n > 0 \tag{2.3}$$

In section 3, we will assume endogenous population growth.

2.3 Production of Final Good, and Labour Demand

We assume the productive sector produces whatever individuals demand. The production process for the final good takes one period. In each period, final production is equal to the total demand in the next period

$$Y_t = D_{t+1}$$

From this last condition, we get the total labour demand. At the beginning of each period, the productive sector demands labour in such a way that supply of goods equals demand in the next period. Substituting demand and supply expressions

$$\mu \left(\gamma L_t\right)^{\alpha} h_t^{\alpha+\beta} = w_t L_t$$

and solving for L_t we get the total labour demand of the economy

$$L_t = \left(\frac{\mu \gamma^{\alpha} h_t^{\alpha+\beta}}{w_t} \right)^{\frac{1}{1-\alpha}} \tag{2.4}$$

Given h and w, if the productive sector demands less than L_t, we have excess supply of the final good $Y_t > w_t L_t$. Labour is very productive and demand for goods is insufficient to absorb supply. To provide an incentive for demand, the productive sector needs to employ more workers, which generates more income and, consequently, greater demand.

[7] For example, in Spain this relation is rather stable -although less than in other European countries- except during particular periods such as when women were incorporated into the labour market.

This result is shown in figure 1, where we have drawn the production and cost functions for particular values of h_t and w_t. There are no incentives to employ more than L_t because firms would suffer losses. In this case, there is a continuum of excess demand, $Y_t < w_t L_t$, and the level of employment (and therefore production) does not increase because the demand that is generated would be unsatisfied. Very strong and rigid diminishing returns to labour in productive capacity emerge. It is necessary to introduce new investments while firms fire workers in order not to suffer losses[8] . Increasing productive efficiency -which would shift the production function in figure 1 upwards- would allow the excess demand to be satisfied and employment and production would also be increased[9] . Therefore, the L_t which satisfies (2.4) is the equilibrium level of employment.

(Figure 1 see below)

In the dynamic analysis we will show that L_t may not equal labour supply, so we could get unemployment $L_t < A_t$. Similarly, we could get $L_t > A_t$, that is, the demand for labour by the productive sector could be rationed. Because our objective is to explain the dynamic relationship between growth and unemployment, this last case will not be analysed. Therefore, we make the necessary assumptions in order that $L_t \leq A_t$, and also we assume that the market for the final good is always in equilibrium[10] .

2.4 Wage Dynamics

The labour market is assumed to be in disequilibrium and the wage dynamics is determined by this disequilibrium. In particular, we assume a Phillips equation, where the wage is

[8] We are considering the profits that firms get by only producing final goods, $B^o = Y_t - w_t L_t$. Total profits are received both from final goods and knowledge production.

[9] Later on, we will show that this is one of the reasons why employment depends positively on technological change.

[10] This is a usual assumption in the related literature (see Pohjola, 1981).

increasing with the rate of employment

$$\frac{w_{t+1}}{w_t} = h(l_t), \qquad h'(l) > 0$$

where $l_t = \frac{L_t}{A_t}$ is the rate of employment. In particular, we consider the following nonlinear curve[11] :

$$\frac{w_{t+1}}{w_t} = \exp(-a + bl_t), \; b > a > 0 \tag{2.5}$$

The functional relationship between the wage dynamics and the rate of employment is illustred in figure 2.

(Figure 2 see below)

This modelling[12] is based on the ideas of Phillips (1958) and Lipsey (1960) regarding the non linearity of the Phillips curve[13] ("excess demand conditions are much more inflationary than excess supply conditions are desinflationary" as stated by Clark et al. (1996)) and the keynesian ideas about downward wage rigidity due to market imperfections analyzed in the *New Keynesian Economics* literature (Mankiw and Romer, 1991). As far as these dynamics is concerned, we try to introduce, in a simple way, the labour market disequilibrium caused by different imperfections which explain wage and price rigidity from the optimizing behaviour of individuals (eficiency wage, insiders-outsiders and the wage bargain between employers and unions). In particular, and in the same line as models of

[11] Although the function that is normally known as the "Phillips curve" is $\varphi(l) = \frac{w_{t+1}-w_t}{w_t}$, abusing language somewhat, we also use the term "Phillips curve" to refer to the function $h(l)$. Both have the same properties because $h(l)$ is a translation of $\varphi(l)$: $h(l) = 1 + \varphi(l)$.

[12] For simplicity, Goodwin (1967) introduces a linear approximation to $h(l_t)$, but, as he said, the linear approximation is "quite satisfactorily for moderate movements of l near to the point 1". Also, the Goodwin model presents an economic inconsistency: l could be greater than one. Desai el al. (2003) prove that by introducing a nonlinear Phillips curve or assuming that a fraction of capitalist profits are not invested $l < 1$ is guaranteed.

[13] There are a number of empirical papers from the 1990s, concerning the nonlinearities or *asymmetries* between wage changes and unemployment. See Chada, Masson and Meredith (1992); Laxton, Meredith and Rose (1995); Clark, Laxton and Rose (1996).

wage bargaining, the labour market situation is interpreted as a measure of the workers' bargaining power. Near to full employment firms raise wage rates in order to attract the most suitable workers[14] .

Finally, regarding a y b we make the following assumptions. On the one hand, in order to guarantee that wages do not rise when $l = 0$, it is necesary that $a > 1$, that is, $e^{-a} < 1$. On the other hand, under full employment, the growth of wages must be high, so that $b - a$ is large, in particular we require $e^{b-a} > 1$. Therefore we assume $b > a > 0$. Usually these parameters are characteristic of labour market (Pohjola, 1981). It will be discussed later.

3 Dynamic Analysis

3.1 The dynamics of l_t

We begin with the analysis of the dynamics of the employment rate l. The analysis shows how complex behaviour can emerge from the rather simple economic structure described above. When sufficient nonlinearities are present and the labour market is in desequilibrium, the interaction between unemployment and the accumulation of knowledge can lead to growth cycles that exhibit more complex behaviour than those obtained in neoclassical growth models. In fact, this is an appropiate qualitative description of the observed oscillations in the real world.

[14] Introducing imperfections in the labour market through wage dynamics is usual in disequilibrium models with money (Chiarella and Flaschel, 1999 and Chiarella et al., 2000) and regime switching models (Ito, 1978, 1980; Picard, 1983). Generally, the theoretical elements of bargaining models are modelled in terms of a "wage curve" (Blanchard and Oswald, 1995) which captures the inverse relation between employment and wages instead of their growth rate. However, Chiarella et al. (2000) assert: "the theory based level form formulations of such wage and price equations should be reducible to rates of growth, possibly considering demand as well as cost pressure terms". So, in their work (chapt. 5) they reformulate the wage curve in terms of growth rates, introducing theoretical elements of wage bargaining models, insiders-outsiders and hysteresis in a Phillips Curve. We think that the formulation in growth rates, as in the model of Chiarella et al., provides a better fit to the formal structure of our model.

Beginning with the decision regarding the production of final goods we get:

$$\frac{Y_{t+1}}{Y_t} = \frac{D_{t+2}}{D_{t+1}}$$

Substituting the expressions for production and demand, and using (2.1) and (2.5) we obtain the following nonlinear dynamic equation for the employment rate:

$$\frac{l_{t+1}}{l_t} = \frac{1}{1+n} \exp\left(\frac{(\alpha+\beta)\delta\,(1-\gamma)+a}{(1-\alpha)}\right) \cdot \exp\left(\frac{-bl_t}{(1-\alpha)}\right)$$

Letting $r = \frac{1}{1+n} \exp\left(\frac{(\alpha+\beta)\delta(1-\gamma)+a}{(1-\alpha)}\right)$ and $s = \frac{b}{(1-\alpha)}$ the equation is reduced to a one-dimensional, nonlinear discrete time dynamical system:

$$l_{t+1} = f(l_t) = r \exp\left(-sl_t\right) l_t \tag{3.1}$$

Figure 3 shows $f(l_t)$ for a selection of values of r (where we assume that $b + \alpha > 1$ and $r < es$ in order to guarantee that $L_t < A_t$ is always satisfied, that is $l < 1$).

(Figure 3 see below)

Equation (3.1) is known as the Ricker-Moran equation (Ricker, 1954; Moran, 1950; Cook, 1965; Macfadyen, 1963), and it has been employed in ecology, especially in the study of fish populations. The dynamics of this equation is well documented in the ecological and mathematical literature (May, 1976). Because its dynamical behaviour has been well studied, we will briefly summarize the main results (see May, 1975; May and Oster, 1976).

Table 1 summarizes the dynamics for each value of r (the parameter s affects the value of equilibrium points, but does not affect the behaviour of the dynamical system).

(Table 1 see below)

The transition to chaotic behavior can be assessed in the bifurcation diagram (figure 4). This shows how increasing r gives rise to a sequence of bifurcating stable points of period

2^n. Note that although this bifurcation process produces an infinite sequence of cycles of period 2^n $(n \to \infty)$ it is bounded by some parameter value, in particular, $r_c \approx 14.767075$. Beyond the limit point r_c, we enter in the "chaotic" regime characterized[15] by a finite number of attracting fixed points, an infinite number of repelling fixed points, and an uncountable number of points (initial conditions) whose trajectories are totally aperiodic. Periodic doubling is an example of a "route to chaos."- the way in which the dynamics of a system change as a parameter is changed, leading ultimately to the appearance of chaos.

(Figure 4 see below)

Figure 5 shows the erratic behaviour which is a characteristic of a regime of chaos generated by the Ricker-Moran equation for a value of r in the chaotic regime[16] , $r = 18$.

(Figure 5 see below)

Let us interpret the dynamics of l in terms of the original parameters. The parameter b affects the employment rate but it does not affect the dynamic behaviour. The greater the value of b, that, the greater is $s = \frac{b}{1-\alpha}$, the lower the employment rate. Because s can be interpreted as reflecting workers' bargaining power, since, given a, an increase in b (that is in s) implies a greater growth of the wage (Pohjola, 1981), the model implies (in the same way as in bargaining wage models (Layard and Nickell, 1985,1986)), that a lower unemployment rate goes along with greater bargaining power. Taking into account the parameters that determine $r = \frac{1}{1+n} \exp\left(\frac{(\alpha+\beta)\delta(1-\gamma)+a}{(1-\alpha)}\right)$, the employment rate and the possibility of chaos increase with the effectiveness of investment in human capital δ,

[15] A sufficient condition for the existence of topological chaos, may be established by the Li-Yorke theorem "Period Three Implies Chaos" (1975).

[16] Taking standard values for the parameters, $\alpha = 0.7, \beta = 0.1, \gamma = 0.7, \delta = 0.1$ and $n = 0.02$ (Lucas, 1988; Barro and Sala-i-Martin, 1995), and $a = 0.8$ (a must be greater than 0) the possibility of chaos is "reasonable" from an economic point of view.

the fraction of time devoted to knowledge accumulation $(1-\gamma)$, the external effect β, the production function returns α and the parameter a. As was discussed earlier, in economies with excess demand, increasing knowledge raises employment because of the increase in demand, and thus in production, more than the increase in productivity. For this reason, the employment rate depends positively on the knowledge parameters[17]

The parameter r is formed by the knowledge production parameters and the Phillips curve's parameter a; weighted by the rate of population growth. For one hand, given a, the parameter r can be interpreted as reflecting the medium level of development of the knowledge sector. Hence, the model predicts that l and the probability of chaos increase with the level of development of the knowledge sector[18] . Because most developed societies have a high level of development of the knowledge sector, the model also predicts that higher levels of economic development are to be associated with higher probabilities of complex dynamics. On the other hand, the parameter r could be also interpreted to reflect labour market rigidity since a determines the elasticity of the wage's growth to the employment rate. As is known, the more sensitive growth wage to changes of the employment rates the higher flexibility in the labour market. Since an increase in a (that is in r) implies a lower elasticity, the model predicts that the more imperfect the labour

[17] Most of the economic literature has shown the negative relation between growth and unemployment (Pissariades, 1990; Bean and Pissarides, 1993), except for the positive relation between unemployment and growth (technological unemployment) in "creative destruction" models (Aghion and Howitt, 1992) based on the ideas of Schumpeter (1934). Bean et al. assessing the empirical evidence, suggest that the relationship between both phenomena is negative in the short run, and it could be either positive, negative or null in the long run. The sign is ambiguous because both variables can be endogenously and jointly determined, which implies that the relationship between them depends on exactly which economic structure we are considering.

[18] However, although empirical evidence shows that the rate of employment and the irregularity of its dynamic behaviour, usually depend positively on the level of development, some developed economies have shown high rates of unemployment (even though not as high as developing countries).

market[19] the greater the possibility of complex dynamics[20] . These results agrees with the ideas of Keynes in the General Theory (1936, chap.22), and Schumpeter, in The Theory of Economic Development (1934, chap.VI), regarding the economic development of capitalist societies. As the level of development increases rigidities and market imperfections begin to interfere with this development, oscillations become more irregular.

As far as population growth is concerned, we find that the employment rate and its irregular behaviour decrease with population growth. Given L, the greater is the population size A, the lower is the employment rate, $l = \frac{L}{A}$. The historical evolution of population growth (Kremer, 1993; Galor and Weil, 2000) shows a consistent fall in fertility rates in developed economies. So, in line with the previous paragraph, a higher level of development is associated with lower population growth and irregular dynamic behaviour.

3.2 The dynamics of y_t

The results obtained in the last section concern the employment rate. They can, however, be referred to the rate of economic growth:

$$\frac{y_{t+1}}{y_t} = \left(\frac{h_{t+1}}{h_t}\right)^{\alpha+\beta} \left(\frac{l_{t+1}}{l_t}\right)^{\alpha} (1+n)^{\alpha-1} = z \cdot \left(\frac{l_{t+1}}{l_t}\right)^{\alpha} \tag{3.2}$$

where $z = e^{\delta(1-\gamma)(\alpha+\beta)} (1+n)^{\alpha-1}$. So, the rate of economic growth depends on the evolution of the employment rate and the parameter z. The qualitative conclusions obtained concerning the dynamics of the employment rate apply directly to the growth rate.

The numerical simulations show that, whenever $z < 1$, the growth of knowledge productivity does not absorb population growth, and the per capita production of the economy y_{t+1} tends to zero. This is because we have assumed than h depends on the fraction of time

[19] The rigidity of the labour market is explained from different imperfections such as cost of mobility, informational imperfections, mismatch between the workers looking for jobs and the vacancies available, minimum wages and union wage setting, etc.

[20] The parameter b also determines the elasticity of the wage, but it does not affect the dynamic behaviour of the unemployment rate.

that individuals devote to knowledge production instead of the total number of workers employed in this sector[21] ,[22] .

However, in order for the economy to disappear, the values which must be taken by the rate of population growth are unrealistic[23] . By considering realistic values of n, the economy would disappear only if it does not invest in knowledge ($\gamma = 1$) or if it does at an infinitesimal level[24] . So, in the same way as traditional models of endogenous growth, sustained growth is due to knowledge production, in particular, to the productivity δ and the fraction of time devoted to knowledge accumulation $(1 - \gamma)$.

Equation (3.2) also shows that the rate of economic growth increases with the knowledge external effect, β. This result is in agreement with Lucas (1988) for high levels of risk aversion. As far as worker's bargaining power b goes, lower levels of the rate of economic growth are associated with higher bargaining power; this is because greater bargaining power implies greater unemployment, and so, less resources are used[25] .

If $z > 1$, per capita income shows a long term positive growth trend that is sustained by knowledge production, and its dynamic behaviour depends on the dynamics of the

[21] This assumption eliminates the so-called "scale effect" in the literature of economic growth: the population size affects human capital accumulation positively and, therefore, it also has a positive effect on economic growth. In our model, if we consider that human capital accumulation depends on the total number of workers employed in this sector (Romer, 1990; Grossman and Helpman,1991; Aghion and Howitt, 1992) the scale effect would cause a overflowing on human accumulation, economic growth and employment when the population's size is large. In particular, economies with large populations would not have unemployment. Moreover, we must choose the initial population size, which is too arbitrary. So we have chosen modelization (2.1).

[22] The inverse relationship between population and economic growth rates reflects the Malthusian idea that the greater is population growth the lower is per capita income.

[23] See Kremer (1993) for the evolution of world population growth rate since one million years AC. until 1990. The greatest rate of growth was 2.01% in 1960. So, for example, for the parameters values that we have considered (see footnote 16) and $n = 2\%$, $z = 1.01822$.

[24] As we will see in the next section, if we assume that endogenous population growth, n depends on income growth, this result will coincide with the Malthusian idea concerning the stagnation of the economy. As the population rises, in a non industrialized economy and with limited resources, at some moment the amount of food falls below the subsistence level. This leads to a situation in which the economy is stagnant, and where the population no longer rises.

[25] This result coincides with other growth models that introduce the unemployment assumption (Davieri and Tabellini, 2000; Alonso, Echevarría and Tran, 2002)

employment rate[26] . The qualitative conclusions obtained about the dynamics of the employment rate apply directly to per capita income y_t. Again, both the flexibility of the labour market and the level of development of knowledge sector determine the complex dynamics of per capita income. For example, for a value of r in the chaotic regime $r = 18$ (figure 6) the trajectory of y_t is quite irregular. This behaviour provides an excellent match to the behaviour of the series of per capita income[27] .

(Figure 6 see below)

Finally, although the model generates a positive income growth trend, in figure 6 we can see periods in which per capita income decreases $\frac{y_{t+1}}{y_t} < 1$, which is consistent with what we see in the real world. Most of the neoclassical theory of growth focuses on explaining the positive trend of income growth, and it does not normally take into account the existence of the periods in which income decreases, and in those cases when it does the decreases are generated exogenously. The singularity of our model is that the nonlinear nature of the model generates both the growth trend and the possibilty of decreases in income without requiring exogenous shocks.

3.2.1 Wage Dynamics Extensions

Looking at the trajectories of employment and per capita income, the amplitudes of the oscillations of the simulated time series are too large when compared with their true (real world) behaviour. The reasons for this result are the simple nonlinear modelizations in some of the economic relationships that we have assumed, for example the wage dynamics. In this section we show how, by introducing more realistic assumptions about the non

[26] For $z = 1$, the positive growth trend disappears. However, the parameter values for which $z = 1$, in particular the value of n, are not realistic for the same reason that we have already explained for $z < 1$.

[27] In figure 6, $a = 0.62$. The rest of parameters values are the standard values (footnote 16).

lineality of the Phillips curve, in particular for l values with neither full employment nor total unemployment, the size of the oscillations decreases and are close to full employment. In particular, we consider the following alternative formulation of the Phillips Curve:

$$\frac{w_{t+1}}{w_t} = h(l_t) = \exp\left(-a + bl_t^{\sigma} + c\,(1 - l_t)^{-\varepsilon}\right) \tag{3.3}$$

with $a, b, c > 0$, $0 < \sigma < 1$, $0 < \varepsilon < 1$ The resulting behaviour is illustred in figure 7.

(Figure 7 see below)

We have made the following assumptions. First, we assume there is a large interval, corresponding to $(\varepsilon_1, 1 - \varepsilon_2)$ with ε_1 and ε_2 small, where wages are very insensitive to employment changes. This modelization considers, in a more precise way, wage behaviour far from either full employment or total unemployment, and it is based on Rose [28] (1967). Secondly, we assume a vertical tangency $l = 0$ in order to capture the rapid fall of wages[29] when the rate of unemployment is very close to 100% (l close to 0). Finally, we assume a vertical asymptote at $l = 1$. This asymptote is designed to reflect the fact that near to full employment it is more and more difficult to hire workers[30] and the labour market pressure accelerates the growth of wages; which near to full employment would be huge.

[28] Rose states: "In some neigbourhood of the l at which unemployment is balanced by unfilled vacancies, frictions and imperfections weaken the responsiveness of wage inflation to changes in l". He introduces a kind of nonlinearity in Phillips Curve with the purpose of preventing local explosive dynamics. The qualitative behaviour which he obtains is similar to the dynamic behaviour that we get with the curve assumed here.

[29] Some macroeconomic growth models with money and disequilibrium (Chiarella et al., 2000; Chiarella and Flaschel, 2000) assume the so-called "kinked" Phillips Curve. This curve is an extreme case of the nonlinear Phillips Curve because it introduces downward rigidity in nominal wages; wages are constant if the growth rate is negative. They affirm that this curve is more realistic than assuming the possibility of wage decreases. The objective of this work is to extend the habitual nonlinearity of the Phillips Curve, a "kinked curve", and to use numerical simulations to show how the dynamical behavior of the model changes drastically being able to generate chaotic paths.

[30] This assumption is very usual both in theoretical work which introduces the Phillips Curve (Phillips, 1958; Lipsey, 1960; Samuelson and Solow, 1960); Rose, 1967; Chiarella and Flaschel,1996) and in empirical work (Chada, Masson y Meredith, 1992; Laxton, Meredith and Rose, 1995; Clark, Laxton and Rose, 1996).

Taking into account the Phillips curve (3.3), the employment rate follows

$$l_{t+1} = f(l_t) = \frac{1}{1+n} \exp\left(\frac{(\alpha+\beta)\delta(1-\gamma) - q(l_t)}{(1-\alpha)}\right) \tag{3.4}$$

where now $q(l_t)$ is

$$q(l_t) = -a + bl_t^{\sigma} + c(1-l_t)^{-\varepsilon}$$

In figure 8 we have drawn the intertemporal evolution of l_t (for the parameter values of note 16) and setting the parameters of the curve[31] (8) at $a = 1.716, b = 1.455, \sigma = 0.01, \varepsilon = 0.1$. The resulting l_t dynamics are complex and irregular, but now, since we are close to full employment values with unemployment rates closed to the real ones, they provide an excellent fit to empirical evidence[32] .

(Figure 8 see below)

In the same way as in the previous section, complexity is transferred to per capita production dynamics. As is shown in figure 9, the intertemporal evolution of y_t follows a cyclical growth with a positive trend, but with the possibility of per capita production decreases at some periods. The main difference now is that oscillations have smaller amplitude.

(Figure 9 see below)

In conclusion, as we postulated at the outset, the simple adaptation of the Phillips Curve to theoretical patterns that fit better its actual behaviour is enough to generate more realistic intertemporal evolutions of the employment rate and per capita production.

[31] As in the previous section, the parameter values of the Phillips curve have been chosen so that the case $l > 1$ is excluded.

[32] The dynamics behaviour of (3.4) is qualitatively similar to the function (3.1). Both are unimodal functions which generate cyclical temporal evolution of the rate of employment. The difference is that now the oscillations are much more close to full employment values.

4 Endogenous population growth

In this section we introduce into the model endogenous population growth. Following the standard models of endogenous population (Becker et al., 1990; Jones, 2001; Galor and Weil, 2000), it is assumed that labour supply is determined through micro-founded fertility choices of individuals, in which households choose the number of children. Every individual is considered to be an "average" couple or family[33] . At every point of time t each family choice determines the number of children b, who will be born in the next period $t+1$.

Let us assume that the average family's preferences are represented by the following standard log-linear utility function:

$$U(c_{t+1}, b_{t+1}) = c_{t+1}^{1-\epsilon} b_{t+1}^{\epsilon}, \qquad 0 < \epsilon < 1$$

where c is the per capita consumption and ϵ meassure the preference for the children. The cost of childrearing E_t includes both the cost of raising a child regardless of quality, θ_1, and the education cost[34] $e_t = \theta_2 h_t^{\rho}$:

$$E_t = \theta_1 + \theta_2 h_t^{\rho}, \quad 0 < \rho < 1$$

θ_1 is considered as a fixed "maintenance" cost (e.g. food, clothes) and the education cost[35]

[33] The concept "average" that we are considering tries to capture the following idea. In the economy there are some families with high income and others with low income, unemployed and employed workers. For simplicity we consider homogenous agents and, so all families solve the same average problem. We may assume some income distribution policy (taxes, transfers) from the high income families to the low income families, or from employed to unemployed.

[34] The endogenous population growth literature habitually assumes that the childrearing costs are measured in time units. Due to the theoretical structure of the model, we assume that these costs are measured in either units of final good or income. As we will see, it does not affect the qualitative results of the model.

e_t depends on the medium level of knowledge[36] .

Given these assumptions, each family's static optimization problem[37] is

$$\begin{aligned} \max U(c_{t+1}, b_{t+1}) &= c_{t+1}^{1-\epsilon} b_{t+1}^{\epsilon} \qquad (4.1) \\ s.t.\ c_{t+1} + (\theta_1 + \theta_2 h_t^{\rho}) b_{t+1} &= y_t \end{aligned}$$

The last equation is the income constraint, where $y_t = \frac{Y_t}{A_t}$ is the per capita income[38] . Solving problem (4.1) we get that the optimal number of children for an average family at each time[39] t

$$b_{t+1} = \frac{y_t \epsilon}{\theta_1 + \theta_2 h_t^{\rho}} \qquad (4.2)$$

b can be interpreted as the fertility rate, which depends positively on the per capita income and negatively on the medium level of knowledge. This relationship shows the so-called quantity-quality trade off in the literature of endogenous growth (Galor and Weill, 2000). For high enough levels of knowledge development, the return from education rises inducing parents to shift quantity for quality of children. As individuals receive all the income from

[35] The literature of endogenous population habitually assumes a linear education cost in h (Galor and Weill, 2000), that is, $\rho = 1$. By considering $\rho < 1$ we have the typical cost function ($E'(h) > 0$ y $E''(h) < 0$). This assumption could be interpreted as the expected education cost decreasing as the children grow and they become adults.

[36] Schultz (1975) claimed that new technology will create a demand of the killed workers to analyze new production process. So, the education cost would depend on technological progress interpreted in our model as knowledge h.

[37] Some models of endogenous population growth introduce altruism in the parent's utility function (Becker et al. 1990). This assumption implies both of the dynastic utility functions considered, and solves dynamic optimization problems of Bellman equation type. For simplicity, we consider a static problem which keeps the theoretical foundation of the endogenous population growth literature. Jones (2001) also considers a similar static optimization problem. By considering some very simple assumptions: "the more standard dynamic optimization problem reduces to the sequence of static problems given above" (see Jones, 2001, p. 5)

[38] In line with the model developed in the previous section, we assume exogenous consumption; income not devoted to childrearing is devoted to consumption of goods.

[39] We consider every period (generation) as a productive period. In the endogenous population models every period usually is considered a period of life. Every family lives one, two or three periods at the most (childhodd, parenthood, old). But according to the logic of production, investment, employment and wages that we are assuming, the periods will be shorter than periods of life. This is not a problem from a theoretical point of view. If a period of life is equal to x productive period, $\frac{b}{x}$ will be the average number of children per family in every period of life.

labour supplied, $y_t = w_t l_t$, the fertility rate can be written as:

$$b_{t+1} = \frac{w_t l_t \epsilon}{\theta_1 + \theta_2 h_t^{\rho}}$$

This equation will allow us to analyze the relationship between unemployment, growth and population growth[40]

Finally, in order to obtain the population growth rate, we assume a constant mortality rate[41] π. Taking this rate into account the population growth rate n is given by:

$$n_t = \frac{A_{t+1}}{A_t} - 1 = \frac{N_{t+1}}{N_t} - 1 = -\pi + \frac{1}{\lambda} b_{t+1} \quad (4.3)$$

In the next subsection we introduce this rate into the model and analyse its dynamic behaviour.

4.1 The dynamics with endogenous n

The economic growth model with unemployment and endogenous population growth is:

$$\begin{aligned} l_{t+1} &= f(l_t) = \frac{1}{1+n_t} l_t \exp\left(\frac{(\alpha+\beta)\delta(1-\gamma)}{(1-\alpha)}\right) [h(l_t)]^{\frac{1}{1-\alpha}} \quad (4.4) \\ h_{t+1} &= \exp(\delta(1-\gamma)) h_t \\ \frac{w_{t+1}}{w_t} &= h(l_t) = \exp\left(-a + b l_t^{\sigma} + c(1-l_t)^{-\varepsilon}\right) \\ \frac{y_{t+1}}{y_t} &= \exp(\delta(1-\gamma)(\alpha+\beta)) \left(\frac{1}{1+n_t}\right)^{1-\alpha} \left(\frac{l_{t+1}}{l_t}\right)^{\alpha} \\ n_t &= -\pi + \frac{w_t l_t \epsilon}{\lambda(\theta_1 + \theta_2 h_t^{\rho})} \end{aligned}$$

Because of the high analytical complexity of the model we limit its study to some numerical simulations (by using MATLAB). Like in the previous sections, the parameter

[40] Some models of endogenous population growth (Galor and Weill, 1996) obtain that fertility depends negatively on wages instead of positively. These models try to capture the effect of female labour incorporation on fertility. Because the childrearing cost is measured in time units the wage represents the opportunity cost of raising children. So, the higher is the wage the greater is the opportunity cost of having children.

[41] Blanchard (1985) states that "Evidence on mortality rates suggests low and approximately constant probabilities from age 20 to age 40", after this, mortality rates depends inversely and exponentially on individual age (see "Gomperty's Law", Wetterstrand, 1981).

values have been choosen excluding $l > 1$. We set $\epsilon = 0.02$, $\theta_1 = 0.3$, $\theta_2 = 0.01$, $\rho = 0.6$, $\pi = 0.01, \lambda = 1.6$, and the initial values $y_0 = 1$, $h_0 = 10, l_0 = 0.9$. The rest of parameters values[42] are the standard values (footnote 16). Next we discuss the main results.

First, the dynamic analysis shows the endogenous emergence of a logistic behaviour in the population growth rate describing its historical evolution[43] (see figure 10).

(Figure 10 see below)

The evolution is characterized by a stage (named habitually post Malthusian[44]) where the population growth rate increases and, after a transition phase, a modern growth stage follows where n decreases and tends to stagnate. As was explained above, households choose the number of children facing a quantity-quality trade off. For low levels of knowledge development the returns to education are small, and income growth raises the fertility rate. But, for high levels of knowledge development, the return to education rises inducing parents to have fewer, but more high-quality, children. These substitution and income effects generate a demographic transition, that is, an economy transits from a high fertility and low knowledge accumulation stage to a low fertility and rapid knowledge accumulation stage.The knowledge growth acceleration in relation to y can be noticed in figure 11.

(Figure 11 see below)

Besides the logistic behaviour of population growth, the behaviour of n in each stage

[42] As will be discussed later, one of the empirical testing problems is parameter value calibration. Until now, there are no empirical studies that reveal their values and they are chosen ad hoc in the theoretical studies. We have chosen these values in order to present the dynamics results. We get a wide enough interval of values for the temporal evolution of the variables satisfying $l < 1$. All the simulations made show the same behaviour in this interval of values which, due to the restriction of labour market disequilibrium, could be relatively small.

[43] The large number of simulations carried out showed that this is a robust property of the model for acceptable changes in parameter values.

[44] Although it has not been included in our model, initially there would be a Malthusian stage, where technological progress is slow and population growth prevents any sustained rise in income per capita.

seems extremely interesting . Figure 10 shows the behaviour of n during the different phases. In the stage previous to the transition n oscillates regularly or periodically, in the transition stage it shows a very regular behaviour, and in the last stage it oscillates in an erratic way. The reasons for this result could be the simple modelization in the birth rate that we have assumed. The model attempts to be representative of a developed economy, and some assumptions are not appropriate for explaining the transition from an agricultural economy to an industrial one. More realistic assumptions regarding the birth rate (e.g. endogenous mortality) and the behaviour of model's variables during the different stages would generate more realistic intertemporal evolutions of the population growth rate[45] .

This type of dynamic evolution is not exclusive of the population growth rate. In figure 12 we have drawn the intertemporal evolution of l and we show the same behaviour that was generated in n: regular oscillations during the previous stage to the transition, stable behaviour during the transition and unstable in the subsequent stage. The evolution of per capita production shows the same behaviour (figure 11). So, we find that during the transition the relevant variables evolve in a regular manner. But beyond this regularity there is a hidden structural change: the change of trend of the population growth.

(Figure 12 see below)

Because there is not enough empirical evidence, testing this result seems difficult. However, we could interpret it with theoretical arguments. The cyclical (but regular) initial stage would correspond to an economy with slow technical progress (underdeveloped) that takes off to sustained growth stage. A growing economy has more opportunities.

[45] For example, in the same line as demographic and industrial transition models (Lucas, 1998; Hansen and Prescott, 1999; Kögel and Prskawetz, 2001) we could model a productive sector for every stage: agricultural and industrial. Moreover instead of a Phillips Curve it should consider an appropriate wage dynamics in the agricultural stage.

So, the rigidities that cause instability have an "outlet" and are less "burdensome". This could cause the regular behaviour of the variables during the transition. Later, when the economy achieves a high level of development, rigidities and imperfections stress and fluctuations are very unstable. Regarding to the employment rate behaviour, during the transition it seems that the labour force fits labour market needs. Current research on the impact of demography on growth shows that changes in the population's age structure play a significant role (Canning and Bloom, 1999; Gómez and Hernández de Cos, 2003). Although age structure has not been analized in our model, we could relate it with population growth. This relationship is direct if population begins to grow because the young population changes[46] . Then, a possible justification for the stability of employment during the transition could be the following. At the beginning of the transition, a young population (resulting from the typical "baby boom" of post Malthusian stage) joins the labor force. A young labour force satisfies the needs of the labour market more easily (e.g. training, geographic and labor mobility) than an adult population (corresponding to a low population growth), characteristic of developed economies (the modern economic growth stage). So, the dynamic result could be related to the population's age structure if it smooths fluctuations through a young labour force during the transition stage.

Finally, introducing endogenous population growth does not affect the result of the previous section about how changes of the model's parameter values affect employment and per capita income dynamics. Regarding the parameters of endogenous population rate of growth, because of the inverse relationship between population and economic growth: a) the greater is the value of child preference ϵ the lower is per capita income, and b) the lower are both the childrearing's cost and mortality rate the greater is per capita income.

[46] The other reason for population growth is the decrease in the mortality rate.

In conclusion, the introduction of endogenous population growth enriches the theoretical results of the model. The model endogenously generates the demographic transition to a developed economy, and explains both why there is a demographic transition and how it is produced.

5 Conclusions

The objective of this research has been to elaborate an endogenous growth model which allows us to examine the interaction between economic growth, labour market behaviour and population growth. We have shown that this interaction is a dynamic process, where each variable affects the other.

The main outcomes of our research are the following. First, the dynamic analysis shows how the per capita production, population growth rate and employment rate can show stable cycles of different periods and a chaotic path depending on both the knowledge sector's level of development and the rigidity of the labour market. In particular, the higher the level of knowledge development and the rigidity of the labour market the greater the possibility of complex dynamics. Second, along the lines of wage bargaining models (Layard and Nickell, 1985, 1986), we find that the higher the worker's bargaining power, the lower the employment rate, the per capita production and the population growth.

Thirdly, in addition to cyclical behaviour, the dynamic analysis points to a positive income growth trend sustained by knowledge accumulation. The economy disappears only if it does not invest in knowledge or if it does at an infinitesimal level. In the latter case, the growth of knowledge productivity does not absorb the population growth and per capita production tends to zero. Thus, the inverse relation between population and economic growth reflects the Malthusian idea that the greater is the population growth, the lower is the per capita income. Finally, when we assume endogenous population growth by means of optimal fertility choices, the model endogenously generates a logistic behaviour in the population growth rate describing its historical evolution. The demographic transition reverses the positive relationship between economic development and population growth, and fertility rates permanently decline. However, besides this structural change, we obtain that during the transition, the relevant variables evolve regularly; in particular, it seems that labour force fits the labour market needs. As some endogenous population research states, this dynamic result could be related to the population's age structure.

To summarize, the elaboration of endogenous growth models which consider both endogenous population growth and the possibility of unemployment enriches considerably the theoretical results of the traditional endogenous growth models. Moreover, although the model has not been elaborated to policy analysis, some conclusions can be extracted. On the one hand, the model predicts that incentives for investment in knowledge will encourage economic growth and employment, but at the same time will increase instability. However, policies which improve the flexibility of the labour market will decrease instability. On the other hand, in the same way as in models of wage bargaining, we find that greater levels of workers' bargaining power are associated with lower levels of both the employment rate and per capita production. Thus, policies that reduce workers' bargaining power will have a positive effect on economic growth, increasing the employment rate. Lastly, with regard to demographic variables, an increase in the population growth rate leads to a decrease in the growth rate of income, and policies which reduce incentives for fertility may positively affect long run economic growth. We find these preliminary results encouraging enough to perform further work, both at the theoretical and empirical levels. These will be possible extensions of our research.

References

Aghion, P., Howitt, P., 1992. A Model of Growth through Creative Destruction. Econometrica, 60, 323-351.

Aghion, P., Howitt, P., 1998. Endogenous Growth Theory. MIT Press, Cambridge Mass.

Alonso, A., Echevarría, C., Tran, K., 2002. Long Run Economic Performance and the Labour Market. Southern Economic Journal, 70, 905-919.

Barro, R.J., Sala-i-Martin, X., 1995. Economic Growth. McGraw-Hill, New York.

Bean, C., Pissarides, C., 1993. Unemployment, consumption and growth. European Economic Review, 37, 837-859.

Becker, G.S, Murphy, K.M., Tamura, R.F., 1990. Human Capital, Fertility, and Economic Growth. Journal of Political Economy, 98, S12-S37.

Blanchard, O.J., 1985. Debt, Deficits, and Finite Horizons. Journal of Political Economy, 93, 2, 223-247.

Blanchard, O.J., Oswald, A.J., 1995. An Introduction to the Wage Curve. The Journal of Economics Perspectives, 9, 153-167.

Bloom, D.E., Canning, D., 1999. Economic Development and Demographic Transition: The Role of Cumulative Causality. CAER II Discussion Paper, 51.

Bloom, D.E., Canning, D., Sevilla, J., 2001. Economic Growth and Demographic Transition. NBER Working Paper, 8685.

Clark, P., Laxton, D., Rose, D., 1996. Asymetry in the U.S. Output-Inflation Nexus. IMF Staff Papers, 43, 216-251.

Cook, L.M., 1965. Oscillation in the Simple Logistic Growth Model. Nature, 207, 316.

Chadha, B., Masson, P., Meredith, G., 1992. Models of Inflation and the Costs of Disinflation. IMF Staff Papers, 39, 395-431.

Chiarella, C.P., Flaschel, G., 1996. Real and Monetary cycles in models of Keynes-Wicksell type. Journal of Economic Behavior & Organization, 30, 327-351.

Chiarella, C.P, Flaschel, G., 1999. The Dynamics of Keynesian Monetary Growth: Macro Foundations. Cambridge University Press.

Chiarella, C., Flaschel, G., 2000. High order disequilibrium growth dynamics: Theorical aspects and numerical features. Journal of Economics Dynamics and Control, 24, 935-963.

Chiarella, C.P, Flaschel, G., Groh, G., Semmler, W., 2000. Disequilibrium, Growth and Labor Market Dynamics: Macroperspectives. Springer-Verlag, Heidelberg.

Daveri, F., Tabellini, G., 2000. Unemployment, Growth and Taxation in Industrial Countries. Economic Policy, 30, 49-104.

Desai, M., Henry, B., Mosley, A., Pemberton, M., 2005. A clarification of the Goodwin model of the growth cycle. Journal of Economics Dynamics and Control, Article in Press.

Galor, O., Weil, D.N., 1996. The Gender Gap, Fertility, and Growth. American Economic Review, 86, 374-387.

Galor, O., Weil, D.N., 2000. Population, technology, and growth: From the Malthusian regime to the demographic transition and beyond. American Economic Review, 90, 806-828.

Gómez, R., Hernández de Cos, P., 2003. Demographic Maturity and Economic Performance: The effect of Demographic Transitions on Per Capita GDP Growth. Documento de trabajo 0318, Banco de España, Servicio de Estudios.

Goodwin, R.M. 1967. A Growth Cycle. In: Feinstein, C.H. (Ed)., Socialism, Capitalism and Economic Growth, Cambridge University Press, pp. 54-58.

Grossman, G. M., Helpman, E., 1991. Innovation and Growth in the Global Economy. Cambridge MA, MIT Press.

Hansen,G., Prescott, E.C., 1999. From Malthus to Solow. Federal Reserve Bank of Minneapolis. Research Department Staff Report 257.

Ito, T., 1978. A note on Disequilibrium Growth Theory. Economics Letters, 1, 45-49.

Ito, T., 1980. Disequilibrium Growth Theory. Journal of Economic Theory, 23, 380-409.

Jones, I.C., 2001. Was a Industrial Revolution Inevitable? Economic Growth over the Very Long Run. Advances in Macroeconomics, 1, n^o 2, art. 1.

Keynes, J.M., 1936. The General Theory of Employment, Interest, and Money. Macmillan, Londres.

Kögel, T., Prskawetz, A., 2001. Agriculture productivity growth and escape from the Malthusian trap. Journal of Economic Growth, 6, 337-357.

Kremer, M., 1993. Population Growth and Technological Change: One Million B.C. to 1990. Quaterly Journal of Economics, 108, 681-716.

Laxton, D., Meredith, G., Rose, D., 1995. Asymmetric effects of economic activity on Inflation: Evidence and policy implications. IMF Staff Papers, 42, 2.

Layard, R., Nickell, S., 1985. The Causes of British Unemployment. National Institute Economic Review, 111, 62-85.

Layard, R., Nickell, S., 1986. Unemployment in Britain. Economica, 53 (Supplement), S121-S169.

Li, T.Y., Yorke, J.A., 1975. Period Three Implies Chaos. American Mathematical Monthly, 82, 271-271.

Lucas, R.E. Jr., 1988. On the Mechanics of Development Planning. Journal of Monetary Economics, 22, 3-42.

Lucas, R.E. Jr., 1998. The Industrial Revolution: Past and Future. Mimeo, University of Chicago. Published in: Lucas, R.E.Jr., (Ed), Lectures on Economic Growth Cambridge:

Harvard University Press, 2002.

Lipsey, R.G., 1960. The Relation between Unemployment and he Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957: A Further Analysis. Economica, 27, 1-31.

Macfadyen, A., 1963. Animal Ecology, London: Pitman.

Mankiw, N.G., Romer, D., (Eds), 1991. New Keynesian Economics, Vol. I, II. MIT Press, Cambridge.

May, R.M., 1975. Biological Populations Obeying Difference Equations: Stable Points, Stable Cycles, and Chaos. Journal of Theoretical Biology, 51, 511-524.

May, R.M., 1976. Simple Mathematical Models with Very Complicated Dynamics. Nature, 261, 459-467.

May, R.M., Oster, G.F., 1976. Bifurcations and dynamic complexity in simple ecologicals models. American Naturalist, 110, 573-599.

Moran, P.A.P., 1950. Some Remarks on Animal Population Dynamics. Biometrics, 6, 250-258.

Phillips, A.W., 1958. The Relation between Unemployment and he Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957. Economica, 25,100, 283-299.

Picard, P., 1983. Inflation and Growth in a Disequilibrium Macroeconomic Model. Journal of Economic Theory, 30, 266-295.

Pissarides, C. A., 1990. Equilibrium unemployment theory, Basil Blackwell, Oxford.

Pohjola, M.J., 1981. Stable and Chaotic Growth: the Dynamics of a Discrete Version of Goodwin's Growth Cycle Model. Zeitschrift für Nationalökonomie, 41, 27-38.

Ricker, W.E., 1954. Stock and Recruitment. Journal Fisheries Research Board of Canada, 11, 559-623.

Romer, P., 1990. Endogenous Technological Change. Journal of Political Economy, 98, part II, S71-S102.

Rose, H., 1967. On the non-linear theory of the employment cycle. Review of Economic Studies, 34, 153-173.

Samuelson, P.A., Solow, R.M, 1960. Analytical Aspects of Anti-Inflation Policy. The American Economic Review, 50, 177-194.

Schumpeter, J.A., 1934. The Theory of Economic Development. Cambridge MA, Harvard University Press.

Wetterstrand, W.H., 1981. Parametric Models for the Life Insurance Mortality Data: Gomperty's Law over Time. Transc. Soc. Actuaries, 33, 159-179.

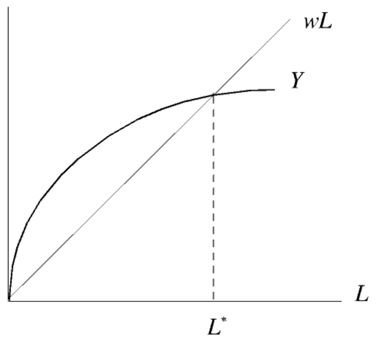

Figure 1: Production and cost functions

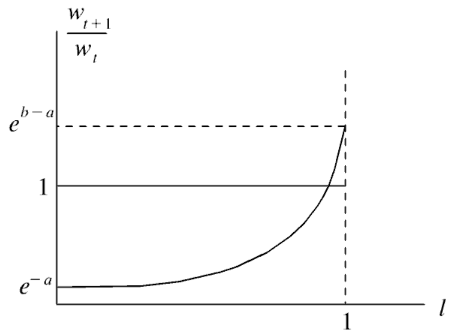

Figure 2: Wage dynamics and rate of employment

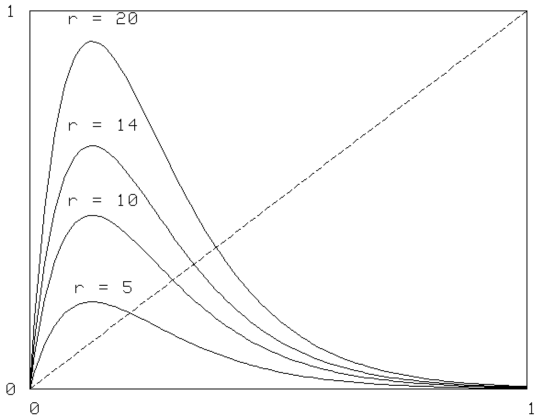

Figure 3: f for different values of r and $s = 9$

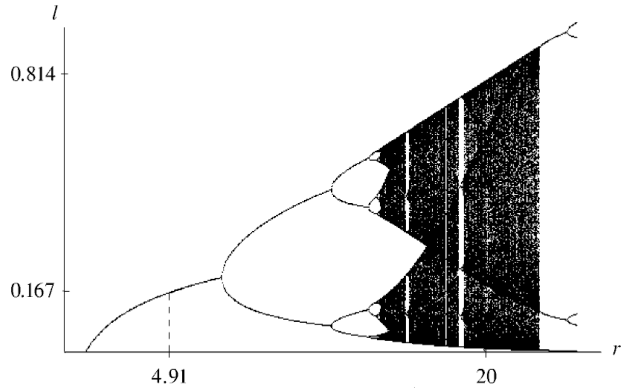

Figure 4: Bifurcation diagram $s = 9$

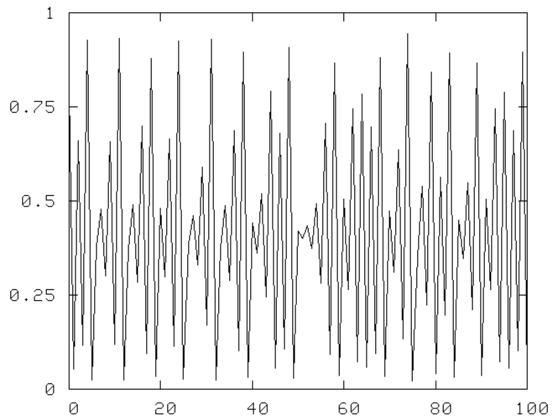

Figure 5: Chaotic time series of l

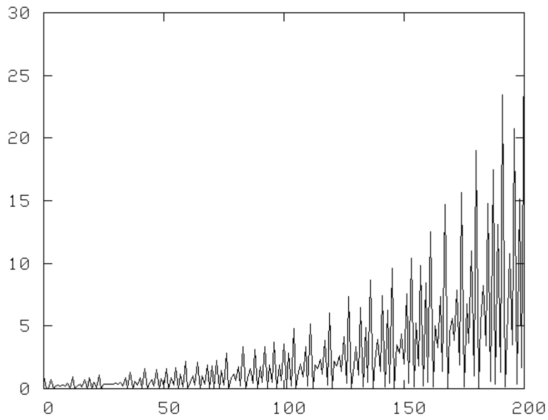

Figure 6: Chaotic time serie of y

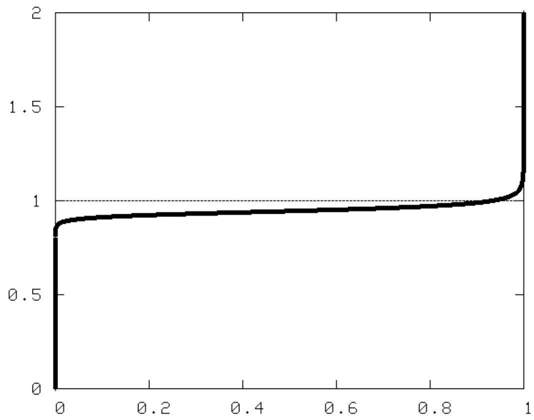

Figure 7: Phillips Curve (9)

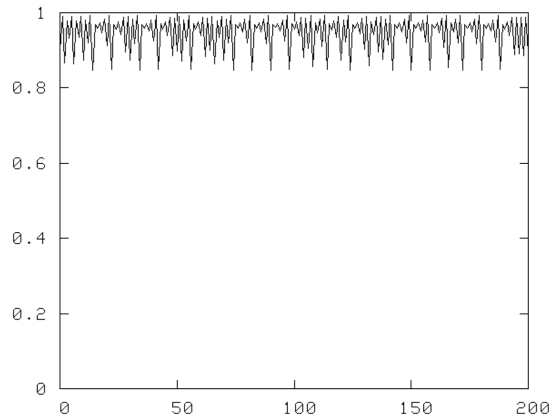

Figure 8: Intertemporal evolution of l with Phillips Curve (9)

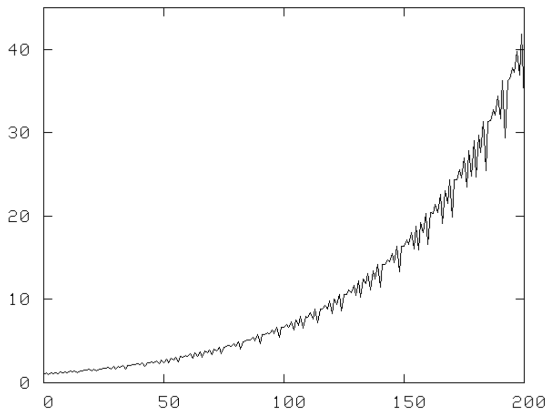

Figure 9: Intertemporal evolution of y with Phillips Curve (9)

Dynamics Behavior	Values of r
Stable equilibrium point	$1 < r < e^2$
Stable cycle of period 2	$e^2 < r < 12.5039$
Stable cycle of period 4	$12.5039 < r < 14.2392$
Stable cycle of period 8	$14.2392 < r < 14.6582$
Stable cycles of period 16, 32, 64,...	$14.6582 < r < 14.7611$
Chaos	$r > 14.7611$

Table 1. Dynamics behaviour of system (7)

Novedades

DIVISIÓN DE ADMINISTRACIÓN PÚBLICA

Rivera Urrutia, Eugenio, *La construcción del gobierno electrónico como problema de innovación institucional: la experiencia mexicana*, DTAP-174

Bravo Pérez, Héctor Manuel, *et al., Evaluación económica del convenio de distribución de aguas superficiales del lago de Chapala y su efecto en el bienestar social,* DTAP-175

Bravo Pérez, Héctor Manuel, Juan Carlos Castro Ramírez, *Construcción de una matriz de contabilidad social con agua…* DTAP-176

Bracho González, Teresa y Giulianna Mendieta, *El proceso de focalización y las estrategias estatales de selección de beneficiarios: El caso del Programa Escuelas de Calidad*, DTAP-177

Arellano, David y Walter Lepore, *Publicness y Nueva Gestión Pública: hacia una recuperación de los valores de lo público,* DTAP-178

López Ayllón, Sergio y Alí Bernardo Haddou Ruiz, *Rendición de cuentas en los órganos reguladores autónomos: algunas consideraciones sobre el diseño institucional de las autoridades reguladoras en México,* DTAP-179

Sour, Laura, *Pluralidad y transparencia en el proceso de aprobación presupuestal al interior de la Cámara de Diputados*, DTAP-180

Cabrero, Enrique, *Los retos institucionales de la descentralización fiscal en América Latina,* DTAP-181

Merino, Mauricio, *La profesionalización municipal en México,* DTAP-182

Arellano, David, *¿Reforma organizacional de gobierno por diseño genérico? El Nuevo Institucionalismo Económico en acción…* DTAP-183

DIVISIÓN DE ECONOMÍA

Torres Rojo, Juan M., *et al., Índice de peligro de incendios forestales* de largo plazo, DTE-358

J. M. Torres *et al*., *Bequest Motive for Conservation in Forest Production Communities,* DTE-359

Hernández, Fausto y Brenda Jarillo Rabling, *Is Local Beautiful? Decentralization in Mexico in the Presence of Elite Capture,* DTE-360

Unger, Kurt, *El desarrollo económico y la migración mexicana: el TLCAN e impactos en las regiones,* DTE-361

Gómez Galvarriato, Aurora, Rafael Dobado and Jeffrey Williamson, *Globalization, De-Industrialization and Mexican Exceptionalism, 1750-1879,* DTE-362

Gómez Galvarriato, Aurora and Aldo Musacchio, *Larger Menus and Entrepreneurial Appetite: An Empirical Investigation of Organization Choice in Mexico,* DTE-363

Mayer, David y Carlos Bazdresch, *Hacia un consenso para el crecimiento económico en México…* DTE-364

Mayer, David y Ma. Fernanda López Olivo, *Transmisión intergeneracional de habilidades cognitivas por niveles socioeconómicos…,* DTE-365

Mayer, David, *Dinámica geográfica de productividad e innovación en la manufactura mexicana*, DTE-366

Scott, John, *Seguro Popular: Incidence Analysis*, DTE-367

DIVISIÓN DE ESTUDIOS INTERNACIONALES

González González, Guadalupe, *México ante América Latina: Mirando de reojo a Estados Unidos*, DTEI-132

Ortiz Mena L.N., Antonio Ortiz y Ricardo Sennes, *Brasil y México en la economía política internacional,* DTEI-133

Minushkin, Susan y Matthew Adam Kocher, *Trade and Investment Policy Preferences and Public Opinion in Mexico,* DTEI-134

Ortiz Mena L.N., Antonio, *México ante el sistema monetario y comercial internacional: lecciones de Bretton Woods a la actualidad*, DTEI-135

Meseguer Covadonga *et al., The Diffusion of Regulatory Reforms in Pension Systems: Latin America in Comparative Perspective,* DTEI-136

Schiavon, Jorge A., *La relación especial México-Estados Unidos: Cambios y continuidades en la Guerra y Pos-Guerra Fría,* DTEI-137

Ortiz Mena, Antonio, *The Domestic Determinants of Mexico's Trade Strategy,* DTEI-138

Kocher, Matthew Adam and Stathis N. Kalyvas, *How free is "Free Riding" in Civil Wars? Violence, Insurgency, and the Collective Action Problem,* DTEI-139

Chabat, Jorge, *Mexico: The Security Challenge,* DTEI-140

Kydd, Andrew, *The Ball is in your Court: Mediation and Blamecasting*, DTEI-141

DIVISIÓN DE ESTUDIOS JURÍDICOS

Posadas, Alejandro, *Canada Trade Law & Policy after NAFTA and the…*, DTEJ-8

Hernández, Roberto, *Alcances del "juicio oral" frente a la Reforma Integral a la Justicia Penal propuesta por presidencia*, DTEJ-9

Magaloni, Ana Laura, *El impacto en el debate sobre la reforma judicial de los estudios empíricos del sistema de justicia: el caso del estudio del Banco Mundial sobre le Juicio Ejecutivo Mercantil,* DTEJ-10

Bergman, Marcelo, *Do Audits Enhance Compliance? An Empirical Assessment of VAT Enforcement,* DTEJ-11

Pazos, María Inés*, Sobre la semántica de la derrotabilidad de conceptos jurídicos,* DTEJ-12

Elizondo Carlos, Luis Manuel Pérez de Acha, *Separación de poderes y garantías individuales: La Suprema Corte y los derechos de los contribuyentes,* DTEJ-13

Fondevila Gustavo, *Estudio de percepción de usuarios del servicio de administración de justicia familiar en el Distrito Federal,* DTEJ-14

Pazos, Ma. Inés, *Consecuencia lógica derrotable: análisis de un concepto de consecuencia falible,* DTEJ-15

Posadas, Alejandro y Hugo E. Flores, *Análisis del derecho de contar con un juicio justo en México*, DTEJ-16
Posadas, Alejandro, *La Responsabilidad Civil del Estado /Análisis de un caso hipotético*, DTEJ-17
López, Sergio y Posadas Alejandro, *Las pruebas de daño e interés público en materia de acceso a la información. Una perspectiva comparada,* DTEJ-18

DIVISIÓN DE ESTUDIOS POLÍTICOS

Cermeño Rodolfo, Sirenia Vázquez, *What is Vote Buying? The Limits of the Market Model,* DTEP-179
Schedler Andreas*, Electoral Authoritarianism Concept, Measurement, and Theory,* DTEP-180
Negretto L. Gabriel, *Confronting Pluralism: Constitutional Reform in Mexico After Fox,* DTEP-181
Beltrán Ulises, *Contextual Effects on the Individual Rationality: Economic Conditions and retrospective Vote,* DTEP-182
Nacif Benito, *¿Qué hay de malo con la parálisis? Democracia y gobierno dividido en México,* DTEP-183
Langston Joy, *Congressional Campaigning in Mexico,* DTEP-184
Nacif Benito, *The Fall of the Dominant Presidency: Lawmaking Under Divided Government in Mexico,* DTEP-185
Lehoucq, Fabrice E., *Constitutional Design and Democratic Performance in Latin America,* DTEP-186
Martínez Gallardo, Cecilia and John D. Huber, *Cabinet Turnover and Talent Searches,* DTEP-187
Lehoucq, Fabrice E., *Structural Reform, Democratic Governance and Institutional Design in Latin America*, DTEP-188

DIVISIÓN DE HISTORIA

Pani, Erika, *Saving the Nation through Exclusion: The Alien and Sedition Acts and Mexico´s Expulsion of Spaniards,* DTH-32
Pipitone, Ugo, *El ambiente amenazado (*Tercer capítulo de *El temblor*…), DTH-33
Pipitone, Ugo, *Aperturas chinas (1889, 1919, 1978),* DTH-34
Meyer, Jean, *El conflicto religioso en Oaxaca*, DTH-35
García Ayluardo Clara, *El privilegio de pertenecer. Las comunidades de fieles y la crisis de la monarquía católica*, DTH-36
Meyer, Jean, *El cirujano de hierro (2000-2005),* DTH-37
Sauter, Michael, *Clock Watchers and Stargazers: On Time Discipline in Early-Modern Berlin,* DTH-38
Sauter, Michael, *The Enlightenment on Trial…,* DTH-39
Pipitone, Ugo, *Oaxaca prehispánica*, DTH-40
Medina Peña, Luis, *Los años de Salinas: crisis electoral y reformas*, DTH-41

Ventas

El **Centro de Investigación y Docencia Económicas / CIDE,** es una institución de educación superior especializada particularmente en las disciplinas de Economía, Administración Pública, Estudios Internacionales, Estudios Políticos, Historia y Estudios Jurídicos. **El CIDE publica**, como producto del ejercicio intelectual de sus investigadores, **libros, documentos de trabajo, y cuatro revistas especializadas:** *Gestión y Política Pública*, *Política y Gobierno*, *Economía Mexicana Nueva Época* **e** *Istor*.

Para adquirir alguna de estas publicaciones, le ofrecemos las siguientes opciones:

VENTAS DIRECTAS:

Tel. 5081-4003, 5727-9800 Ext. 6094 y 6091
Fax 5727 9800 Ext. 6314

Av. Constituyentes 1046, 1er piso,
Col. Lomas Altas, Del. Álvaro Obregón,
11950, México, D.F.

VENTAS EN LÍNEA:

Librería virtual: **www.e-cide.com**

Dudas y comentarios:
publicaciones@cide.edu

¡Nuevo!

Adquiera el **CD de las colecciones completas** de los documentos de trabajo de la **División de Historia** y de la **División de Estudios Jurídicos.**