NÚMERO 497

ANTONIO JIMÉNEZ

Coordination Incentives for Information Acquisition with a Finite Set of Players

DICIEMBRE 2010

CIDE

www.cide.edu

Fax: 5727•9800 ext. 6314
Correo electrónico: publicaciones@cide.edu
www.cide.edu

Acknowledgements

I gratefully acknowledge financial support from Conacyt.

Abstract

We consider a class of two-player quadratic games under incomplete information to study the relation between exogenous coordination motives and strategic interactions in information acquisition. The players make decisions in two stages. They decide about information acquisition in the first stage and choose their actions in the second stage. Preferences are such that the optimal action of each player depends on the state of the world and on the action taken by the other player. We show that if the degree of coordination in actions is sufficiently high, then the strategic interaction in the information choice does not have the same coordination motives as the action choice. Consequently, heterogeneous beliefs can be sustained endogenously for our class of games if the degree of complementarity or substitutability is high enough. Our results contrast qualitatively with the case studied by Hellwig and Veldkamp (2009) where the set of players is a continuum.

Resumen

Consideramos una clase de juegos cuadráticos, con dos jugadores, bajo información incompleta, para estudiar la relación entre incentivos de coordinación exógenos e interacciones estratégicas en la adquisición de información. Los jugadores toman decisiones en dos etapas. Deciden sobre la adquisición de información en la primera etapa y eligen sus acciones en la segunda etapa. Sus preferencias son tales que la acción óptima de cada jugador depende del estado del mundo y de la acción tomada por el otro jugador. Demostramos que si el grado de coordinación es suficientemente alto, entonces la interacción estratégica en la elección de información no tiene el mismo incentivo de coordinación que las acciones. En consecuencia, las creencias heterogéneas pueden mantenerse endógenamente para nuestra clase de juegos si el grado de complementariedad o sustituibilidad es suficientemente alto. Nuestros resultados contrastan cualitativamente con el caso estudiado por Hellwig y Veldkamp (2009) donde el conjunto de jugadores es un continuo.

Coordination Incentives for Information Acquisition with a Finite Set of Players*

Antonio Jiménez-Martínez
Centro de Investigación y Docencia Económicas

October 2010

Abstract

We consider a class of two-player quadratic games under incomplete information to study the relation between exogenous coordination motives and strategic interactions in information acquisition. The players make decisions in two stages. They decide about information acquisition in the first stage and choose their actions in the second stage. Preferences are such that the optimal action of each player depends on the state of the world and on the action taken by the other player. We show that if the degree of coordination in actions is sufficiently high, then the strategic interaction in the information choice does not have the same coordination motives as the action choice. Consequently, heterogeneous beliefs can be sustained endogenously for our class of games if the degree of complementarity or substitutability is high enough. Our results contrast qualitatively with the case studied by Hellwig and Veldkamp (2009) where the set of players is a continuum.

JEL Classification Numbers: C72, D82, D83
Keywords: Incomplete information, information acquisition, coordination

Correspondence To:
A. Jiménez-Martínez
Centro de Investigación y Docencia Económicas
División de Economía
Carretera México-Toluca 3655, Lomas de Santa Fe
01210 Mexico City, Mexico
e-mail: antonio.jimenez@cide.edu
antonio.jimm@gmail.com

*I gratefully acknowledge financial support from CONACYT.

1. Introduction

In many environments the optimal action of a decision maker depends on both the underlying fundamentals and the actions taken by other agents. Models which incorporate this feature have been successfully used to analyze a broad class of coordination problems under uncertainty and its welfare implications,[1] usually using perfect Bayes-Nash equilibrium as solution concept.

For such environments, one could intuitively think of two reasons why the decision maker wishes to improve her knowledge of unknown fundamentals. First, this information acquisition allows directly for better predictions of the fundamentals. Second, since the other agents' actions are based on their own expectations of the fundamentals, learning about the fundamentals leads to more precise inferences about those actions. In practice, agents are often able to make decisions on information acquisition about unknown fundamentals before choosing their actions.

This paper studies the relation between the exogenously given coordination motives in actions and the strategic interactions in information acquisition for a tractable class of two-player quadratic games. Our analysis uses a game theoretical framework that allows us to formalize the intuitions given above behind the incentives for information acquisition. Ultimately, this line of research tries to shed light into the open question of whether heterogenous beliefs can be endogenously sustained in strategic environments. This is clearly relevant since many models analyze economic phenomena using the assumption that agents have heterogenous beliefs. Therefore, we should be interested in knowing whether strategic interactions in information acquisition could eliminate the heterogeneity.

In a beautiful recent paper, Hellwig and Veldkamp (2009)—henceforth, HV—have addressed this question using a two-stage, beauty contest game with a large number of ex ante identical small players. In their model, the players decide on costly information acquisition before choosing their actions. HV show that the strategic interactions in information acquisition reproduce exactly the exogenous coordination motives in actions. More precisely, they obtain that the information choice exhibits complementarity when actions are complements and substitutability when actions are substitutes. One concludes that heterogeneous beliefs will not be sustained endogenously in the long run.

For this class of games, a player's best response in actions typically depends on arbitrarily higher-order iterated expectations of the state, that is, what the player expects that any other player expects that any other player expects and so forth. As in Morris and Shin (2002), the assumption that the set of players is a continuum enables HV to use an average expectation operator to keep track of the higher-order iterated expectations of the

[1] Incentives of this nature have been considered, among others, by (i) Cooper and John (1988) to study coordination failures in macroeconomic models, (ii) Morris and Shin (2002), Hellwig (2005), Cornand and Heinemann (2008), and Angeletos and Pavan (2007) to study the effects of public information disclosure on social welfare, (iii) Morris and Shin (2005) to study the welfare consequences of central bank transparency, (iv) Calvó-Armengol and de Martí (2007, 2009) to study efficient information transmission in communication networks, and (v) Calvó-Armengol, de Martí, and Prat (2009) to study endogenous information transmission in networks.

state. However, if one considers instead a finite set of players, then this approach would be appropriate only when the higher-order expectations are very homogeneous across the players. But, if the players begin with heterogeneous beliefs, then the heterogeneity would not necessarily vanish unless one imposes a very restrictive symmetric information structure. As a consequence, an average expectation operator would be ill-suited to keep track of the required higher-order expectations with a finite number of players and a flexible information structure.

In this paper, we depart from the assumptions in HV in two directions. First, we consider a finite set of players and, therefore, do not use an average expectation operator to account for higher-order beliefs. Second, we consider a general class of quadratic games which include the beauty contest game studied by HV as a special case. For tractability purposes, we conduct our analysis by considering a two-player game, though our results continue to hold qualitatively so long as the set of players is finite. As in HV, we consider that the players acquire information through private signals, conditionally independent given the state of world,[2] and allow for either strategic complementarity or substitutability in actions. Yet, we allow for a broader set of external effects. We measure the degree of coordination in actions using the slope of a player's best response with respect to the other player's action.

Our results agree with those in the case studied by HV when the degree of coordination in actions is moderate, Proposition 1. However, for our class of preferences, we also show in Proposition 2 that if the degree of coordination is sufficiently high, then the information choice does not follow the same strategic motives as the actions. More precisely, we identify a threshold for the complementarity degree above which the equilibrium information choices are substitutes. This is also the case if we restrict attention to the version of the beauty contest game considered by HV. Information acquisition is costless in our model, which reinforces the message conveyed by this result. Also, we show that if the degree of substitutability is sufficiently high, then, for some initial information decisions, the choices in information become complementary. The latter result, however, does not follow for the beauty contest game.

The main reason behind the discrepancy of our results for the beauty contest game with respect to those obtained by HV is as follows. When the set of players is finite, the effect of a change in a player's information choice on the higher-order beliefs that any other player uses to determine her optimal action is higher than in the case with a continuum of players. As a consequence, with a finite number of players, the slope of a player's optimal action in her private signal is more sensitive to the information choice of any player.

To understand the role of this observation in our results, consider the two-player case with complementary actions. When the degree of complementarity is moderate, player 1 wishes to improve her knowledge of the state because she is very concerned about matching

[2]In their model, HV allow also for public signals. This feature, however, does not interfere with the analysis of the question studied in this paper.

her action to it. On the other hand, suppose that player 2 improves her information. Then, her optimal action becomes more sensitive to her private signal so that, in order to satisfy the coordination motive, player 1 wishes to make her own optimal action more sensitive to her signal too. Therefore, there are two effects which make player 1 wish to improve her information about the state.

However, when the degree of complementarity exceeds a certain threshold, the two effects identified above become less clear. This, together with the risk-aversion that affects the coordination motive, tends to reverse the incentives of player 1 to complement the information choice of player 2. On the one hand, player 1 becomes now very little interested in matching her action to the state and very interested in matching her action with that of player 2. So, she is barely interested now in learning about the state. On the other hand, player 2's optimal action becomes less sensitive to her private signal regardless of her information choice. In particular, for a degree of complementarity sufficiently high, player 2's optimal action is very close to the optimal action that she would choose when she acquires no information at all. Therefore, player 1 wishes to decrease the sensitivity of her optimal action to her private signal too. One way of satisfying the coordination motive, which is the prevalent one now, is then to acquire very little amount of information. Furthermore, player 1 is risk-averse with respect of the difference between the slopes of the players' optimal actions. Given that the two aforementioned effects do not make it clearly valuable for player 1 to improve her information, her desire to insure against the discrepancy between the optimal actions finally offsets those effects and motivates her to reduce the amount of information that she acquires.

Since Morris and Shin (2002), and Calvó-Armengol and de Martí (2007), it is well known that, for the class of games studied in this paper, quadratic preferences lead to linear best responses in actions. To keep track of the players' higher-order iterated expectations, we follow the approach introduced in the economics literature by Calvó-Armengol and de Martí (2007) and write the linear optimal action in terms of a knowledge index. The knowledge index depends on the covariances between the signals received by each of the players and the state. Fortunately, for this class of games, it is well known that the action strategies are unique at equilibrium. Calvó-Armengol and de Martí (2009) use a key result in team theory due to Radner (1963) to demonstrate this convenient uniqueness result for a beauty contest game.

The rest of the paper is organized as follows. We introduce the model in Section 2. Section 3 analyzes the strategies in actions at equilibrium and the strategic interactions in information acquisition. We discuss various examples of strategic settings where our results are relevant in Section 4 and conclude in Section 5.

2. The Model

2.1. *Actions and Payoffs*

We consider two players, $i = 1, 2$, who make decisions in a two-stage game.

In the first stage, nature selects a state of the world $\theta \in \mathbb{R}$, which is unobservable for the players. Instead, each player i observes a (noisy) private signal realization s_i. Then, each player i can improve her knowledge of θ by choosing the correlation between the random variables which generate the state θ and the signal realization s_i. Information acquisition decisions are taken simultaneously.

In the second stage, each player i chooses an action $a_i \in \mathbb{R}$. Actions are taken simultaneously too. The final payoff u_i to each player i depends on state of the world θ and on the action profile $a = (a_i, a_{-i})$ chosen by the players.

We consider a general class of quadratic preferences, which is similar to that used by Angeletos and Pavan (2007) to analyze the social value and the efficient use of public information in a setting with a continuum of players. Formally, the payoff u_i to each agent i is given by a twice-differentiable, real-valued function U. This payoff function U is common for both players.

Throughout the paper we shall take $i = 1$, without loss of generality, when we need to fix a given player in the analysis. Denoting partial derivatives by subscripts in the usual way, we measure the degree of coordination in the players' actions using parameter

$$\lambda := -\frac{U_{a_1 a_2}}{U_{a_1 a_1}}.$$

Note that λ above gives us the slope of player 1's best response with respect to player 2's action. Also, we use parameter

$$\pi := \frac{U_{a_2 a_2}}{U_{a_1 a_1}}$$

as a measure of the externality generated on player 1 by the action chosen by player 2. It gives us the slope of the strategy over actions chosen by player 2 which is most preferred by player 1.

Assumption 1. —Preferences— For each player $i = 1, 2$ such that $u_i = U$,

(i) U is quadratic;

(ii) $U_{a_i a_i} < 0$;

(iii) $U_{a_{-i} a_{-i}} \leq 0$;

(iv) $-U_{a_i a_{-i}} / U_{a_i a_i} \in (-1, 1)$.

The assumption that U is quadratic guarantees linearity of optimal strategies in actions. Besides, we are assuming that each player's payoff is strictly concave in her own action, which ensures that best responses are well defined, and concave in the other player's action. Given these assumptions, Assumption 1 (iv) is needed to guarantee both existence and uniqueness of perfect Bayes-Nash equilibrium in our information acquisition game. Note that from Assumption 1 (ii) and (iii), it follows $\pi \geq 0$.

Other than the restrictions imposed above to make the analysis tractable, this specification of preferences is quite general and, in particular, allows for strategic complementarity ($\lambda > 0$) and substitutability ($\lambda < 0$) in actions. Higher values of $\lambda > 0$ mean more complementarity and lower values of $\lambda < 0$ mean more substitutability.

The payoff structure of a beauty contest game is a particular case of the preference specification given by Assumption 1 above. In particular, the variant of the beauty contest game used by HV (adapted to our two-player game) is given by the payoff function ($i = 1$)

$$U(a_1, a_2, \theta) = -(1-\lambda)^{-2}\big[a_1 - (1-\lambda)\theta - \lambda a_2\big]^2, \tag{1}$$

where $\lambda \in (-1, 1)$ measures the degree of complementarity/substitutability in the players' actions. This payoff function satisfies Assumption 1 on preferences if $\pi = \lambda^2$.

2.2. *Information Acquisition*

We consider a Gaussian information structure for tractability purposes. In the first stage of the game, nature draws the state realization θ from a normal distribution with mean μ and variance σ^2. The realization θ is unobservable for the players. Accordingly, the (common) priors of the players about θ are given by the normal distribution from which θ is drawn. Also, each player i observes a signal realization $s_i \in \mathbb{R}$.[3]

We assume that each player i can choose in the first stage of the game the informativeness of her signal by choosing the correlation between the random variables that generate θ and s_i. By doing so, player i makes a decision on her own belief revision process, which must be specified according to Bayes' rule.[4] Thus, each player i ends up with some (endogenously chosen) posteriors about θ, which she uses in the second stage to make her decision on actions.

Formally, let us denote by $\tilde{z}$ a random variable with realization z.

Assumption 2. —Information Structure— The random vector $(\tilde{\theta}, \tilde{s}_1, \tilde{s}_2)$ follows a multi-normal distribution with mean (μ, μ, μ) and variance-covariance matrix

$$\begin{pmatrix} \sigma^2 & \rho_1\sigma\gamma & \rho_2\sigma\gamma \\ \rho_1\sigma\gamma & \gamma^2 & \rho_1\rho_2\gamma^2 \\ \rho_2\sigma\gamma & \rho_1\rho_2\gamma^2 & \gamma^2 \end{pmatrix},$$

where $\sigma^2 = \mathrm{Var}[\tilde{\theta}]$, $\gamma^2 = \mathrm{Var}[\tilde{s}_1] = \mathrm{Var}[\tilde{s}_2]$, and $\rho_i \in [-1, 1]$ is the correlation coefficient between $\tilde{\theta}$ and $\tilde{s}_i$ for each player $i = 1, 2$.

[3]We regard a signal as a random variable and a signal realization simply as a particular realization of the random variable.

[4]Modeling endogenous information acquisition by allowing the players to move from a prior distribution to a posterior distribution (under the restriction imposed by Bayes' rule) is quite standard in the literature. For instance, this approach is used by Allen (1983, 1986) in a more abstract setting from the perspective of information demand theory, and by Bergemann and Välimäki (2002) in their work on mechanism design when players are allowed to acquire information.

Notice that, using the correlation coefficient, we can express the covariance between $\tilde{\theta}$ and each $\tilde{s}_i$ as $\mathrm{Cov}[\tilde{\theta}, \tilde{s}_i] = \rho_i \sigma \gamma$. Therefore, assuming $\mathrm{Cov}[\tilde{s}_1, \tilde{s}_2] = \rho_1 \rho_2 \gamma^2$ is the same as requiring

$$\mathrm{Cov}[\tilde{s}_1, \tilde{s}_2] = \frac{\mathrm{Cov}[\tilde{\theta}, \tilde{s}_1]\mathrm{Cov}[\tilde{\theta}, \tilde{s}_2]}{\mathrm{Var}[\sigma^2]},$$

which, in turn, is equivalent to assuming that signals are conditionally independent given θ.

Consider a given player $i = 1, 2$. Using some basic results on normal distributions, we know that the random variable $\tilde{\theta} \,|\, s_i$ follows a normal distribution with mean

$$\mathbb{E}[\tilde{\theta} \,|\, s_i] = \mu + \frac{\rho_i \sigma}{\gamma}(s_i - \mu). \tag{2}$$

Analogously, the random variable $\tilde{s}_{-i} \,|\, s_i$ follows a normal distribution with mean

$$\mathbb{E}[\tilde{s}_{-i} \,|\, s_i] = \mu + \rho_i \rho_{-i}(s_i - \mu). \tag{3}$$

This is the only piece of information about player i's posteriors that we will need in our subsequent analysis.

2.3. *Equilibrium*

In the second stage of the game, each player i chooses an action a_i, for each signal realization s_i that she observes and each action a_{-i} taken by the other player, so as to maximize her expected payoff $\mathbb{E}[U(a_i, a_{-i}, \tilde{\theta}) \,|\, s_i]$. By solving this optimization problem, we obtain player i's best response in actions.

Furthermore, the usual fixed point requirement must be satisfied in a perfect Bayes-Nash equilibrium. So, let $\alpha_i : \mathbb{R} \to \mathbb{R}$ be player i's *optimal action strategy* so that $\alpha_i(s_i)$ is the optimal action chosen by player i upon observing signal realization s_i. Then, the following condition must hold in a perfect Bayes-Nash equilibrium:

$$\alpha_i(s_i) = \arg\max_{a_i \in \mathbb{R}} \mathbb{E}[U(a_i, \alpha_{-i}(\tilde{s}_{-i}), \tilde{\theta}) \,|\, s_i] \quad \text{for each} \quad s_i \in \mathbb{R} \quad \text{and each} \quad i = 1, 2. \tag{4}$$

In the first stage of the game, each player i decides on information acquisition. Let $x_i := \rho_i^2 \in [0, 1]$ be an *information acquisition choice* for player i. Higher values of x_i indicate higher degree of informativeness for the signal chosen by player i. Of course, the expected payoff of each player i in the first stage, provided that both players follow their optimal action strategies in the second stage, depends on the *information acquisition profile* (x_1, x_2). So, let $F : [0, 1]^2 \to \mathbb{R}$ be the function specified as

$$F(x_1, x_2) := \mathbb{E}[U(\alpha_i(\tilde{s}_i), \alpha_{-i}(\tilde{s}_{-i}), \tilde{\theta})]. \tag{5}$$

Definition 1. A *perfect Bayes-Nash equilibrium* of the information acquisition game is a pair of optimal action strategies (α_1, α_2), which satisfy condition (4) above, and an information acquisition profile $(x_1^*, x_2^*) \in [0, 1]^2$ such that, for each player $i = 1, 2$,

$$x_i^* = \arg\max_{x_i \in [0,1]} F(x_i, x_{-i}^*),$$

where F is specified in (5) above.

Our objective in this paper is to study the relation between the pair of parameters (λ, π) and the degree of complementarity/substitutability that the ex ante expected payoff F, defined in (5) above, exhibits. In particular, we study the extent to which the sign of parameter λ affects the sign of the second derivative $F_{x_1 x_2}$. To do so, we need to characterize first the optimal action strategies α_1, α_2.

3. Main Results

Obtaining a closed expression for the function F defined in (5) above is constructive. We first characterize the action strategies at equilibrium and then study the ex-ante expected payoff of the players when they follow those equilibrium action strategies.

3.1. *Equilibrium Action Strategies*

Given our differentiability assumptions and the assumption that $u_i = U$ is strictly concave in player i's own action, Assumption 1 (ii), the requirement that player 1 follows her optimal action strategy in the second stage, specified by condition (4) above, holds if and only if

$$\mathbb{E}\big[U_{a_1}(\alpha_1(s_1), \alpha_2(\tilde{s}_2), \tilde{\theta}) \,\big|\, s_1\big] = 0$$

is satisfied for each $s_1 \in \mathbb{R}$.

It is useful to consider first the complete information case. Suppose that θ is known to the players (consider, e.g., $x_1 = x_2 = 1$). Then, $\alpha_i(s_i) = \tau(\theta)$ for each $s_i \in \mathbb{R}$ and for both players $i = 1, 2$, so that $\tau(\theta)$ is well defined as the unique solution to $U_{a_1}(\tau(\theta), \tau(\theta), \theta) = 0$. Furthermore, since U is quadratic, $\tau(\theta)$ must be linear in θ and, therefore, we can write $\tau(\theta) = \tau_0 + \tau_1\theta$, where $\tau_0, \tau_1 \in \mathbb{R}$.

Now, using a first-order Taylor expansion of $U_{a_1}(\alpha_1(s_1), \alpha_2(s_2), \theta)$ around $(\tau(\theta), \tau(\theta), \theta)$, we obtain

$$\begin{aligned} U_{a_1}(\alpha_1(s_1), \alpha_2(s_2), \theta) =& U_{a_1 a_1}(\tau(\theta), \tau(\theta), \theta)[\alpha_1(s_1) - \tau(\theta)] \\ &+ U_{a_1 a_2}(\tau(\theta), \tau(\theta), \theta)[\alpha_2(s_2) - \tau(\theta)], \end{aligned}$$

wherein we have made use of $U_{a_1}(\tau(\theta), \tau(\theta), \theta) = 0$. Then, it follows that

$$\begin{aligned} &\mathbb{E}\big[U_{a_1}(\alpha_1(s_1), \alpha_2(\tilde{s}_2), \tilde{\theta}) \,\big|\, s_1\big] = 0 \\ &\qquad \Leftrightarrow U_{a_1 a_1}\alpha_1(s_1) - \big(U_{a_1 a_1} + U_{a_1 a_2}\big)\mathbb{E}[\tau(\tilde{\theta}) \mid s_1] + U_{a_1 a_2}\mathbb{E}[\alpha_2(\tilde{s}_2) \mid s_1] = 0. \end{aligned}$$

To ease the notational burden, Let us write $\mathbb{E}_i[\cdot]$ instead of $\mathbb{E}[\cdot \mid s_i]$ and α_i instead of $\alpha_i(s_i)$ when no possible confusion arises. Then, from the expression above it follows that

$$\alpha_1 = (1 - \lambda)\mathbb{E}_1[\tau(\tilde{\theta})] + \lambda\mathbb{E}_1[\alpha_2(\tilde{s}_2)].$$

By iterating recursively, one obtains

$$\begin{aligned}\alpha_1 &= (1-\lambda)\mathbb{E}_1[\tau(\tilde{\theta})] + (1-\lambda)\lambda\mathbb{E}_1\left[\mathbb{E}_2[\tau(\tilde{\theta})]\right] + \lambda^2\mathbb{E}_1\left[\mathbb{E}_2[\alpha_1(\tilde{s}_1)]\right]\\ &= \cdots \\ &= (1-\lambda)\Bigg(\frac{\tau_0}{1-\lambda} + \tau_1\left(\mathbb{E}_1[\tilde{\theta}] + \lambda\mathbb{E}_1\left[\mathbb{E}_2[\tilde{\theta}]\right] + \lambda^2\mathbb{E}_1\Big[\mathbb{E}_2\left[\mathbb{E}_1[\tilde{\theta}]\right]\Big] + \cdots\right)\Bigg),\end{aligned}$$

or, equivalently,

$$\alpha_1 = \tau_0 + (1-\lambda)\tau_1\sum_{k=0}^{\infty}\lambda^k\mathbb{E}_1\mathbb{E}_2\mathbb{E}_1\cdots\mathbb{E}_{p(k)}[\tilde{\theta}], \tag{6}$$

where $\mathbb{E}_1\mathbb{E}_2\mathbb{E}_1\cdots\mathbb{E}_{p(k)}[\tilde{\theta}]$ denotes the $(k+1)$-order iterated expectations of $\tilde{\theta}$. These nested expectations give us what player 1 expects that player 2 expects that player 1 expects, and so on up to the $k+1$ level of iteration, of the unknown state of the world $\tilde{\theta}$. Here, the subindex $p(k)$ equals 1 if k is either zero or even and equals 2 if k is odd. Note that the expression (6) above for $\alpha_1(s_1)$ is well defined given that $\lambda \in (-1,1)$, as required by Assumption 1 (iv).

We can use the distributional results in (2) and (3) to obtain

$$\mathbb{E}_1[\tilde{\theta}] = \mu + \rho_1\left(\frac{\sigma}{\gamma}\right)(s_1-\mu),$$

$$\mathbb{E}_1\left[\mathbb{E}_2[\tilde{\theta}]\right] = \mu + \rho_2(\rho_1\rho_2)\left(\frac{\sigma}{\gamma}\right)(s_1-\mu),$$

$$\mathbb{E}_1\Big[\mathbb{E}_2\left[\mathbb{E}_1[\tilde{\theta}]\right]\Big] = \mu + \rho_1(\rho_1\rho_2)^2\left(\frac{\sigma}{\gamma}\right)(s_1-\mu),$$

so that, by iterating recursively, we get

$$\mathbb{E}_1\mathbb{E}_2\mathbb{E}_1\cdots\mathbb{E}_{p(k)}[\tilde{\theta}] = \mu + \rho_{p(k)}(\rho_1\rho_2)^k\left(\frac{\sigma}{\gamma}\right)(s_1-\mu).$$

At this point, we need a closed expression that keeps track of the discounted $k+1$ order nested expectations of the players in order to meet the fixed point requirement at equilibrium. To do so, we make use of the knowledge index introduced in the economics literature by Calvó-Armengol and de Martí (2007) in their work on communication in networks. So, consider the pair of matrices

$$\phi := \left(\frac{\sigma}{\gamma}\right)(\rho_1\,,\rho_2)_{1\times 2}, \qquad \Omega := \begin{pmatrix}\rho_1\rho_2 & 0\\ 0 & \rho_1\rho_2\end{pmatrix}_{2\times 2}.$$

Then, the expression above for $\mathbb{E}_1\mathbb{E}_2\mathbb{E}_1\cdots\mathbb{E}_{p(k)}[\tilde{\theta}]$ can be rewritten, using those matrices ϕ and Ω, as

$$\mathbb{E}_1\mathbb{E}_2\mathbb{E}_1\cdots\mathbb{E}_{p(k)}[\tilde{\theta}] = \mu + \phi\cdot\Omega^k\cdot e_1(s_1-\mu), \tag{7}$$

where $e_1 = (1, 0)$. By plugging the expression in (7) above into the expression for $\alpha_1(s_1)$ given by (6), we obtain

$$
\begin{aligned}
\alpha_1(s_1) &= \tau_0 + \tau_1\mu + (1-\lambda)\tau_1\,\phi \cdot \sum_{k=0}^{\infty} \lambda^k \Omega^k \cdot e_1(s_1 - \mu) \\
&= \tau_0 + \tau_1\mu + (1-\lambda)\tau_1\,\phi \cdot [I - \lambda\Omega]^{-1} \cdot e_1 \cdot (s_1 - \mu),
\end{aligned}
$$

where I denotes the two by two identity matrix. Note that the infinite sum $\sum_{k=0}^{\infty} \lambda^k \Omega^k = [I - \lambda\Omega]^{-1}$ above is well the defined since we are assuming $\lambda \in (-1, 1)$.

Thus, the slope of player i's equilibrium strategy in actions with respect to her private signal can be written as

$$
m_i := (1-\lambda)\tau_1\phi \cdot [I - \lambda\Omega]^{-1} \cdot e_i,
$$

where e_i is the ith vector of the canonical basis of $\mathbb{R}^2$. Since $[I - \lambda\Omega]$ is a two by two matrix, we can compute its inverse to obtain

$$
m_i = \tau_1 \left(\frac{\sigma}{\gamma}\right) \frac{(1-\lambda)(1+\lambda\rho_{-i}^2)\rho_i}{(1-\lambda^2\rho_1^2\rho_2^2)}, \qquad i = 1, 2.
$$

Hence, using the arguments above, we have shown

Lemma 1. *The action strategy for each player $i = 1, 2$ in a perfect Bayes-Nash equilibrium is given by*

$$
\alpha_i(s_i) = \tau_0 + \tau_1\mu + m_i\,(s_i - \mu), \tag{8}
$$

where $\tau_0, \tau_1 \in \mathbb{R}$, and

$$
m_i = \tau_1 \left(\frac{\sigma}{\gamma}\right) \frac{(1-\lambda)(1+\lambda x_{-i})x_i^{1/2}}{(1-\lambda^2 x_1 x_2)}. \tag{9}
$$

The shape of the slope m_i, as a function of λ, x_1 and x_2, obtained in Lemma 1 above plays a crucial role in our results about equilibrium interactions on the information choice. In a separate appendix we show that the equilibrium action strategy given by Lemma 1 is unique.

3.2. *Information Acquisition Decisions*

To obtain a closed expression for the ex ante expected utility F, we need to use the expressions that we have obtained in Lemma 1 for α_1 and α_2 to compute the expected value of $u_1 = U$, as required by the definition of F given by (5). Using a second-order Taylor expansion of $U(\alpha_1(s_1), \alpha_2(s_2), \theta)$ around $(\tau(\theta), \tau(\theta), \theta)$, we obtain

$$
\begin{aligned}
U(\alpha_1(s_1), \alpha_2(s_2), \theta) &= U(\tau(\theta), \tau(\theta), \theta) + \frac{1}{2} U_{a_1 a_1}[\alpha_1(s_1) - \tau(\theta)]^2 \\
&\quad + \frac{1}{2} U_{a_2 a_2}[\alpha_2(s_2) - \tau(\theta)]^2 + U_{a_1 a_2}[\alpha_1(s_1) - \tau(\theta)][\alpha_2(s_2) - \tau(\theta)],
\end{aligned}
$$

wherein we have made use of $U_{a_1}(\tau(\theta),\tau(\theta),\theta)=U_{a_2}(\tau(\theta),\tau(\theta),\theta)=0$. Therefore, using the definition of F given in (5), we have

$$
\begin{aligned}
F =& \mathbb{E}\big[U(\tau(\tilde\theta),\tau(\tilde\theta),\tilde\theta)\big]+\frac{1}{2}U_{a_1a_1}\mathbb{E}\big[[\alpha_1(\tilde s_1)-\tau(\tilde\theta)]^2\big]\\
&+\frac{1}{2}U_{a_2a_2}\mathbb{E}\big[[\alpha_2(\tilde s_2)-\tau(\tilde\theta)]^2\big]+U_{a_1a_2}\mathbb{E}\big[[\alpha_1(\tilde s_1)-\tau(\tilde\theta)][\alpha_2(\tilde s_2)-\tau(\tilde\theta)]\big].
\end{aligned}
$$

By Assumption 1 (ii), $-U^{-1}_{a_1a_1}$ is a positive constant, so that the sign of F_{x_1,x_2} coincides with the sign of $-U^{-1}_{a_1a_1}F_{x_1,x_2}$. Using this, we propose the normalization $\widehat F := -U^{-1}_{a_1a_1}F$ and proceed with the analysis in terms of $\widehat F$ instead of F. Then, using the definitions of λ and π, from the equation above we can derive the expression for the function $\widehat F$ as

$$
\begin{aligned}
\widehat F =& -U^{-1}_{a_1a_1}\mathbb{E}\big[U(\tau(\tilde\theta),\tau(\tilde\theta),\tilde\theta)\big]-\frac{1}{2}\mathbb{E}\big[[\alpha_1(\tilde s_1)-\tau(\tilde\theta)]^2\big]\\
&-\frac{1}{2}\pi\mathbb{E}\big[[\alpha_2(\tilde s_2)-\tau(\tilde\theta)]^2\big]+\lambda\mathbb{E}\big[[\alpha_1(\tilde s_1)-\tau(\tilde\theta)][\alpha_2(\tilde s_2)-\tau(\tilde\theta)]\big].
\end{aligned}
\tag{10}
$$

So, we need now to analyze the terms inside the expectation operators in equation (10) above. Using the expression provided in Lemma 1 for $\alpha_i(s_i)$, we obtain

$$
\alpha_i(\tilde s_i)-\tau(\tilde\theta)=-\tau_1(\tilde\theta-\mu)+m_i(\tilde s_i-\mu)=(-\tau_1\,,m_i)\cdot\begin{pmatrix}\tilde\theta-\mu\\ \tilde s_i-\mu\end{pmatrix}.
$$

But then it follows that $\alpha_i(\tilde s_i)-\tau(\tilde\theta)$ is normally distributed with zero mean. So, using some basic results on normal distributions, we can compute its variance as

$$
\mathbb{E}\big[[\alpha_i(\tilde s_i)-\tau(\tilde\theta)]^2\big]=\Big[\tau_1^2\sigma^2+m_i^2\gamma^2-2\tau_1x_i^{1/2}m_i\sigma\gamma\Big],\qquad i=1,2,\tag{11}
$$

and the covariance between $\alpha_1(\tilde s_1)-\tau(\tilde\theta)$ and $\alpha_2(\tilde s_2)-\tau(\tilde\theta)$ as

$$
\begin{aligned}
&\mathbb{E}\big[[\alpha_1(\tilde s_1)-\tau(\tilde\theta)][\alpha_2(\tilde s_2)-\tau(\tilde\theta)]\big]\\
&\qquad=\Big[\tau_1^2\sigma^2+(x_1x_2)^{1/2}m_1m_2\gamma^2-\tau_1(x_1^{1/2}m_1+x_2^{1/2}m_2)\sigma\gamma\Big].
\end{aligned}
\tag{12}
$$

By plugging expressions (11) and (12) obtained above into (10), and by using the expression for the slope m_i $(i=1,2)$ obtained in Lemma 1, we get

$$
\begin{aligned}
\widehat F =& -U^{-1}_{a_1a_1}\mathbb{E}\big[U(\tau(\tilde\theta),\tau(\tilde\theta),\tilde\theta)\big]+\frac{(2\lambda-\pi-1)(\tau_1\sigma)^2}{2}\\
&+\frac{(1-\lambda)^2(\tau_1\sigma)^2}{2}\left[-x_1q_1^2-\pi x_2q_2^2+2x_1q_1-2\left(\frac{\lambda-\pi}{1-\pi}\right)x_2q_2+2\lambda x_1x_2q_1q_2\right],
\end{aligned}
$$

where $q_i:[0,1]^2\to\mathbb{R}$ denotes the function specified as

$$
q_i(x_1,x_2):=\frac{1+\lambda x_{-i}}{1-\lambda^2x_1x_2},\qquad i=1,2.
$$

All that remains then is to compute the second derivative $\widehat{F}_{x_1x_2}$ from the expression above. By doing the algebra, we finally obtain

$$\begin{aligned}\widehat{F}_{x_1x_2} =&\lambda \left[\frac{(1-\lambda)\tau_1\sigma}{1-\lambda^2 x_1x_2}\right]^2 \Big[(2-\pi)\lambda^2 x_1x_2q_1q_2 \\ &+ \lambda\Big(x_1q_1 - \frac{\lambda-\pi}{1-\lambda}x_2q_2 + (1-\pi)x_2q_2^2\Big) + q_2 - \frac{\lambda-\pi}{1-\lambda}q_1 - \pi q_1q_2\Big].\end{aligned} \tag{13}$$

Also, for the particular case of the beauty contest game with the preference specification given by (1), we have $\pi = \lambda^2$ and, therefore, the second order derivate above becomes

$$\begin{aligned}\widehat{F}^{\text{bc}}_{x_1x_2} =&\lambda \left[\frac{(1-\lambda)\sigma}{1-\lambda^2 x_1x_2}\right]^2 \Big[- x_1x_2q_1q_2\lambda^4 - x_2q_2^2\lambda^3 \\ &+ (2x_1x_2q_1q_2 - x_2q_2 - q_1q_2)\lambda^2 + (x_1q_1 + x_2q_2^2 - q_1)\lambda + q_2\Big].\end{aligned} \tag{14}$$

With the expressions for the ex-ante expected utility of player 1 in (13) and (14) above at hand, we can state our main results.

Proposition 1. *Assume 1 and 2. Then, $\widehat{F}_{x_1x_2}(x_1,x_2) = 0$ for each $(x_1,x_2) \in [0,1]^2$ when $\lambda = 0$. Moreover, for each $\pi \geq 0$ there exists some $\epsilon > 0$ such that if $\lambda \in (0,\epsilon)$, then $\widehat{F}_{x_1x_2}(x_1,x_2) > 0$ for each $(x_1,x_2) \in [0,1]^2$ while if $\lambda \in (-\epsilon, 0)$, then $\widehat{F}_{x_1x_2}(x_1,x_2) < 0$ for each $(x_1,x_2) \in [0,1]^2$.*

Proof. The first claim in the proposition follows directly from the specification of the function $\widehat{F}_{x_1x_2}$ given by (13). As for the second claim, take a given $(x_1,x_2) \in [0,1]^2$ and consider the function $h : [-1,1] \times \mathbb{R}_+ \to \mathbb{R}$ specified as

$$\begin{aligned}h(\lambda,\pi) :=&(1-\lambda)\left[x_1x_2q_1q_2(2-\pi)\lambda^2 + x_1q_1\lambda + (1-\pi)x_2q_2^2\lambda + q_2 - \pi q_1q_2\right] \\ &- (\lambda-\pi)\left[x_2q_2\lambda + q_1\right].\end{aligned}$$

Then, for each $\lambda \in (-1,1)$, we have

$$\widehat{F}_{x_1x_2} = \left[\frac{(1-\lambda)\tau_1\sigma}{1-\lambda^2 x_1x_2}\right]^2 \left[\frac{\lambda}{1-\lambda}\right] h(\lambda,\pi).$$

Also, we obtain

$$h(0,\pi) = q_2 - \pi q_1q_2 + \pi q_1 = 1.$$

The result follows since $h(0,\pi) > 0$, $\widehat{F}_{x_1x_2} = 0$ for $\lambda = 0$, $\widehat{F}_{x_1x_2}$ is a continuous function in λ, and $\lambda/(1-\lambda) > 0$ if $\lambda \in (0,1)$ while $\lambda/(1-\lambda) < 0$ if $\lambda \in (-1,0)$. □

Hence, in our model, the information acquisition choice has the same strategic motives as the actions when the degree of coordination is moderate, $\lambda \in (-\epsilon,\epsilon)$ for some $\epsilon > 0$. This is true for a class of games more general than a beauty contest game. The result above agrees with the main result in HV for the case with a continuum of players, provided that the degree of coordination is not too high.

However, the relation in motives obtained in Proposition 1 can be reversed when the degree of coordination is sufficiently high. Proposition 2 below gives us precisely this result. Our results are driven by the shape of the slope of the players' optimal actions. Of particular importance is the observation that the slope of a player's optimal action (in her private signal) is quite sensitive to the information choices of both players. In contrast, this role of a player's information choice on the sensitivity of any player's optimal action to her signal is crucially mitigated in a game with a continuum of players.

To illustrate the role of the influence of the players' information choices on the sensitivities of their optimal actions to their private signals, consider the case of complementary actions, $\lambda \in (0,1)$. Note that, using the expressions for m_1 and m_2 given by Lemma 1, one obtains

$$\frac{\partial m_2}{\partial x_2} = \tau_1 \left(\frac{\sigma}{\gamma}\right) \frac{(1-\lambda)(1+\lambda x_1)(x_2^{-1}+\lambda^2 x_1)x_2^{1/2}}{2(1-\lambda^2 x_1 x_2)^2},$$

$$\frac{\partial m_1}{\partial x_2} = \tau_1 \left(\frac{\sigma}{\gamma}\right) \frac{(1-\lambda)\lambda(1+\lambda x_1)x_1^{1/2}}{(1-\lambda^2 x_1 x_2)^2},$$

and

$$\left|\frac{\partial m_1}{\partial x_2} - \frac{\partial m_2}{\partial x_2}\right| = \tau_1 \left(\frac{\sigma}{\gamma}\right) \frac{(1-\lambda)(1+\lambda x_1)}{2x_2^{1/2}(1-\lambda^2 x_1 x_2)^2}\left[1-\lambda x_1^{1/2} x_2^{1/2}\right]^2.$$

When λ is close to zero, player 1 is mainly concerned about matching her action with the objective $\tau(\theta)$. This effect clearly makes valuable for her to increase x_1, irrespective of player 2's information choice. Also, even though λ is small, player 1 has also a motive for aligning her optimal action with that of player 2, at least to a certain extent, since $\lambda > 0$.

On the other hand, we see from the expression for $\partial m_2/\partial x_2$ above that an increase in x_2 causes a significative change in m_2 when λ is close to zero. Therefore, it is valuable for player 1 to change m_1 in such a way so as to respond to the variation in m_2. From the expression for $\partial m_1/\partial x_2$ above, we observe that m_1 changes already in the required direction due simply to the increase in x_2. However, this induced change is small since λ is close to zero. In other words, the difference $|\partial m_1/\partial x_2 - \partial m_2/\partial x_2|$ is relatively large in this case. In particular, note that

$$\lim_{\lambda \to 0^+} \left|\frac{\partial m_1}{\partial x_2} - \frac{\partial m_2}{\partial x_2}\right| = \tau_1 \left(\frac{\sigma}{\gamma}\right) \frac{1}{2x_2^{1/2}}.$$

As a consequence, to reach the required change in m_1, player 1 must also complement herself that change on m_1 induced by the increase in x_2. To do this, she needs to increase x_1 as well. Thus, we identify two effects which make player 1 wish to increase x_1 when player 2 increases x_2. This is the logic behind the result in Proposition 1.

Suppose now that λ is close to one. Then, m_2 approaches zero and player 2's optimal action approaches $\mathbb{E}[\tau(\tilde{\theta})] = \tau_0 + \tau_1\mu$. In other words, player 2 behaves as if she acquires no information at all. Furthermore, we see from the expression for $\partial m_2/\partial x_2$ above that any increase in x_2 causes almost no change in m_2. On the other hand, since λ is close to one, player 1 has little interest now in choosing an action close to the objective $\tau(\theta)$. She is mainly concerned about matching m_1 with m_2. One way of achieving this is to behave as if she does not acquire any information at all either. This effect makes it valuable for her to decrease x_1. In this case, note that

$$\lim_{\lambda\to 1^-}\left|\frac{\partial m_1}{\partial x_2}-\frac{\partial m_2}{\partial x_2}\right|=0,$$

so that, in the limit, player 1 does not need to increase x_1 for her optimal action to meet that of player 2. Furthermore, recall that, given our class of preferences, player 1 is risk-averse with respect to the difference of actions. In other words, she is risk-averse with respect to $|m_1-m_2|$. So, player 1 wishes to insure herself against the risk of m_1 deviating from m_2 and, furthermore, she knows that her optimal action gets close to the one chosen by player 2 when she acquires very little information. As a consequence, she finds valuable to reduce x_1 when player 2 increases x_2. This is the intuition behind the results in Proposition 2.

Proposition 2. *Assume 1 and 2. Then,*

(i) for each $\pi\in[0,1)$ there exists some $\kappa_\pi\in(0,1)$ such that if $\lambda\in(\kappa_\pi,1)$, then $\widehat{F}_{x_1x_2}(x_1,x_2)<0$ for each $(x_1,x_2)\in(0,1)^2$,

(ii) for each $\pi\in[0,1)$ there exists some $\varepsilon_\pi,\delta_\pi>0$ and some $\kappa_\pi\in(-1,0)$ such that if $\lambda\in(-1,\kappa_\pi)$, then $\widehat{F}_{x_1x_2}(x_1,x_2)>0$ for each $0\le x_1<\varepsilon_\pi$ and each $(1-\delta_\pi)<x_2\le 1$,

(iii) for the beauty contest game given by the payoff function in (1), there exists some $\kappa\in(0,1)$ such that if $\lambda\in(\kappa,1)$, then $\widehat{F}^{bc}_{x_1x_2}(x_1,x_2)<0$ for each $(x_1,x_2)\in(0,1)^2$.

Proof. Take a given $(x_1,x_2)\in(0,1)^2$ and consider the function $h:[-1,1]\times\mathbb{R}_+\to\mathbb{R}$ specified as

$$\begin{aligned}h(\lambda,\pi):=&(1-\lambda)\left[x_1x_2q_1q_2(2-\pi)\lambda^2+x_1q_1\lambda+(1-\pi)x_2q_2^2\lambda+q_2-\pi q_1q_2\right]\\&-(\lambda-\pi)\left[x_2q_2\lambda+q_1\right].\end{aligned}$$

Then, we have

$$\widehat{F}_{x_1x_2}=\left[\frac{(1-\lambda)\tau_1\sigma}{1-\lambda^2x_1x_2}\right]^2\left[\frac{\lambda}{1-\lambda}\right]h(\lambda,\pi)$$

for $\lambda\in(-1,1)$. It can be checked that $h(0,\pi)=1$ for each $\pi\in[0,1)$.

(i) Take a given $\pi\in[0,1)$. Then,

$$\begin{aligned}h(1,\pi)&=(\pi-1)[x_2q_2+q_1]\\&=(\pi-1)\left[\frac{x_2(1+x_1)+(1+x_2)}{1-x_1x_2}\right]<0.\end{aligned}$$

Since $h(0,\pi)>0$, $h(1,\pi)<0$, and $h(\cdot,\pi)$ is continuous in λ, there is some $\kappa_\pi\in(0,1)$ such that $h(\lambda,\pi)<0$ for each $\lambda\in(\kappa_\pi,1)$. The result follows since $\lambda/(1-\lambda)>0$ for each $\lambda\in(0,1)$.

(ii) Take a given $\pi \in [0,1)$, and consider $x_1 = 0$ and $x_2 = 1$. Then,

$$h(-1,\pi) = 2\left[-(1-\pi)q_2^2 + q_2 - \pi q_1\right] - (-1-\pi)\left[-q_2 + q_1\right] = \pi - 1 < 0.$$

Since $h(0,\pi) > 0$, $h(-1,\pi) < 0$ for $x_1 = 0$ and $x_2 = 1$, and $h(\cdot,\pi)$ is continuous in λ, in x_1, and in x_2, then there is some $\varepsilon_\pi, \delta_\pi > 0$ and some $\kappa_\pi \in (-1,0)$ such that $h(\lambda,\pi) < 0$ for each $\lambda \in (-1,\kappa_\pi)$, each $0 \leq x_1 < \varepsilon_\pi$, and each $(1-\delta_\pi) < x_2 \leq 1$. The result follows since $\lambda/(1-\lambda) < 0$ for each $\lambda \in (-1,0)$.

(iii) Take a given $(x_1, x_2) \in (0,1)^2$ and consider the function $g : [-1,1] \to \mathbb{R}$ specified as

$$g(\lambda) = -x_1x_2q_1q_2\lambda^4 - x_2q_2^2\lambda^3 + (2x_1x_2q_1q_2 - x_2q_2 - q_1q_2)\lambda^2 + (x_1q_1 + x_2q_2^2 - q_1)\lambda + q_2.$$

Then, we have

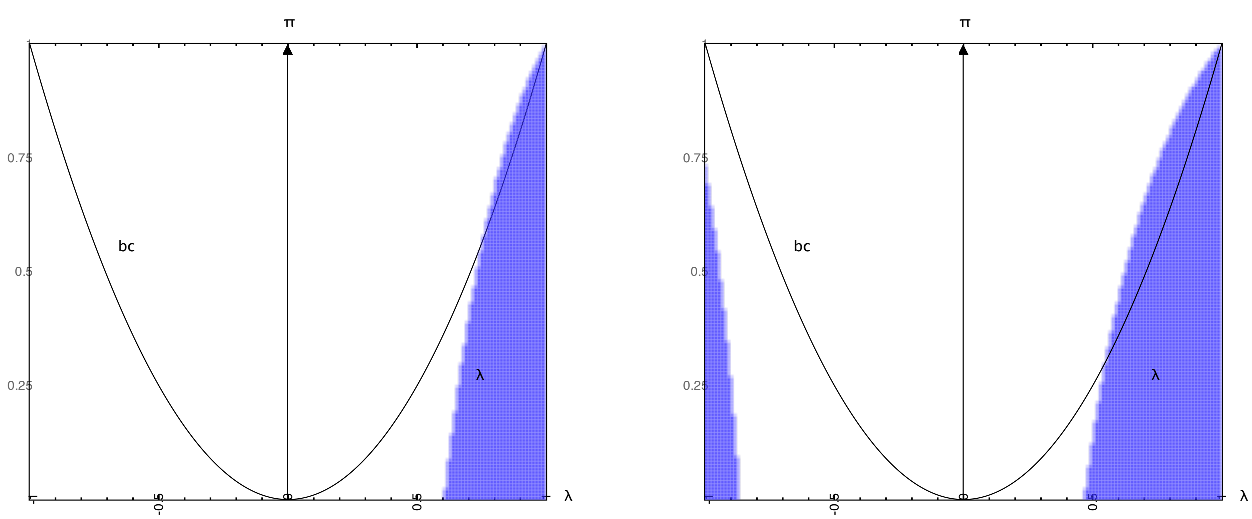
$$\widehat{F}^{\text{bc}}_{x_1x_2} = \lambda\left[\frac{(1-\lambda)\sigma}{1-\lambda^2x_1x_2}\right]^2 g(\lambda),$$

so that the sign of $\widehat{F}^{\text{bc}}_{x_1x_2}$ coincides with the sign of g for each $\lambda \in (0,1)$. It can be checked that

$$\begin{aligned} g(1) &= (x_1x_2 - 1)q_1q_2 + (x_1 - 1)q_1 - (x_2 - 1)q_2 \\ &= \frac{(x_1 - 1) - (x_1 + 3)x_2}{1 - x_1x_2} < 0. \end{aligned}$$

The result follows since $g(0) > 0$, $g(1) < 0$ and g is continuous in λ. $\square$

Proposition 2 (i) says that information choices are substitutes if the degree of complementarity in actions is sufficiently high. Also, starting from a situation in which a given player i acquires little amount of information while the other player acquires a large amount of information, a sufficiently high level of substitutability in actions implies that player i's information choice turn to complement that of the other player.



(a) $x_1 = x_2 = 0.5$ (b) $x_1 = 0.1$, $x_2 = 0.9$

Figure 1. The shadowed region indicates the values for (λ, π) such that the information choice does not

have the same coordination motives as the action choice. The line bc is $\pi = \lambda^2$ and corresponds to the beauty contest game.

The result provided by Proposition 2 (iii) contrast sharply the main result in HV for a game with a continuum of players. In particular, the result in Proposition 2 (iii) is restricted to the class of beauty contest games considered by HV. We obtain that information acquisition choices are substitutes if the degree of complementarity in actions is high enough.

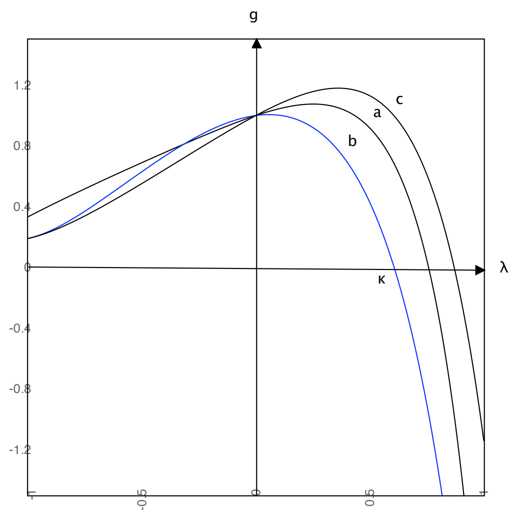


Figure 2. Graph of the function g used in the proof of Proposition 2 (iii) for (a) $x_1 = 0.5$, $x_2 = 0.5$, (b) $x_1 = 0.2$, $x_2 = 0.8$, and (c) $x_1 = 0.8$, $x_2 = 0.2$. The parameter κ identified in Proposition 1 is displayed for case (b).

Most of our results hold regardless the external effect captured by parameter π. However, this external effect plays also an important role in the analysis of the question studied in this paper. Notice that the shape of the shadowed region in Figure 1 seems to suggest that the discrepancy between the strategic motives in information and in actions (i) is facilitated when the external effect decreases and (ii) disappears when the external effect is high enough so that $\pi = 1$.

4. Applications

From the results provided by Proposition 2, one concludes that heterogeneous beliefs maybe sustained endogenously in settings where the degree of complementarity or substitutability is high enough. Presumably, a wide class of models might meet the conditions leading to the results in Proposition 2. To illustrate this, we discuss in this section some strategic settings where the results in Proposition 2 could apply.

4.1. *Investment Complementarities*

Consider a model of production externalities where a_i is interpreted as the amount of investment chosen by investor i. The payoff function for investor $i = 1$ is given by

$$U(a_1, a_2, \theta) = R(a_2, \theta)a_1 - c(a_1),$$

where $c(a_1)$ is a twice-differentiable cost function and $R(a_2, \theta)$ is a twice-differentiable return function that measures the externality to investor 1 caused by the adequacy of agent 2's investment with respect to the underlying state. Assume $c'' > 0$, $R_{a_2}, R_\theta > 0$, $R_{a_2 a_2} < 0$, and $R_{a_2}/c'' < 1$. Thus, we are considering that the investment externality has the form of a complementarity.

Using our results, one obtains that heterogenous beliefs will be endogenously sustained if the ratio R_{a_2}/c'' exceeds a certain threshold. Also, we have $\pi = -R_{a_2 a_2}/c''$ so that the crucial threshold decreases as $R_{a_2 a_2}$ increases. In other words, more concave return functions (with respect to the other investor's action) facilitate the persistence of heterogeneous beliefs.

4.2. *Cournot Duopoly*

Consider a model of Cournot competition where a_i is interpreted as the quantity of the good offered by firm i. The market price of the good is given by $P = d_0 + d_1\theta - d_2(a_1 + a_2)$ (with $d_0, d_1, d_2 > 0$) and the cost for firm $i = 1$ is given by $c_0 a_1 + c_1 a_1^2$ (with $c_0, c_1 > 0$). Then, the payoff function for firm $i = 1$ can be expressed as

$$U(a_1, a_2, \theta) = \big[(d_0 - c_0) + d_1\theta - (d_2 + c_1)a_1 - d_2 a_2\big] a_1.$$

Here we have $\lambda = -d_2/2(d_2 + c_1) < 0$ so that actions are substitutes. The assumptions of our model are satisfied since $c_1 > 0$ implies $\lambda > -1$. Also, we have $\pi = 0$. Using our results, we obtain that, in this setting, it is very unlikely that heterogeneous beliefs prevail. To obtain the result in Proposition 2 (ii), one must start from an initial situation where firm 1 acquires little amount of information while firm 2 is very well informed. Furthermore, one needs that λ be close enough to -1. However, we see that $-1/2$ is the lowest value that λ can achieve in this duopoly.

4.3. *Bertrand Duopoly*

Consider a model of Bertrand competition with heterogenous goods where a_i is interpreted as the price set by firm i. The market demand for firm $i = 1$ is given by $Q_1 = e_0 + e_1\theta - e_2(a_1 - a_2)$ (with $e_0, e_1, e_2 > 0$) and its cost is given by $c_0 Q_1 + c_1 Q_1^2$ (with $c_0, c_1 > 0$). Then, the payoff function for firm $i = 1$ can be expressed as

$$U(a_1, a_2, \theta) = \big[e_0 + e_1\theta - e_2(a_1 - a_2)\big]\big[a_1 - c_0 - c_1\big(e_0 + e_1\theta - e_2(a_1 - a_2)\big)\big].$$

We obtain $\lambda = (1 + 2c_1e_2)/2(1 + c_1e_2) \in (0, 1)$ so that actions are complementary. We also have $\pi = c_1e_2/(1 + c_1e_2)$. Therefore, we see that the product c_1e_2 is the key parameter to analyze whether heterogenous beliefs will be sustained endogenously. For this to happen, our results tell us that c_1e_2 must be sufficiently high but below a certain bound. As c_1e_2 increases, both λ and π increase and get closer to one. Thus, the result in Proposition 2 (i) will be obtained when c_1e_2 is high enough, so that λ exceeds the required threshold, but not too high so as to avoid that π gets too close to one. We see that in this Bertrand model, complementarity in pricing decisions and the external effect have offsetting implications on the strategic interaction in the information choice.

5. Concluding Remarks

This paper investigated the relation between endogenous coordination in information acquisition and incentives in actions for a tractable class of games with complementarity or substitutability in actions, externalities, and a fairly general information structure.

Our analysis highlighted the differences in the nature of the interactions when the set of players is finite with respect to the case with a continuum of players. From a methodological viewpoint, keeping track of the particular higher-order beliefs of the players through a knowledge index leads to conclusions different to those obtained by using an average expectation operation.

Our restriction to two-player games is dictated by the need of tractability. With a larger number of players, computing the required inverse of matrix $[I - \lambda\Omega]$ is exceedingly challenging and one must resort to computational numerical methods. This inverse is a crucial ingredient in the slope of a player's optimal action with respect to her private signal. However, the form of the inverse of matrix $[I - \lambda\Omega]$ is not affected by increasing the number of players. Neither is affected the form of ratio between polynomial functions (of parameter λ) of the slope of a player's optimal action. Therefore, our results continue to hold qualitatively so long as the number of players is finite.

As the number of players in our game increase, our results converge to the main result obtained by HV and, therefore, the information choice tend to inherit the same motives as the action choice. This follows simply from the fact that the higher-order average expectation operators approximate the average of higher-order expectations when the number of players tends to infinity.

References

Allen, B. (1983): "Neighboring Information and Distributions of Agents' Characteristics Under Uncertainty," *Journal of Mathematical Economics*, 12, 63-101.

Allen, B. (1986): "The Demand for (Differentiated) Information," *Review of Economic Studies*, 23, 311-323.

Angeletos, G.-M., and A. Pavan (2007): "Efficient Use of Information and Social Value of Information," *Econometrica*, 75, 4, 1103-1142.

Bergemann, D., and J. Välimäki (2002): "Information Acquisition and Efficient Mechanism Design," *Econometrica*, 70, 3, 1007-1033.

Calvó-Armengol, A., and J. de Martí (2007): "Communication Networks: Knowledge and Decisions," *American Economic Review*, 97, 2, 86-91.

Calvó-Armengol, A., and J. de Martí (2009): "Information Gathering in Organizations: Equilibrium, Welfare, and Optimal Network Structure ," *Journal of the European Economic Association*, 7, 1, 116-161.

Calvó-Armengol, A., de Martí, J. and A. Prat (2009): "Endogenous Communication in Complex Organizations," mimeo.

Cooper, R., and A. John (1988): "Coordinating Coordination Failures in Keynesian Models," *Quarterly Journal of Economics*, 103, 441-463.

Cornand, C., and F. Heinemann (2008): "Optimal Degree of Public Information Dissemination," *The Economic Journal*, 118, 528, 718-742.

Hellwig, C. (2005): "Heterogeneous Information and the Welfare Effects of Public Information Disclosures," UCLA Working Paper.

Hellwig, C., and L. Veldkamp (2009): "Knowing what Others Know: Coordination Motives in Information Acquisition," *Review of Economic Studies*, 76, 1, 223-251.

Morris, S., and H. S. Shin (2002): "Social Value of Public Information," *American Economic Review*, 92, 5, 1521-1534.

Morris, S., and H. S. Shin (2005): "Central Bank Transparency and the Signal Value of Prices," *Brooking Papers of Economic Activity*, 2, 1-66.

Radner, R. (1962): "Team Decision Problems," *Annals of Mathematical Statistics*, 33, 857-881.

Novedades

DIVISIÓN DE ADMINISTRACIÓN PÚBLICA

María del Carmen Pardo, *Los mecanismos de rendición de cuentas en el ámbito ejecutivo de gobierno*, DTAP-245

Sergio Cárdenas, Separados y desiguales: Las escuelas de doble turno en México, DTAP-244

Sergio Cárdenas, *Obstáculos para la calidad y la equidad: La corrupción en los sistemas educativos*, DTAP-243

Sergio Cárdenas, Ignacio Lozano, Miguel Torres y Katsumi Yamaguchi, *Identificando beneficiarios de programas gubernamentales*, DTAP-242

Ma. Amparo Casar, Ignacio Marván y Khemvirg Puente, *La rendición de cuentas y el poder legislativo*, DTAP-241

Lizbeth Herrera y José Ramón Gil García, *Implementación del e-gobierno en México*, DTAP-240

Laura Sour, *Gender Equity, Enforcement Spending and Tax Compliance in Mexico*, DTAP-239

Laura Sour y Fredy Girón, *Electoral Competition and the Flypaper Effect in Mexican Local Governments*, DTAP-238

Ma. Amparo Casar, *La otra reforma*, DTAP-237

Judith Mariscal y Federico Kuhlmann, *Effective Regulation in Latin American Countries. The cases of Chile, Mexico and Peru*, DTAP-236

DIVISIÓN DE ECONOMÍA

Alejandro López, *Poverty and Commercialization of Non-timber Forest Products*, DTE-486

Alejandro López et al., *Natural Resource Dependence in Rural Mexico*, DTE-485

Fausto Hernández, *Obstáculos al desarrollo del sistema financiero en México*, DTE-484

Rodolfo Cermeño y Benjamín Oliva, *Incertidumbre, crecimiento del producto, inflación y depreciación cambiaria en México*, DTE-483

Kurt Unger, *Mercado y autoconsumo. Vocación agropecuaria de los municipios de Guanajuato*, DTE-482

David Mayer, *Divergences and Convergences in Human Development*, DTE-481

Arturo Antón y Fausto Hernández, *VAT Collection and Social Security Contributions under Tax Evasion: Is There a Link*?, DTE-480

Eric Zenón y Juan Rosellón, *Expansión de las redes de transmisión eléctrica en Norteamérica: Teoría y aplicaciones*, DTE-479

María José Roa, *Racionalidad, uso de información y decisiones financieras*, DTE-478

Alexander Elbittar y Sonia Di Giannatale, *King Solomon's Dilemma: An Experimental Study on Implementation*, DTE-477

División de Estudios Internacionales

Irina Alberro and J. Schiavon, *Shaping or Constraining Foreign Policy?*, DTEI-202
Jorge Schiavon, *La diplomacia local de los gobiernos estatales en México (2000-2010)*, DTEI-201
Luis Fernández y J. Schiavon, *La coordinación en la política exterior de Brasil y México*, DTEI-200
Alejandro Anaya, *Internalización de las normas internacionales de derechos humanos en México*, DTEI-199
Rafael Velázquez y Karen Marín, *Política exterior y diplomacia parlamentaria: El caso de los puntos de acuerdo durante la LX Legislatura*, DTEI-198
Jorge Schiavon y Rafael Velázquez, *La creciente incidencia de la opinión pública en la política exterior de México: Teoría y realidad*, DTEI-197
Jorge Chabat, *La respuesta del gobierno de Calderón al desafío del narcotráfico: Entre lo malo y lo peor*, DTEI-196
Jorge Chabat, *La Iniciativa Mérida y la relación México-Estados Unidos*, DTEI-195
Farid Kahhat y Carlos E. Pérez, *El Perú, Las Américas y el Mundo*, DTEI-194
Jorge Chabat, *El narcotráfico en las relaciones México-Estados Unidos*, DTEI-193
Jorge Schiavon y Rafael Velázquez, *La creciente incidencia de la opinión pública en la política exterior de México: Teoría y realidad*, DTEI-197
Rafael Velázquez y Karen Marín, *Política exterior y diplomacia parlamentaria: El caso de los puntos de acuerdo durante la LX Legislatura*, DTEI-198
Alejandro Anaya, *Internalización de las normas internacionales de derechos humanos en México*, DTEI-199

División de Estudios Jurídicos

Gustavo Fondevila, *Estudio de percepción de magistrados del servicio de administración de justicia familiar en el Distrito Federal*, DTEJ-47
Jimena Moreno, Xiao Recio Blanco y Cynthia Michel, *La conservación del acuario del mundo*, DTEJ-46
Gustavo Fondevila, *"Madrinas" en el cine. Informantes y parapolicías en México*, DTEJ-45
María Mercedes Albornoz, *Utilidad y problemas actuales del crédito documentario*, DTEJ-44
Carlos Elizondo y Ana Laura Magaloni, *La forma es fondo. Cómo se nombran y cómo deciden los ministros de la Suprema Corte de Justicia de la Nación*, DTEJ-43
Ana Laura Magaloni, *El ministerio público desde adentro: Rutinas y métodos de trabajo en las agencias del MP*, DTEJ-42
José Antonio Caballero, *La estructura de la rendición de cuentas en México: Los poderes judiciales*, DTEJ-41
Marcelo Bergman, *Procuración de justicia en las entidades federativas. La eficacia del gasto fiscal de las Procuradurías Estatales*, DTEJ-40
Ana Elena Fierro, *Transparencia: Herramienta de la justicia*, DTEJ-39
Ana Elena Fierro y Adriana García, *¿Cómo sancionar a un servidor público del Distrito Federal y no morir en el intento?*, DTEJ-38

División de Estudios Políticos

Andreas Schedler, *The Limits to Bureaucratic Measurement. Observation and Judgment in Comparative Political Data Development*, DTEP-224

Andrea Pozas and Julio Ríos, *Constituted Powers in Constitution-Making Processes. Supreme Court Judges, Constitutional Reform and the Design of Judicial Councils*, DTEP-223

Andreas Schedler, *Transitions from Electoral Authoritarianism*, DTEP-222

María de la Luz Inclán, *A Preliminar Study on Pro and Counter Zapatista Protests*, DTEP-221

José Antonio Crespo, *México 2009: Abstención, voto nulo y triunfo del PRI*, DTEP-220

Andreas Schedler, *Concept Formation in Political Science*, DTEP-219

Ignacio Marván, *La revolución mexicana y la organización política de México. La cuestión del equilibrio de poderes, 1908-1932*, DTEP-218

Francisco Javier Aparicio y Joy Langston, *Committee Leadership Selection without Seniority: The Mexican Case*, DTEP-217

Julio Ríos Figueroa, *Institutions for Constitutional Justice in Latin America*, DTEP-216

Andreas Schedler, *The New Institutionalism in the Study of Authoritarian Regimes*, DTEP-215

División de Historia

Sergio Visacovsky, *"Hasta la próxima crisis". Historia cíclica, virtudes genealógicas y la identidad de clase media entre los afectados por la debacle financiera en la Argentina (2001-2002)*, DTH-68

Rafael Rojas, *El debate de la Independencia. Opinión pública y guerra civil en México (1808-1830)*, DTH-67

Michael Sauter, *The Liminality of Man: Astronomy and the Birth of Anthropology in the Eighteenth Century*, DTH-66

Ugo Pipitone, *Criminalidad organizada e instituciones. El caso siciliano*, DTH-65

Ugo Pipitone, *Kerala, desarrollo y descentralización*, DTH-64

Jean Meyer, *Historia y ficción, hechos y quimeras*, DTH-63

Luis Medina, *La Comanchería*, DTH-62

Luis Medina, *La organización de la Guardia Nacional en Nuevo León*, DTH-61

Luis Medina, *El Plan de Monterrey de 1855: un pronunciamiento regionalista en México*, DTH-60

Mónica Judith Sánchez, *Liberal Multiculturalism and the Problems of Difference in the Canadian Experience*, DTH-59

Ventas

El **CIDE** es una institución de educación superior especializada particularmente en las disciplinas de Economía, Administración Pública, Estudios Internacionales, Estudios Políticos, Historia y Estudios Jurídicos. El Centro publica, como producto del ejercicio intelectual de sus investigadores, **libros, documentos de trabajo,** y **cuatro revistas especializadas:** *Gestión y Política Pública*, *Política y Gobierno*, *Economía Mexicana Nueva Época* e *Istor*.

Para adquirir cualquiera de estas publicaciones, le ofrecemos las siguientes opciones:

VENTAS DIRECTAS:	VENTAS EN LÍNEA:
Tel. Directo: 5081-4003 Tel: 5727-9800 Ext. 6094 y 6091 Fax: 5727 9800 Ext. 6314 Av. Constituyentes 1046, 1er piso, Col. Lomas Altas, Del. Álvaro Obregón, 11950, México, D.F.	Librería virtual: **www.e-cide.com** Dudas y comentarios: **publicaciones@cide.edu**

¡¡Colecciones completas!!

Adquiere los **CDs de las colecciones completas** de los documentos de trabajo de todas las divisiones académicas del CIDE: **Economía**, **Administración Pública**, **Estudios Internacionales**, **Estudios Políticos**, **Historia** y **Estudios Jurídicos.**






¡Nuevo! ¡¡Arma tu CD!!

Visita nuestra Librería Virtual www.e-cide.com y selecciona entre 10 y 20 documentos de trabajo. A partir de tu lista te enviaremos un CD con los documentos que elegiste.