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Federal against Local Governments: Political  
Accountability under Asymmetric  
Information

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## Abstract

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*We model a situation in which the voters are or not fooled by the local or/and the federal government, and their capability of accounting the behavior of the governments, when they are not fully informed about which are services/goods that each level have to provide, and hence they are not able to know which of the levels have failed, the federal or the local, if only one has failed. Also, we do not assume that the voters know the type of the parties, bad or good —willing to divert resources or not, roughly—. We propose two mechanisms, one in which the voters have information about the competencies, the other when not. Expectably, in the first situation from the mechanism it is possible to infer the types of all the parties, but in the second only in rare situations it reveals the types of all the parties. However, if there are good parties in both levels of governance, the second mechanism select two good parties. That is, there is accountability, although not that perfect as it is possible in the first situation. In the other situations, that is, if in only one level there are good parties, or in both levels all the parties are of bad type, the mechanism predicts the obvious result: Or it will be only one good party in office, not knowing which one is good and which one is bad, or it will be only bad parties in office. Strikingly enough, in some situations, the voters may also learn, through the mechanism, the competencies.*

## Resumen

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*Modelamos en este trabajo una situación en la cual los votantes son o no engañados por el gobierno, sea local o federal, y, dadas estas circunstancias, su capacidad para requerir rendición de cuentas por medio del voto, cuando los votantes no tienen información de el/los bien/es que son proveídos por cada uno de los niveles de gobierno.*

*Tampoco asumimos que los votantes conocen el tipo de los partidos, buenos o malos —dispuestos a no desviar recursos o dispuestos a ello—. Proponemos dos mecanismos de voto, uno en el que los votantes sí conocen las respectivas competencias, y otro en el que no. Como era de esperarse, en el primer mecanismo es posible, por medio del voto, inferir el tipo de los partidos, pero en el segundo, en general, sólo en raras circunstancias esto es posible. Sin embargo, si hay más de un partido de tipo bueno en cada nivel, esto es posible. En otras situaciones, esto es, si sólo en un nivel hay partidos de tipo bueno, o en ambos niveles sólo hay partidos de tipo malo, obtenemos el resultado obvio: Habrá siempre un partido bueno y uno malo en el gobierno, o bien habrá siempre partidos de tipo malo en el gobierno. Sorprendentemente, en algunas situaciones, los votantes, por medio del mecanismo pueden también inferir las respectivas competencias.*

# Federal against Local Governments: Political Accountability under Asymmetric Information

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## Abstract

We model a situation in which the voters are or not fooled by the local or/and the federal government, and their capability of accounting the behavior of the governments, when they are not fully informed about which are services/goods that each level have to provide, and hence they are not able to know which of the levels have failed, the federal or the local, if only one has failed. Also, we do not assume that the voters know the type of the parties, bad or good —willing to divert resources or not, roughly—. We propose two mechanisms, one in which the voters have information about the competencies, the other when not. Expectably, in the first situation from the mechanism it is possible to infer the types of all the parties, but in the second only in rare situations it reveals the types of all the parties. However, if there are good parties in both levels of governance, the second mechanism select two good parties. That is, there is accountability, although not that perfect as it is possible in the first situation. In the other situations, that is, if in only one level there are good parties, or in both levels all the parties are of bad type, the mechanism predicts the obvious result: Or it will be only one good party in office, not knowing which one is good and which one is bad, or it will be only bad parties

in office. Strikingly enough, in some situations, the voters may also learn, through the mechanism, the competencies.

## 1 Introduction

Decentralization reforms have been in place in many developing nations during the last decade. Tanzi (1996) argues that these reforms go hand by hand with democratization of political regimes where elections take place regularly. This recent push towards the greater use of decentralized provision of public goods has been built to a significant degree on the notion that having government closer to the people will lead to better governance.<sup>1</sup> Thus one of the key questions in choosing the tier of government at which certain goods and services should be provided is the extent to which government can be held accountable for its actions.

In turn, one fundamental element for accountability in a federation with multiple tiers of authorities, often regarded as an assumption in political agency models, is the citizens' exact knowledge about different responsibilities of each level of power. If they are not acquainted with them, then the alleged better governance may not take place.

To illustrate this briefly, suppose there is a two-tier government, two-public-good country. Federal government is supposed to produce good A, and the local one, good B. If voters do not recognize which good is supposed to be produced by each level of government, they may be punishing the wrong level of government through vote, distorting the usual incentives for good behavior while in office and, consequently, having government closer to people may not necessarily lead to better governance, "just because, among other reasons, accountability may not be possible and hence there may be room for easy corruption."

For convenience, political models for Federations have assumed that people are acquainted with the responsibility of different levels of government,<sup>2</sup> but at the end this is an empirical issue and it may not always be the case, especially in less developed countries, where the fiscal federalism reforms have been deficient in defining responsibilities for federal and local administrations, and education attainment is rather low. For these reasons in this

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<sup>1</sup>We owe this concept to Oates (1972).

<sup>2</sup>Even the famous Oates' decentralization theorem.

paper we relax the usual assumption of acquaintance with the duties of the federal and local authorities.

To further motivate our study we first conduct a national survey in a developing nation, Mexico, which is being decentralizing its public activities, to determine empirically whether people distinguishes responsibilities among different levels of government. It turned out that in this country citizens do not know who is responsible for the provision of distinct public goods and services. To say it once again, we argue that this fact damages political accountability and hence better governance.

“The principal contribution of this paper is to model that situation described in the precedent paragraphs which, as far as we know, has not been studied in the literature so far. Analogously to the Holmstrong’s result and, depending on who, strikingly or expectably, our model predicts the existence of an equilibrium that, in spite of the fact that the voters are ignorant in relation to which level of governance has failed, they can induce them (the governments) to not fail. The strategy is quite simple but effective: If only one of levels fails, the voters punish both levels and, consequently, none of the levels speculate with the fact that the voters do not know certainly which level has failed and logically they decide not to fail (theorem 2 below). Nevertheless, that strategy has a negative counterpart, which is that they are possibly taking off of office a very good government.” This trade-off —to face the risk of taking off a very good government or to accept a corrupt government at some level (federal or local) jointly with a good one in the other level—, is a very interesting question that, although very close to the subject of our work, deserves and needs a metha model of our model, a topic left for future research.””

With this result in mind, we then modeled this situation and compare it with the ideal one. Results from the model are interesting.. and contrast with the existing literature as...

This paper is organized as follows. Section 1 briefly reviews the literature, which in turn motivates the survey. Section 2 describes the survey’s results which stress that this feature has to be modeled. Section 3 models theoretically the situation where citizens are not aware of the tier of government responsible for producing a specific public good or service.

## 2 Literature Review and Motivation

Politics is about picking the right people as well as giving them the right incentives (BESLEY :::?). Elections have been identified in the literature as a fundamental vertical mechanism of accountability (or incentive), while separation of power an essential horizontal one (O'Donnell, 1991).

Modern democracies have introduced a mixed of both types of mechanisms to enhance political accountability and thus good governance. On the one hand, elections give voters some control over politicians who abuse their power, thus creating incentives for good behavior (BESLEY, MANIN, Przeworski, etc). On the other, separation of powers has been identified as a potential importance in generating representation<sup>3</sup> and in rendering accounts not only to citizens but also to one another.

Federations, for example, not only divide the power between legislative, executive and judicial power, but also between national and local governments. This last political division can be seen as a political multi-agency problem between citizens (principal) and government (multi-agent as there are at least two levels of government).

This phenomenon of many jurisdictions has not been studied extensively. Existing literature on this issue has concentrated in analyzing competition among different tiers of authorities. These are based on a pioneering work by Holmstrom (1982) for firms. He models a group of agents which are asked to perform tasks where the outcome depends on some unobserved variable which is common to all. As agents' effort is unobservable it becomes optimal to condition the incentives given to one agent on the outcomes achieved by others. BESLEY ??? suggests that this setting can be extended to sub-national governments as there is the possibility for voters to use policy making in other -horizontal- jurisdictions as a benchmark for policy makers in their own. Holmstrom mechanisms are applied by Besley and Case (1995) to the context of decentralized political competition where a number of local governments are being asked to perform a similar task against a backdrop of correlated private information<sup>4</sup>.

But with multiple jurisdictions voters face an additional informational

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<sup>3</sup>Perssons et. al. (1997) show that if the separation of powers is organized in a specific form, the government as a whole will be induced to reveal to citizens the true conditions under which it operates and that this information, in turn, will enable citizens to enforce representation through retrospective voting.

<sup>4</sup>Besley (???) surveys this literature of yardstick competition among local governments.

problem: not only do they have to observe and evaluate the outcome but also they need first to know exactly the distinct responsibilities a specific level of government has. This is an important issue because the big push toward the use of decentralized provision of public goods has been built to a significant degree on the notion that having government closer to the people will lead to better governance (Oates, 1972). Thus one of the key questions in choosing the tier of government at which certain goods and services should be provided is the extent to which government can be held accountable for its actions.

Therefore if citizens are not acquainted with the responsibilities of each tier of authority, they may be punishing the wrong politician. To illustrate this briefly, suppose there is a two-tier government, two-public-good country. Federal government is supposed to produce good A, and the local one, good B. If voters do not recognize which good is supposed to be produced by each level of government, they may be punishing the wrong level of government through vote, distorting the usual incentives for good behavior while in office and, consequently, having government closer to people may not necessarily lead to better governance.

Political agency models have assumed a unitary government, but modern democracies tend to have multiple-governments. This has not been addressed by the existing literature. This is the main contribution of this paper.

The importance of this is that if responsibilities among governments are undefined or citizens do not recognize them, then decentralization may even deter better governance and economic performance. So for elections in a multiple government environment to be an effective instrument for accountability, this problem has to be dealt with.

It may be said that the assumption of unitary government is a convenient one and that it does not affect the results. Our model suggests the contrary. Also it may be argued that the assumption is realistic as it is easy for voters to identify the correctly the assignment of each level of government. We argue that at least for less developed countries (LDCs) this is not the case. There are several arguments to suspect so. First, these countries have just recently turned into democracies and citizens may not be used to vote and even less to distinguish duties among different levels of government (CITAR PAPER). Second, hand by hand with democracy, decentralization movements have taken place; these are many times oriented by political pressures and have in fact undefined the assignments among the tiers of authorities as most public activities have been deconcentrated but the last responsibility remains at the



federal government level <sup>5</sup>. Finally, LDCs have a long tradition of low degree of educational attainment, which make it difficult even to know that there exist several authorities.

As said, political models, for simplicity, have assumed that people are acquainted with the responsibility of different levels of governments<sup>6</sup>, but at the end this is an empirical issue. For this reason, a survey on this was carried out. Next section describes its results briefly so as to motivate the formal political accountability model for multiple tiers of government.

### 3 Responsibility Knowledge Survey

To be able to determine whether voters distinguish duties of each tier of administration, a survey was carried out nationally in Mexico<sup>7</sup>. We chose this developing country because it is a Federation with three tiers of government, federal, state and municipal orders; and, because a decentralization process has been under way since 1996

People were asked about different responsibilities such as the authority is in charge of building schools, provide health and primary education, national defense, design of poverty programs, public safety in streets, recollecting garbage, lightening in streets; and, on the other hand, authority in charge of collecting different taxes such as corporate and personal income, property, VAT and excises.

On average, only 6 percent of citizens know the right assignment of each level of government. What is more, 32 per cent did not know there exists three tiers of government in Mexico. On the revenue side, 88 per cent of people think that all different taxes are levied by the federal government only. The survey is rich for analyzing different topics, as one can infer by level of income, level of education, region, etc <sup>8</sup>.

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<sup>5</sup>In many Latin American countries a change in responsibilities necessarily imply a modification of the Constitution, and this has not been the case in some countries, including Mexico (see Hernández and Iturribarría, 2004; IADB, 1997).

<sup>6</sup>In fact, these models assume unitary government, i.e., one level of government.

<sup>7</sup>For a complete description of the methodology, results and surver analysis, see Hernández and Torres (2004).

<sup>8</sup>An interesting result, for example, is that in education and health as the level of education increases, the acquaintance of the level of government in charge of the public service rises up to some level (college degree). Afterwards, it decreases since people do not

For this paper’s purposes the surveys suggest that citizens are not familiar with the level of government they should punish through vote when some public good has been under produced or its quality is low. In other words, voters may be punishing the wrong level of authority deterring the advantages of political accountability<sup>9</sup>. Hence, better governance through decentralization is not guaranteed.

Thus, based on this empirical result, next section builds a model where asymmetric information exists from the duties of authorities’ point of view.

## 4 The Models

In order to theoretically highlight the differences between a situation where the voters have perfect information (with respect to whom has failed, the federal or the local levels, but only one) with the situation in which they do not know who has failed when only one has failed, we present first the model for the former case. We will model both situations using the concept of an Extensive Game with Imperfect Information according to the presentation given in Osborne and Rubinstein (1998). For shortness, the game defined to describe the first situation will be called with perfect information —although, as the voters do not know the type of the parties, it does not have, making honor to the true, prefect information—, the other with imperfect information.

First, we will introduce the common ingredients of the models with perfect and imperfect information.

### 4.1 Common ingredients of the models with perfect and imperfect information

*The general set-up.*

First, we will give a semi informal view of the ingredients of the two games. Time is discrete and the horizon is infinite. In the economy, there will

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care about the quality of the service because they sent their children to a private school and receive their health care from private hospitals.

<sup>9</sup>Political accountability has been attributed to improve economic performance. First, because it....And, second, because it refers to the ability of voters to select the most able candidate, where ability can refer to some mix of integrity, technical expertise or other intrinsic features valued by voters at large. For a review, see Perssons and Tabellini (2004).

be  $n_F + n_L + 2$  actors, which we describe in what follows. We assume there are  $n_F$  federal parties,  $n_L$  local parties but, there will be two players by party: Each party may be of two types, *good or bad* — concepts that we will define below— Parties will be indexed by  $p \in P = \{1, \dots, n_F, n_F + 1, \dots, n\}$  ( $n_F > 1$ , or  $n > n_F + 1$ , otherwise there is no need of theory), so that if  $p \leq n_F$ ,  $p$  stands for a party in the federal level, otherwise it stands for a party in the local level; one of them is in charge of the federal government and another one is in charge of the local government. Another actor will be the Nature or Chance who will choose the types of the parties and, finally, the voters, who will be assumed as one agent, a fact that we justify below.<sup>10</sup> Either in the federal government or in the local government, the corresponding party faces, in each period, the possibility of doing what was promised when it was voted in the previous period, or not to do that.<sup>11</sup> In a given period, the parties are such that they are willing to divert resources, unless they care about their future, that is, unless they care about the possibility of being voted again in the next period, or later. During a period that a party in office is taking its action, it gets more utility (intertemporal, or instantaneous utility) if it does not take the promised action than if it takes the promised action. On the other hand, if it does not do what was promised, the voters may decide to take it out of office for the next period: Here is the heart of the trade-off faced by the parties. This language is only a way to encompass a variety of situations, as summarizing: 1) The party in charge directly diverts resources that, in the other case (if the resources are not diverted), go to the voters (society), resources that were promised to give them in the elections (when the corresponding government was elected); 2) any kind of action that was promised (in the elections) to be taken when in office, which when taken provokes instantaneous-utility to the voters, otherwise des-utility to them and additional utility to the party: That is, the bad action may be simply, a lie. Of course, not all the actions taken in a given level generates the same instantaneous-utility. However, for simplicity, we assume that the two possible actions (to do what was promised, not to do that — to divert resources or not, to lie or not—) can be measured in monetary current

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<sup>10</sup>We follow, in this sense, the same assumption as in Pearson, Tabellini and Roland (1997), in which work they assume the voters as one player. Any how, we have our own arguments in order to justify the assumption of a representative voter. See the footnote ?

<sup>11</sup>Here a ‘period’ is referred to the time passed by between two elections. Later, when formalizing the model, for technical and expository reasons, will be used another indexation. See, in the section 2, the definition of the set of histories.

benefits that a party may get in each period, which will be the same all the periods, as follows. *Legally*, if the party does not divert resources and it is in the federal level, it gets  $0 < x_{nd}^F$  (think of salaries of the politicians), and a quantity that it gets if it diverts resources will be  $x_d^F$ , with  $x_{nd}^F < x_d^F$ , and similarly for the local level, we assume  $0 < x_{nd}^L < x_d^L$ . That is, a party that diverts resources, gets more current benefits than in the case when it does not divert resources. Nevertheless, all those quantities are known only by the parties, so the voters are unable to anticipate the possible actions of the parties, they do not know the parties' utilities: The voters only know if a party in office provokes or not des-utility, not the exact quantity diverted, if it were the case. We want to make explicit a fact that is implicit in the previous words: We suppose that to do or not to do what was promised is only a question of will from the part of the governments, not other factor can make them to fail. Now, as the principal aim of this paper is to model a situation in which the voters are or not fooled by the local or/and the federal government, and their capability of accounting the behavior of the governments, we assume, without loss of generality, that there is no *conflict of voters' intertemporal preferences*, that is, we suppose, roughly speaking, that they prefer, in a given period, not to be fooled than to be fooled. They cannot do anything to correct the behavior of the parties in the current period, but, if they are fooled, they may take the corresponding party in charge out of office for the next period, by means of the elections.<sup>12</sup> Therefore, besides the parties and the Nature, the other actor of the economy will be denoted by  $V$ , a representative voter, let's say.<sup>13</sup> There is imperfect information in

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<sup>12</sup>Considerations in relation to the judicial power and its concomitants, the policy and the prison systems, and its connection with the accountability issue are left out of the model. Our intuition claims: Those ingredients should not modified substantially the results of our model, provided that those elements do not function perfectly, a reasonable assumption in almost every country. But, on the other hand, if introduced, certainly it would complicate over the top the exposition.

<sup>13</sup>Of course, as it will be clear in the section 2, our model is not totally general, in the sense that we are assuming preferences, both for the voters as for the parties, that are additively time separable, with some corresponding discount factors. Also, in the case of the voters, we will assume that all of them have the same discount factor. But, on the other hand, the model is not that restrictive as the previous comment may suggest. We model the 'concerning about the future' by means of the discount factor for both, the parties and the voters. It may be done it in a more general way, but it would be less transparent. The representative voter may be thought as an 'ideal voter' in the sense of that it would be a totally rational voter who has some concern also on the future generations. Our approach, then, can be both thought as a first insight on what could be the shape of the equilibria

various senses in the two games we define, one with imperfect information because there are simultaneous moves and because the voters do not know the types of the parties, and one with imperfect information, because the voters are additionally not informed on who has failed, when only one of the parties has failed. From now on, as said it before, we will name the later as the game with imperfect information and the former as the game with perfect information. That the voters are imperfectly informed on the parties' types and on who has failed when both levels fails, are the key elements of the principal game we propose in order to model the problem in this paper studied. The reason is clear: If the voters were informed on the types of the parties in office, they would be able, in not a few reasonable circumstances— if reality, instead of theory, is in mind—, to predict with almost certainty, who is willing to fail and who is not and then no conflict of accountability would be present and thus no speculation is necessary —if theory is in mind, the voters are able to predict with certainty, as will be clear once the game be formally specified — , but if in addition the voters do not know who has failed when only one has failed, the situation is much worse, something intuitive and clearly reflected in our results. The cases when both parties fail or none of them fail do not entail conflict of accountability, clearly, so we suppose in that case that the voters know the node where they are going to move. On the contrary, the parties know the types of each other. That relies on the belief that the politicians use to have much better information about themselves than the rest of the society: Furthermore, and indeed a fact, if only one party has failed, the other one in office infers the type of the other. The timing of the game is then the following. First, the types of the parties are simultaneously chosen. That will formally be specified by introducing the Nature or Chance as a player, in a way that in each election period, we define a signaling game, not a Bayesian game, so that not exogenous distribution of types are defined. Then, the parties in office simultaneously move, that is, they take or not the promised action. The player  $V$  observes those actions and it decides which are the parties in office for the next period. In the following period, the Chance moves, then the parties in office —they do not necessarily coincide with the parties in office in the previous period—, once again, choose simultaneously their actions, then the player  $V$  observes those actions and decides again which are the parties for the next period,

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in a more general positive model, or as a normative or prescriptive model, from which it is possible to infer a good strategy, in order to obtain an efficient accounting.

and so on. Therefore, as was advanced at the beginning of this section, in the the economy there are  $n + 2$  actors, the player  $V$ , the player  $C$  and  $n$  parties. On the other hand, letting  $N$  be the set of players, then we will have  $N = \{(1, G), \dots, (n_F, G), \dots, (n, G), (1, B), \dots, (n_F, B), \dots, (n, B), V, C\}$ , with the convention that given a vector  $(i, G)$  with  $1 \leq i \leq n_F$  the player  $(i, G)$  is a party of the good type in the federal level, and if  $n_F < i \leq n$ , the player  $(i, G)$  is a party of the good type in the local level, and  $1 \leq i \leq n_F$  the player  $(i, B)$  is a party of the bad type in the federal level, and if  $n_F < i \leq n$  the player  $(i, B)$  is a party of the bad type in the local level. Roughly, a party will be of bad type if it has a very short vision of future, and consequently it is optimal for it to divert resources. Lastly, we remark a very important characteristic of our model: We do not neither enter in the details of how the voters decide, in a given period, which party enters the next period —among the good parties—, when they have decided to take the party in office off for the next period, nor we specify the possible actions' correlation between the federal and local levels. Let us explain. In a first place, to enter in those details would necessary make too narrow the scope of the paper — we should focus in a country-society in particular, with the need of a specification of the historical-political and in general social-historical characteristics of the country in question—. Secondary, fortunately enough, there is no need of that. We are able to send the main message of this paper, without entering in that details. Further, if there are two parties in each level, we provide a complete answer. Formal definitions follows.

*The Actions in each time indexed period*

Chance *The player C*

The player  $C$  chooses an action from the set  $A^C = \{G, B\} \times \{G, B\}$ . The first coordinate of a vector  $(f, l)$  stands for the type of the party in the federal level, the second is the type of the party in the local level.  $G$  means that the corresponding party is a good one,  $B$  means that it is of bad type. The definitions of good and bad will be given below, when we specify the preferences of the parties.

AP *The parties*

The parties in office choose an action from the set  $A^i = \{x_{nd}^i, x_d^i\}$  for  $i \in \{F, L\}$ . In a given indexed period, if a party takes  $x_{nd}^i$  in the level  $i \in \{F, L\}$ , then it has not decided to divert resources, getting then  $x_{nd}^i$  current money benefits; if it takes  $x_d^i$ , then it has decided to divert resources, getting then  $x_d^i$  current money benefits.

AV *The player V*

The representative voter chooses an action from the set  $A^V = \{1, \dots, n_F\} \times \{n_F + 1, \dots, n\}$ . The first coordinate  $x^1$  of a given  $x = (x^1, x^2) \in A^V$  is the decision of  $V$  about which party will be in the federal level in the next period of governance and the second is the decision of  $V$  about which party will be in the local level in the next period of governance. In a given indexed period, to take  $(i, j) \in A^V$ , means that  $V$  has decided that the party  $i$  will be in the federal level in the next period of governance and the party  $j$  will be in office in the local level in the next period of governance.

H *The set of histories*

Let denote by  $H$  the set of histories, which is defined as follows.  $H = \{\emptyset\} \cup (\cup_{t=0}^{\infty} H_t)$ , where  $H_0 = A^C$ , and defining  $\tilde{N}_i = \{4k + i \mid k \in \tilde{N}\}$  for  $i \in \{0, 1, 2, 3\}$  (where  $\tilde{N} = \{0, 1, 2, \dots\}$  is the set of the natural numbers), we inductively define

$$H_t = \begin{cases} H_{t-1} \times A^F & \text{if } t \in \tilde{N}_L \\ H_{t-1} \times A^L & \text{if } t \in \tilde{N}_V \\ H_{t-1} \times A^V & \text{if } t \in \tilde{N}_C \\ H_{t-1} \times A^C & \text{if } t \in \tilde{N}_F \end{cases} \quad \text{for } t \geq 1, \text{ where } (\tilde{N}_F, \tilde{N}_L, \tilde{N}_V, \tilde{N}_C) =$$

$(\tilde{N}_0, \tilde{N}_1, \tilde{N}_2, \tilde{N}_3)$ . Observe that this definition is suggesting that, the ‘elections’ are celebrated every three periods, and that, after the elections, first, moves the party in the federal level and then the party in the local level. These comments will become evident once we define the player function.

P *The player function*

We assume, without loss of generality, that at  $t = 1$  the party 1 is moving in the federal level and the party  $n_F + 1$  in the local level and that after the actions of  $V$ , moves the Chance, then moves the party in the federal level, then the party in the local level moves, and so on. Formally, let denote by  $P$  the function  $P : H \setminus Z \rightarrow N$  (where  $H \setminus Z$  is the set of finite histories, so that  $Z$  denotes the set of the terminal histories), which is given by  $P(\emptyset) = C$ ,  $P(h) = 1$  for  $h \in H_0$ ,  $P(h) = n_F + 1$  for  $h \in H_1$ , and  $P(h) = V$  for  $h \in H_2$ . Now, generalizing, for a given  $h = (a_l)_{l=0}^{l=t}$ , we have that

if  $t \in \tilde{N}_F$ , then  $P(h) = i$  if  $a_{t-1} = (i, j) \in A^V$

if  $t \in \tilde{N}_L$ , then  $P(h) = j$ , if  $a_{t-2} = (i, j)$  with  $(i, j) \in A^V$

if  $t \in \tilde{N}_V$ , then  $P(h) = V$  and

if  $t \in \tilde{N}_C$ , then  $P(h) = C$ .<sup>14</sup>

Now, as we said in the general description of the model, we will not enter on how the social-historical characteristics that may influence the voters' election, in new elections. This voting process, without doubt, entails complex social and probabilistic considerations that heavily depend on the country in question, and, as a consequence, it is a topic outside of this paper.

### *The preferences*

#### UP *The player V*

The instantaneous or intertemporal utility function is defined as follows. As we said it when defining the set of histories, the elections are celebrated every three time indexed periods. In view of this last convention, the instantaneous preference is naturally defined as a function of the moves or actions made by the players in the corresponding time indexed periods in between two elections. For example, consider a history  $h \in H_3$ . That is,  $h = (a_0, a_1, a_2, a_3) \in A^C \times A^F \times A^L \times A^V$ . Now, according to the our informal description of the preferences, we assume:  $(a_0, x_{nd}^F, x_{nd}^L, a_3) \succ (a_0, x_d^F, x_{nd}^L, a_3) \succeq (a_0, x_{nd}^F, x_d^L, a_3) \succ (a_0, x_d^F, x_d^L, a_3)$  for all  $(a_0, a_3) \in A^C \times A^V$ .<sup>15</sup> Observe that the conditions  $(a_0, x_{nd}, x_{nd}, a_3) \succ (a_0, x_d, x_{nd}, a_3)$  and  $(a_0, x_{nd}, x_d, a_3) \succ (a_0, x_d, x_d, a_3)$  mean that  $V$  strictly prefers to be fooled by no government, than to be fooled by only one level, and to be fooled by only one level than to be fooled by both levels, the federal and the local. The condition  $(a_0, x_d, x_{nd}, a_3) \succeq (a_0, x_{nd}, x_d, a_3)$  is consistent with our assumption that the money that a party may get is the same in both levels, the federal and the local. We will assume this preference in all periods of governance (the three time indexed periods). With these considerations in place, we define  $f^V : A^C \times A^F \times A^L \times A^V \rightarrow \Re$  given by  $f^V(a_0, a_1, a_2, a_3) = -(a_1 + a_2)$ , which represents our assumptions over the actions in each period of governance. Finally, for a given history  $h = (a_t)_{t=0}^{l=\infty} \in Z$  and  $\delta^V \in (0, 1)$ , we define

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<sup>14</sup>Notice that if  $t = 0$ , our assumptions are formally expressed as if  $a_{-1}^1 = 1$  and  $a_{-1}^2 = n_F + 1$ . Also observe, once again, that the order of the governments at the outset of the game is arbitrary, in the sense that we may assume, with only a notational change, that  $a_{-1}^1 = 2$  and  $a_{-1}^2 = n_F + 2$ , or any other convention.

<sup>15</sup>Here the symbol  $\succ$  stands for a preference relation (strictly preferred). ( $\succeq$  stands for the weakly preferred relation)



$U^V : Z \rightarrow \mathfrak{R}$  given by

$$U^V(h) = \sum_{t \in \tilde{N}_F} (\delta^V)^{\frac{t}{4}} f^V(a_t, a_{t+1}, a_{t+2}, a_{t+3}). \quad (\text{uv1})$$

UP *The parties*

Take  $t \in \tilde{N}_F$  and  $(a_t, a_{t+1}, a_{t+2}, a_{t+3}) \in A^C \times A^F \times A^L \times A^V$ . Now, consider the player in the federal level. We assume, then, that  $(a_t, x_d^F, a_{t+2}, a_{t+3}) \succ (a_t, x_{nd}^F, a_{t+2}, a_{t+3})$  for all  $(a_t, a_{t+2}, a_{t+3}) \in A^C \times A^L \times A^V$ . That is, as explained before, a party in governance prefers to divert resources than not to do so, if it does not care about the future. For a player in the local level, similarly, we assume that  $(a_t, a_{t+2}, x_d^L, a_{t+3}) \succ (a_t, a_{t+2}, x_{nd}^L, a_{t+3})$  for all  $(a_t, a_{t+2}, a_{t+3}) \in A^C \times A^L \times A^V$ . Define, then, the function  $f^F : A^C \times A^F \times A^L \times A^V \rightarrow \mathfrak{R}$  given by  $f^F(a_0, a_1, a_2, a_3) = a_1$ . Now, consider a party  $j \in \{1, \dots, n_F\}$  and define the following function,  $I^{jF} : Z \rightarrow \{(b_t)_{t \in \tilde{N}_F} | b_t \in \{0, 1\}\}$  given by  $I_t^{jF}((a_l)_{l=0}^{l=\infty}) = \begin{cases} 1 & \text{if } a_{t-1}^1 = j \\ 0 & \text{if } a_{t-1}^1 \neq j \end{cases}$  for all  $t \in \tilde{N}_F$ ,  $t > 1$ , where  $(a_l)_{l=0}^{l=\infty} \in Z$ . The function  $I^{jF}$  is simply such that, at a given period  $t \in \tilde{N}_F$ , it takes 1 if the party  $j$  is in office in the federal level, and zero if not. Therefore, the preference of the player  $j$  when it is in office in the federal level is defined by the utility function  $U^F : Z \rightarrow \mathfrak{R}$  given by

$$U^{jF}(h) = \sum_{t \in \tilde{N}_F} (\delta^{jF})^{\frac{t}{4}} f^F(a_t, a_{t+1}, a_{t+2}, a_{t+3}) I_t^{jF}(h), \quad (1)$$

where  $\delta^{jF} \in (0, 1)$  for all  $j \in \{1, \dots, n_F\}$ . Similarly, for  $j \in \{n_F + 1, \dots, n\}$  we define  $I^{jL} : Z \rightarrow \{(b_t)_{t \in \tilde{N}_F} | b_t \in \{0, 1\}\}$  given by  $I_t^{jL}((a_l)_{l=0}^{l=\infty}) = \begin{cases} 1 & \text{if and } a_{t-1}^2 = j \\ 0 & \text{if and } a_{t-1}^2 \neq j \end{cases}$  for all  $t \in \tilde{N}_F$ ,  $t > 2$ , and  $f^{jL} : A^F \times A^L \times A^V \rightarrow \mathfrak{R}$  given by  $f^{jL}(a_0, a_1, a_2, a_3) = a_2$  and, finally  $U^{jL} : Z \rightarrow \mathfrak{R}$  given by

$$U^{jL}(h) = \sum_{t \in \tilde{N}_F} (\delta^{jL})^{\frac{t}{4}} f^{jL}(a_t, a_{t+1}, a_{t+2}, a_3) I_t^{jL}(h), \quad (2)$$

where  $\delta^{jL} \in (0, 1)$  for all  $j \in \{n_F + 1, \dots, n\}$ .

As commented before, we have by assumption  $I_0^{1F}(h) = 1$  and  $I_0^{n_F+1L}(h) = n_F + 1$  and therefore,  $I_0^{fF}(h) = I_0^{lL}(h) = 0$  for all  $f \neq 1$  and  $l \neq n_F + 1$ .

### The information structure

#### IP The information partition of the parties

For a player  $j \in \{1, \dots, n_F, \dots, n\}$  we will have that, if  $h = (a_l)_{l=0}^{l=t}$  is such that  $t \in \tilde{N}_F$  and  $P(h) = j$ —that is, the player  $j$  is in office in the federal level—, then the information set containing  $h$  is a singleton (notice that the player  $C$  is who has moved previously). In other words, the corresponding player knows the move of the player that has moved previously. On the other hand, if  $t \in \tilde{N}_L$ , then  $h = (a_l)_{l=0}^{l=t}$  is such that  $a_t \in A^F$  (because the corresponding player that has moved previously is in office of the federal government, and the corresponding player that has to move is in office of the local government), and therefore the corresponding information set in question will contain two histories,  $(a_0, \dots, a_{t-1}, x_{nd}^F)_{l=0}^{l=t}$  and  $(a_0, \dots, a_{t-1}, x_d^F)_{l=0}^{l=t}$ . To say it again, the corresponding player does not know the move of the player that moved previously. Informally, the players in charge of the federal and local government are playing simultaneously. Formally, we define  $\tilde{I}^j(t, F) = \left\{ \bar{h} \subset H \mid \bar{h} = \left\{ (a_l)_{l=0}^{l=t} \right\} \text{ if } t \in \tilde{N}_F, \text{ and } a_{t-1}^1 = j \right\}$  with  $j \in \{1, \dots, n_F\}$ , and  $\tilde{I}^j(t, L) = \left\{ \bar{h} \subset H \mid \bar{h} = \left\{ (a_1, \dots, a_{t-1}, a) \mid a \in A^F \right\} \text{ if } t \in \tilde{N}_L, \text{ and } a_{t-2}^2 = j \right\}$  with  $j \in \{n_F + 1, \dots, n\}$ .  $\left\{ \left\{ \tilde{I}^j(t, F) \right\} \mid t \in N_F \right\}$  is then the information partition of the player  $j \in \{1, \dots, n_F\}$  and  $\left\{ \left\{ \tilde{I}^j(t, L) \right\} \mid t \in N_L \right\}$  is the information partition of a player  $j \in \{n_F + 1, \dots, n\}$ .

## 4.2 The Model with Perfect Information

#### IVp The information partition of the player $V$

For the player  $V$ , in this section, we assume that the unique imperfection in information is in relation to the types of the parties. The player  $V$  cannot distinguish between histories that differ in a movement of the player  $C$ . Formally, we define, for a given  $t \in \tilde{N}_V$ ,  $A_t = \left\{ (a_j)_{j \in \tilde{N}_C, j \leq t} \mid a_j \in A^V \right\}$ ,  $M_t = \left\{ l \in \tilde{N}_V \mid l < t \right\}$ , and let  $|M_t|$  be the cardinality of  $M_t$ , and finally, for a given  $\bar{a}_t = (\bar{a}_j) \in A_t$  and  $x_t = (x_l^t) \in (A^F \times A^L)^{|M_t|}$ , the set  $I(\bar{a}_t, x_t)$  as  $\left\{ (a_l)_{l=0}^{l=t} \in H \mid \begin{array}{l} (a_{l-1}, a_l) = x_l^t \text{ for } l \in M_t \\ (a_j)_{j \in A_t} = (\bar{a}_j)_{j \in A_t} \end{array} \right\}$ .

$\tilde{I}^V = \left\{ \left\{ I(\bar{a}_t, x_t) \right\} \mid \bar{a}_t \in A_t, x_t \in (A^F \times A^L)^{|M_t|}, t \in \tilde{N}_V \right\}$  is then the

information partition of the player  $V$ . That is, the player  $V$  knows everything but the moves of the player  $C$ .

### Strategies

The set of strategies for a player  $i \in N$  is given by  $S^i = \left\{ s : \tilde{I}^i \rightarrow A^i \right\}$ , where  $\tilde{I}^i$  and  $A^i$  are the the information partition and the set of actions of the player  $i$  respectively.

The model with perfect information is  $\left\langle N, H, P, \left( \tilde{I}^i \right)_{i \in N \setminus \{C\}}, \tilde{I}^{VI}, (\succeq_i)_{i \in N \setminus \{C\}} \right\rangle$ , where each  $\succeq_i$  is defined as above.

## 4.3 The model with imperfect information

The only thing that changes is the information partition of the player  $V$ .

IVi The only information that the player  $V$  has is its own action. Formally, we define  $2^{M_t} = \{m \mid m \subset M_t\}$ , and for  $x = (x^1, x^2) \in (A^F \times A^L)^{|M_t|}$  with  $x^1 \in (A^F \times A^L)^{|m_t|}$  and  $x^2 \in (A^F \times A^L)^{|M_t| - |m_t|}$ . Then we define,  $X^1(m_t) = \left\{ x \in (A^F \times A^L)^{|m_t|} \mid \begin{array}{l} x^l = (x_{nd}^F, x_d^L) \text{ or} \\ x^l = (x_d^F, x_{nd}^L) \end{array} \text{ for } l \in m_t \right\}$ ,  $X^2(m_t) = \left\{ x \in (A^F \times A^L)^{|M_t| - |m_t|} \mid \begin{array}{l} x^l \neq (x_{nd}^F, x_d^L) \text{ and} \\ x^l \neq (x_d^F, x_{nd}^L) \end{array} \text{ for } l \in M_t \setminus m_t \right\}$  and  $I(m_t, (\bar{a}_j), x^2) = \cup_{x^1 \in X^1(m_t)} I((\bar{a}_j), (x^1, x^2))$ . Observe that for a given  $(m_t, (\bar{a}_j), x^2)$  the set  $I(m_t, (\bar{a}_j), x^2)$  contains all the histories that, given the set  $m_t$ , the player  $V$  has moved according to  $(\bar{a}_j)$ , and the parties have moved according to  $x^2$ , outside of the set  $m_t$ . Now observe that for each  $(m_t, (\bar{a}_j), x^2)$  we have a different information set. Then, for a given  $m_t$  and  $(\bar{a}_j)$ , we define  $I(m_t, (\bar{a}_j)) = \{I(m_t, (\bar{a}_j), x^2) \mid x^2 \in X^2(m_t)\}$ , obtaining all the different information sets consistent with them. Then, for a given  $t$ ,  $I(t) = \{I(m_t, (\bar{a}_j)) \mid (\bar{a}_j) \in A_t, m_t \in 2^{M_t}\}$  is the set of all different information sets at time  $t$ . Obviously, then,  $\tilde{I}^{VI} = \left\{ I(t) \mid t \in \tilde{N}_V \right\}$  is the information partition of the player  $V$  in this case. The game then in the case of imperfect information is given by

$$\left\langle \bar{N}, H, P, \left( \tilde{I}^i \right)_{i \in N \setminus \{V, C\}}, \tilde{I}^{VI}, (\succeq_i)_{i \in N \setminus \{V, C\}} \right\rangle, \quad (3)$$

where each  $\succeq_i$  is defined as above.

The definition of strategies are the same as before.

A very important remark here is in order. Implicit in the information structure of the game with imperfect information defined above, is that the voters have conscience of their ignorance about the competencies of the two levels. A model including the possibility of that the voters have not conscience of their ignorance is left for future research. Our intuition is that a good model would predict that ‘any outcome is possible, and that the outcome is probabilistic.’

## 5 The results

First, we need the following classification: A party is of good type at a given moment  $t \in \tilde{N}$  in which it is in office if, given that the other party is deciding to never divert resources, its strategy specifies not to divert resources at that moment and for ever in the future; otherwise, it is of bad type at that moment. As we will see, a bad party will be such that it has a very small discount factor  $\beta^i$  or, informally, if it is very impatient or it has a very short vision of future. As said it before, to avoid to be restricted to a country in particular, we will define the strategies of the voters in a general way, so that, if there are good parties in both levels, they will be in office at some time in the future, but if there is more than one good party in each level, we will not specified which ones will be. The strategies that we propose make neither a serious look backward to the history nor a serious look forward to the future and, furthermore, it neither entails a high level of knowledge of mathematical tools, nor a high level of rationality in none of the possible meanings of that word. The ultimate need it is not a non credible collective rationality, but only a serious attitude against lies and corrupt politicians: Do not forgive a lie or a deviation from the law, and nothing else.

Now we come to the results. First, we will gave it informally.

Consider the model with perfect information with  $n_F > 1$  or  $n_L > 1$ . If the voters do not confirm a party unless it has not diverted resources, and do not put again a party in office unless there are no parties left to see its behavior, then as a best response the parties behave in a way such that a good party does not divert resources, and a bad party does, being this profile of strategies a Nash equilibrium. Given those strategies, the result represents simply that, if in a given level there are no parties of good type, no good parties will be in office for ever. On the other hand, if there is at least one, the

first good one elected at some period, will be confirmed for ever. Although very intuitive, the result entails the essence of the mechanism proposed in this paper: If the voters are willing to punish severely and systematically, if there are good parties, a good will be in office for ever, in a finite number of period lower than the number of parties are in the given level. It may seem at a first glance that the second result above is also a trivial one. Nothing so far to the correct interpretation of the issue under study: The voters do not know, *a priori*, the types of the parties, but, if they punish, in a finite number of periods, a good party will be in office, that is, they learn.

In order to formally express our first result, we define the following objects. Take  $h_t = (a_l)_{l=1}^t \in I(t) \in \tilde{I}^i$ , with  $i = P(I_t) \in \{1, \dots, n\}$ , then

$s^{(i,j)}(P)(I_t) = \begin{cases} x_{nd}^j & \text{if } j = G \\ x_d^j & \text{if } j = B \end{cases}$ . Now, given  $h_t = (a_l)_{l=1}^t \in I(t) \in \tilde{I}^V$  define

$D(F, h_t) = \left\{ a_l^1 \in \{1, \dots, n_F\} \mid l \in \tilde{N}_C, a_{l+2} = x_d^F \text{ and } 0 \leq l \leq t \right\}$  and  $D(L, h_t) =$

$\left\{ a_l^2 \in \{n_F + 1, \dots, n\} \mid l \in \tilde{N}_C, a_{l+3} = x_d^L \text{ and } 0 \leq l \leq t \right\}$ , then  $s^V(P)(I(t)) =$

$(i, j)$  if  $(a_{t-1}, a_t) = (x_{nd}^F, x_{nd}^L)$  and  $a_{t-3} = (i, j)$ ,  $s^V(P)(I(t)) = \begin{cases} (i, \tilde{j}) & \text{with } \tilde{j} \notin D(L, h_t) \text{ if } D(L, h_t) \\ (i, n_F + 1) & \text{if } D(L, h_t) = \{n_F + 1, \dots\} \end{cases}$

if  $(a_{t-1}, a_t) = (x_{nd}^F, x_d^L)$  and  $a_{t-3} = (i, j)$ ,  $s^V(P)(I(t)) = \begin{cases} (\tilde{i}, j) & \text{with } \tilde{i} \notin D(F, h_t) \text{ if } D(F, h_t) \neq \{1, \dots, n_F\} \\ (1, j) & \text{if } D(F, h_t) = \{1, \dots, n_F\} \end{cases}$

$(a_{t-1}, a_t) = (x_d^F, x_{nd}^L)$  and  $a_{t-3} = (i, j)$  and finally  $s^V(P)(I(t)) = \begin{cases} (\tilde{i}, \tilde{j}) & \text{with } (\tilde{i}, \tilde{j}) \notin D(F, h_t) \times D(L, h_t) \\ (1, \tilde{j}) & \text{with } \tilde{j} \notin D(L, h_t) \text{ if only } L \\ (i, n_F + 1) & \text{with } \tilde{i} \notin D(F, h_t) \text{ if } C \end{cases}$

if  $(a_{t-1}, a_t) = (x_d^F, x_d^L)$  and  $a_{t-3} = (i, j)$ .

Then we have the following:

**Theorem 1** Take the model  $\left\langle N, H, P, \left( \tilde{I}^i \right)_{i \in N \setminus \{C\}}, (\succeq_i)_{i \in N \setminus \{C\}} \right\rangle$ , and suppose the voters are patient enough, then any profile of the form

$((s^{(i,j)}(P))_{(i,j) \in N \setminus \{C, V\}}, s^V(P))$  is a Nash equilibrium. We have two cases:

1) If in a given level  $i \in \{F, L\}$ , there are no good parties, never enter a good party at the given level (trivial); 2) If there is at least one good party at a given level  $i \in \{F, L\}$ , the first good party chosen remains for ever in office from the moment it is elected, which may be at  $t = 0$  or  $t = T_i < n_i$ .

Three comments. Just for simplicity, we assume that the voters does not observe the history before the outset of the game. We may assume that the voters did not learn anything before the moment they are observing the

game, that is, before  $t = 0$ . This would be a consistent way to interpret the proposed strategy at  $t = 0$ . On the other hand, a way to include the history before the outset, is simply to observe the parties that has failed before  $t = 0$ , and do not consider them among the ones that can be selected for the next period. But, From when?, What if all were observed? to our taste, the simplest assumption is to think that the voters have not learned anything before  $t = 0$ .

The second comment is in order to highlight the significance of the first theorem of this paper. Notice that if there are good parties in a given level, that good party that is going to enter in office, will not speculate with the fact that the voters do not know its type, just because, if it does and it divert resources, it will be taken off of office for ever, if it does exist another good party, or at least for a number of periods not larger than the number of bad parties there are in the level.

The third comment is related to the patience of the voters. The point that we want to make is very important, although simple, because it contains all the flavor of the message of this paper: If  $V$  is too impatient, no learning and no punishment, and the consequent bad behavior of all the parties, good and bad ones, is a possible Nash equilibrium, a very intuitive outcome and, perhaps, easy to find in some countries' history in the real world, since systematic punishment to the bad parties does not seem to be the most common voting behavior. It seems that the voters take into account other things than the only fact that a party has diverted resources in the past, and forgive that bad behavior, reelecting again and again parties that had diverted resources or had lied again and again, perhaps having in mind that the persons in that parties in present times, are different of the ones in the past.

To illustrate the point more formally and sharply, suppose that  $\beta^V = 0$  and consider the following strategy of the player  $V$ :  $s^V(I_t) = (1, n_F + 1)$  for all  $I_t \in I^V$ , and consider the following strategies for the parties:  $s^{(i,j)}(0)(I_t) = x_d^j$ , for all  $h_t = (a_l)_{l=1}^t \in I(t) \in \tilde{I}^i$ , with  $i = P(I_t) \in \{1, \dots, n\}$  and all  $j \in \{G, B\}$ . Now suppose that all the parties are of bad type, but the party 1 and the party  $n_F + 1$ . Clearly, the profile  $((s^{(i,j)}(0))_{(i,j) \in N \setminus \{C, V\}}, s^V)$  is a Nash equilibrium, in which the unique two good parties are in office, but both are diverting resources for ever (the proof of this result is trivial). The only two good parties are in office for ever, but they divert resources for ever.

Now we come to the model with imperfect information, assuming that  $n_F > 1$  or  $n_L > 1$ .

The result is the following. The strategy of the voters. Take a history such that the actions of the last parties in the history are either that both has diverted resources or only one has, then the voters elect the first two parties that both have not diverted resources in the same period of governance, if there exist.

If not, we have various situations. The details are better understood once they are formalized, thus now we only give the general idea. If along the history all the parties have diverted resources and all were matched with all, the voters elect the first pair. If not, the voters elect the two parties that were the first ones elected such that only one diverted resources along the history, if all were matched with all. If not all were matched with all, they elect for the federal level the one that was first voted among all of them, if it exists, and for the local level the one that has the lower label among the ones that were not matched with the one selected for the federal level. If along the history all the parties voted were matched with all the parties of the local level, but some parties were matched with no parties of the local level, the voters elect the one that has the lower label among of them for the federal level, and the party  $n_F + 1$  for the local level.

Lastly, if the history is such that the last two parties both have not diverted resources, they are reelected.

The best response of the parties to that strategy is the following. A bad party type will always divert resources, and a good type, if it is the first one not diverting resources, does not divert resources, otherwise it does.

The process has various outcomes, depending upon the number of good parties that there are in each level. The simplest case is when in no level there are good parties: The process, similarly to the previous theorem, selects two bad parties to be in office for ever, the first ones. The other simple case is when there is at least a one good party in each level. The process select the first two good parties to be in office for ever. The last case is when in only one level there are good parties. In this last case the process selects the firsts two parties such that when in office only one of them diverted resources. Strikingly enough, in some cases, the voters not only learn the types of some parties and are able to select a good party if it exists, but also they can infer the competencies of each level, as the example below will show clearly.

The dark side of our mechanism is that good parties that know that they will not be in office behave badly. True, and that comment opens the fundamental questions: Is our mechanism the best one? Of course, first we

need to specify in what sense a mechanism is the best or a best. Let us say the best mechanism is a systematic strategy of the voters that select two good parties, if they exist, in the lower number of periods. We mean by systematic strategy, a mechanism that does not contemplate the possibility of choosing two good parties by chance. It is clear that, by chance, in only one shot that is possible. The second question: Is it possible to design a mechanism such that select two good parties, if there exists, but also such that any good party will not divert resources? For this last question we have some comments. The main point is suggested by the questions: If along a history there are two different pairs of parties that have not diverted resources, Which one will be selected? or, if the pairs are turned over an over, Which are the incentives for them to not divert resources? The possible answer to those dilemmas is the following: Just turns them period by period, so any good party will be in office again and again, but not period by period —period of governance, is clear—. Obviously, in that situation, for a party to be of good type, it must be more patient than in our original mechanism. But, that digression suggests heavily that there may be no a general definition of a best mechanism, because there is a strong trade off between to ask for a minimal number of periods for a good party to be selected, to ask for the least demanding degree of patience, or to ask for a mechanism such that a good party never divert resources. We leave these questions for future research.

Another question is, Do there exists a mechanism that will reveal the type of all the parties? In general, the answer to that question is no. Simply think of a situation in which in one level there are only good parties, and in the other there are only bad parties: Here, whatever the mechanism is prescribing, the outcome observed by the voters is always that only one party divert resources, thus no information is possible to learn by changing the parties in office.

Before we formalize the mechanism, let us present a simple example, although probably the most common situation in the real world, which will give us all the intuition behind the strategies proposed as a mechanism that not only forces to some parties to reveal its types, but also that, under some conditions, it permits to the voters to learn the competencies. Imagine two parties by level, and such that in the federal level there is only one good party, the party 1, and in the local level no good parties there are. To fix ideas, imagine that the federal level provides 1 banana in each period, and the local level provides 1 pound of Cafe. Now suppose that  $(a_1, a_2) =$



$(x_{nd}^F, x_d^L) = (0, 1)$ ,<sup>16</sup> that is, the party 1 in the federal level have not diverted resources, but the party 3 in the local level has diverted resources —following our convention in the notation of the previous section—. But recall that the voters do not observe the vector  $(1, 0)$ . They only know that the information set  $\{(1, 0), (0, 1)\}$  has occurred, that is, they know that they did not receive the banana, but they do not know which party has to provide the banana. Our strategy suggests to vote again the party 1 for the federal level —recall that we assume that before the outset of the game the voters did not learn anything—, but the party 4 for the local level, that is,  $s^V(\{(1, 0), (0, 1)\}) = (1, 4)$ . But again the party 1 does not divert resources, and the party 4 does. That is,  $(a_4, a_5) = (0, 1)$ , and again the voters observe the set  $\{(1, 0), (0, 1)\}$  and then  $s^V(a_1, \dots, a_5) = (2, 3)$ . Then, as 2 and 3 are of bad type, they divert resources, and therefore the voters learn that the parties 2 and 3 are of bad type. Therefore, they know that the party 1 is of good type. But the outcome when the parties 1 and 3 played simultaneously was 1 banana and 0 pounds of cafe, then the voters know that the party 1 is providing the bananas, so they also learn that the federal level is responsible of the provision of the bananas. On the other hand, if all the parties are of good type in both levels, the voters, by means of no mechanism, can learn the competencies, although they can learn the types. Anyhow, to learn the competencies of the levels is not the objective of a voting process: It is an educational issue.

Formally. Take  $h_t = (a_l)_{l=1}^t \in I(t) \in \tilde{I}^{VI}$ , then for  $i \in \{1, \dots, n_F\}$  we define the following objects,  $D_1(i, h_t) = \left\{ j \in \{n_F + 1, \dots, n\} \mid \exists l \in \tilde{N}_C, l \leq t, a_l = (i, j), \text{s.t. } (a_{l+2}, a_{l+3}) = \right.$  —notice that if  $D_1(i, h_t) \neq \emptyset$ , then the party  $i$  has been elected along the history  $h_t = (a_l)_{l=1}^t$ , that is, there exist  $l \leq t$  and  $j \in \{n_F + 1, \dots, n\}$ , such that  $a_l = (i, j)$ —,  $D_2(i, h_t) =$

$$\left\{ j \in \{n_F + 1, \dots, n\} \mid \exists l \in \tilde{N}_C, l \leq t, a_l = (i, j), (a_{l+2}, a_{l+3}) = (x_{nd}^F, x_{nd}^L) \right\}, D_3(i, h_t) = \left\{ j \in \{n_F + 1, \dots, n\} \mid \exists l \in \tilde{N}_C, l \leq t, a_l = (i, j), \text{s.t. } (a_{l+2}, a_{l+3}) \in \{(x_{nd}^F, x_d^L), (x_d^F, x_{nd}^L)\} \right\},$$

where  $|D_3(i, h_t)|$  denotes the cardinality of the set  $D_3(i, h_t)$  —as usual, we assume  $|D_3(i, h_t)| = 0$  if  $D_3(i, h_t) = \emptyset$ —,  $A(nd, d, h_t) = \arg \max \{|D_3(i, h_t)| \mid i \in \{1, \dots, n_F\}, 0 < |D_3(i, h_t)|\}$

Also, if  $A(nd, d, h_t) \neq \emptyset$  we define  $l(F, h_t) = \min \left\{ l \in \tilde{N}_C, l \leq t \mid \exists a_l = (i, j), i \in A(nd, d, h_t) \right\}$

and  $i(h_t) = a_{l(F, h_t)}^1$ ,  $D_4(F, h_t) = \left\{ i \in \{1, \dots, n_F\} \mid \exists l \in \tilde{N}_C, l \leq t, \text{ and } j \text{ such that } a_l = (i, j) \right\}$ ,  $D_5(F, h_t) = \{i \in D_4(F, h_t) \mid D_3(i, h_t) \notin \{\{n_F + 1, \dots, n\}, \emptyset\} \text{ and } D_1(i, h_t) = \emptyset\}$

<sup>16</sup>We also assume that  $(x_d^F, x_{nd}^L) = (1, 0)$ .

and  $D_6(h_t) = \left\{ (i, j) \in \{1, \dots, n_F\} \times \{n_F + 1, \dots, n\} \mid \exists a_l = (i, j), l \in \tilde{N}_C, l \leq t, a_{l+2}, a_{l+3} \in \{(x_{nd}^F, x_{nd}^L), (x_{nd}^L, x_{nd}^F)\} \right\}$   
Now observe that if  $D_5(F, h_t) = \emptyset$ , then  $D_3(i, h_t) \in \{\{n_F + 1, \dots, n\}, \emptyset\}$  or  
 $D_1(i, h_t) = \emptyset$ , or both. Note also that if  $D_5(F, h_t) \neq \emptyset$ , then  $A(nd, d, h_t) \neq \emptyset$   
and therefore  $D_3(i(h_t), h_t)^C \neq \emptyset$ . Finally,  $l(nd, h_t) = \min \left\{ k \leq t \mid (a_{k+2}, a_{k+3}) = (x_{nd}^F, x_{nd}^L), k \in \tilde{N}_C \right\}$   
 $l(d, h_t) = \min \left\{ k \mid (a_{k+2}, a_{k+3}) = (x_d^F, x_d^L), k \in \tilde{N}_C \right\}$  and  $l(d, nd, h_t) = \min \left\{ k \mid (a_{k+2}, a_{k+3}) \in \{(x_{nd}^F, x_{nd}^L), (x_{nd}^L, x_{nd}^F)\} \right\}$

Take  $h_t$  such that  $(a_{t-1}, a_t) \in \{(x_{nd}^F, x_d^L), (x_d^F, x_{nd}^L), (x_d^F, x_d^L)\}$ , then  $s^V(I)(I_t) = a_l$  with  $l = l(nd, h_t)$ , if  $\exists i$  such that  $D_2(i, h_t) \neq \emptyset$ .

If  $D_2(i, h_t) = \emptyset$  for all  $i \in D_4(F, h_t)$ , then  $s^V(I)(h_t) = (i(h_t), \min D_3(i(h_t), h_t)^C)$ , if  $D_5(F, h_t) \neq \emptyset$ .

Now, if  $D_5(F, h_t) = \emptyset$ ,  $s^V(I)(h_t) = (\min D_4(F, h_t)^C, n_F + 1)$ , if  $D_4(F, h_t) \neq \{1, \dots, n_F\}$ . But if  $D_4(F, h_t) = \{1, \dots, n_F\}$ , then  $s^V(I)(h_t) = \begin{cases} a_l, l = l(d, h_t) & \text{if } D_6(h_t) = \emptyset \\ a_l, l = l(d, nd, h_t) & \text{if } D_6(h_t) \neq \emptyset \end{cases}$ .

Finally,  $s^V(I)(h_t) = (i, j)$ , if  $(a_{t-1}, a_t) = (x_{nd}^F, x_{nd}^L)$  and  $a_{t-3} = (i, j)$ .

The parties. Define  $i(h_t, first) = \min \{i \in D_4(h_t) \mid i \text{ is of good type}\}$  and take  $h_t = (a_l)_{l=1}^t \in I(t) \in \tilde{I}^i$ , with  $i = P(I_t) \in \{1, \dots, n\}$ , then  $s^{(i,j)}(I)(I_t) = \begin{cases} x_{nd}^j & \text{if } i = i(h_t, first) \\ x_d^j & \text{otherwise} \end{cases}$ . Then we have the following result.

**Theorem 2** Take the model  $\left\langle N, H, P, \left( \tilde{I}^i \right)_{i \in N \setminus \{V, C\}}, \tilde{I}^{VI}, (\succeq_i)_{i \in N \setminus \{V, C\}} \right\rangle$ , and suppose the voters are patient enough. Then any profile of the type  $((s^{(i,j)}(I))_{(i,j) \in N \setminus \{C, V\}}, s^V(I))$  is a Nash equilibrium. We have three cases: a) If there are good parties in both levels, then there is a time  $T = |\{1, \dots, n_F\} \times \{n_F, \dots, n\}|$  and two good parties that will be in office from  $t \leq 4T$ . The voters learn the types of at least the two good parties that will be in office; also, depending on the equilibrium profile, they may learn even the competencies of the levels of governance (as it is evident from our example, it suffices that along the equilibrium path there is an event in which there was only one party diverting resources, and another posterior event in which one of those is matched so that the two parties divert resources); b) If there are no good parties, no good parties will be in office, and the first two parties are in office for ever; the voters learn the types of all the parties c) If only in one of the levels there are good types, then the first two parties such that only one of them diverted resources will be in office for ever; the voters learn that in only one level there are good parties and, somehow paradoxically, they may learn the competencies, as in the item (a) of this theorem.

## 6 Technical issues and proofs

The first issue is to characterize when a party is good, that is, we have to characterize when for a party, at a given  $t \geq 0$ , it is better not to divert resources than to do it, given the strategies of the other. Essentially, a party will be of bad type, if it is too impatient, the idea that we formalize in what follows.

**Lemma 1** *There exists a number  $\delta_m(j, x_d, x_{nd}) \in (0, 1]$  such that,*

*L.1.1 if  $\delta_m(j, x_d, x_{nd}) < 1$  and  $\delta < \delta_m(j, x_d, x_{nd})$ , then  $D \succ ND$ ;*

*L.1.2 if  $\delta_m(j, x_d, x_{nd}) = 1$  and  $\delta < \delta_m(j, x_d, x_{nd})$ , then  $D \succ ND$ ;*

*L.1.3 if  $\delta_m(j, x_d, x_{nd}) < 1$  and  $\delta > \delta_m(j, x_d, x_{nd})$ , then  $ND \succ D$ ; and*

*L.1.4 if  $\delta_m(j, x_d, x_{nd}) < 1$  and  $\delta = \delta_m(j, x_d, x_{nd})$ , then  $ND \sim D$ ;*

Proof: Let be  $h(\delta, x_{nd}, x_d, j) = x_{nd} - x_d + \delta x_d - \delta^j x_{nd}$  and  $c(h) = |\{\delta \geq 0 \mid h(\delta, x_{nd}, x_d, j) = 0\}|$ . Observe that  $1 \leq c(h) \leq 2$ , since  $h$  is strictly concave on  $\delta \geq 0$  (given that  $\frac{\partial^2 h(\delta, x_{nd}, x_d, j)}{\partial \delta^2} = -j(j-1)\delta^{j-2}x_{nd} < 0$ , if  $\delta > 0$ ) and  $h(1, x_{nd}, x_d, j) = 0$ . We then define

$$\delta_m(j, x_d, x_{nd}) = \begin{cases} \delta^R & \text{if } c(h) = 2, h(\delta^R, x_{nd}, x_d, j) = 0 \text{ and } \delta^R < 1 \\ 1 & \text{if either } c(h) = 1, \text{ or } c(h) = 2, h(\delta^R, x_{nd}, x_d, j) = 0 \text{ and } \delta^R > 1 \end{cases} \quad (4)$$

Now we will show that this definition of  $\delta_m(j, x_d, x_{nd})$  satisfies *L.1.1-L.1.4* (clearly,  $0 < \delta_m(j, x_d, x_{nd})$ , because  $h(0, x_{nd}, x_d, j) = x_{nd} - x_d < 0$ ). Note that, if  $0 \leq \delta < 1$ , then,  $D \succ ND$  if and only if  $h(\delta, x_{nd}, x_d, j) < 0$ , since  $D \succ ND$  if and only if  $\frac{x_d}{1-\delta^j} > \frac{x_{nd}}{1-\delta}$ , if  $0 \leq \delta < 1$ . Suppose that  $\delta_m(j, x_d, x_{nd}) < 1$ . Thus, we have  $h(\delta_m(j, x_d, x_{nd}), x_{nd}, x_d, j) = 0$ . Therefore, we have  $h(\delta, x_{nd}, x_d, j) < 0$  if and only if  $\delta < \delta_m(j, x_d, x_{nd})$ , since, once again,  $h$  is strictly concave on  $\delta \geq 0$ . This concludes the proof of *L.1.1* and *L.1.3* (the proof of *L.1.4* is totally analogue to *L.1.1* and *L.1.3* and therefore omitted). Suppose now that  $\delta_m(j, x_d, x_{nd}) = 1$ : If  $c(h) = 1$ , we have  $h(\delta, x_{nd}, x_d, j) < 0$  for all  $0 < \delta < 1$ , because  $h$  is strictly concave on  $\delta \geq 0$  (1 is the maximum of  $h$  on  $\delta \geq 0$  and  $h(1, x_{nd}, x_d, j) = 0$ ). For the case when  $c(h) = 2$ ,  $h(\delta^R, x_{nd}, x_d, j) = 0$  and  $\delta^R > 1$  just note that  $h(\delta, x_{nd}, x_d, j) < 0$  for all  $0 \leq \delta < 1$ .

**Theorem 3** Take  $\delta_m(j, x_d, x_{nd})$ . Then,  $\delta_m(j, x_d, x_{nd})$  is a non-increasing function of  $j \in \mathfrak{R}_+$ .<sup>17</sup> More precisely, there exists a natural number  $j(x_d, x_{nd})$  such that

T.1.1 If  $j < j(x_d, x_{nd})$ , then  $\delta_m(j, x_d, x_{nd}) = 1$ ; and

T.1.2 If  $j \geq j(x_d, x_{nd})$ , then  $\delta_m(j, x_d, x_{nd}) < 1$  and  $\frac{\partial \delta_m(j, x_d, x_{nd})}{\partial j} < 0$ ; and

T.1.3  $\lim_{j \rightarrow \infty} \delta_m(j, x_d, x_{nd}) = 0$ .

Proof: Define

$$j(x_d, x_{nd}) = \min \{j \in N \mid x_d < jx_{nd}\}.^{18}$$

Take  $j(x_d, x_{nd})$  and  $\delta^c(j, x_d, x_{nd}) = \left[ \frac{x_d}{jx_{nd}} \right]^{\frac{1}{j-1}}$ . Then, if  $j < j(x_d, x_{nd})$  we have that  $\delta^c(j, x_d, x_{nd}) \geq 1$ . Now observe that  $\left. \frac{\partial h(\delta, x_{nd}, x_d, j)}{\partial \delta} \right|_{\delta=\delta^c(j, x_d, x_{nd})} = 0$  and, therefore (once again, because  $h(\delta, x_{nd}, x_d, j)$  is strictly concave with respect to  $\delta$ ),  $\delta_m(j, x_d, x_{nd}) = 1$ , since we have either  $c(h) = 1$  or  $c(h) = 2$ ,  $h(\delta^R, x_{nd}, x_d, j) = 0$  and  $\delta^R > 1$ , so the item T.1.1 is proven.

To prove that for  $j \geq j(x_d, x_{nd})$  the object  $\delta_m(j, x_d, x_{nd})$  is a differentiable function of  $j$  with  $\frac{\partial \delta_m(j, x_d, x_{nd})}{\partial j} < 0$  we will use the Implicit Function Theorem (see, for instance, bla, bla). Indeed, for  $j \geq j(x_d, x_{nd})$  the object  $\delta_m(j, x_d, x_{nd})$  is also defined by the equation  $h(\delta_m(j, x_d, x_{nd}), x_{nd}, x_d, j) = 0$  since, one more time,  $h(\delta, x_{nd}, x_d, j)$  is strictly concave with respect to  $\delta$ : This last claim is consequence of the fact that,  $\left. \frac{\partial h}{\partial \delta} \right|_{(\delta, j)=(\delta^c(j, x_d, x_{nd}), j)} = 0$  and that  $\delta^c(j, x_d, x_{nd}) < 1$  for  $j \geq j(x_d, x_{nd})$  and  $h(1, x_{nd}, x_d, j) = 0$ . We can also conclude, for the same argument of concavity of  $h$ , that  $\delta_m(j, x_d, x_{nd}) < \delta^c(j, x_d, x_{nd})$  and so  $\delta_m(j, x_d, x_{nd}) < 1$ . Now it leaves to check the assumptions of the Implicit Function Theorem. Clearly,  $h$  is a  $C^\infty$  function of  $(\delta, j)$  and,  $\left. \frac{\partial h}{\partial \delta} \right|_{(\delta, j)=(\delta_m(j, x_d, x_{nd}), j)} = x_d - jx_{nd} [\delta_m(j, x_d, x_{nd})_{nd}]^{j-1} > x_d - jx_{nd} [\delta^c(j, x_d, x_{nd})]^{j-1} = 0$ , since  $\delta_m(j, x_d, x_{nd}) < \delta^c(j, x_d, x_{nd})$ , as it was proven above. Therefore,  $\left. \frac{\partial h}{\partial \delta} \right|_{(\delta, j)=(\delta_m(j, x_d, x_{nd}), j)} < 0$  and thus the assumptions of the implicit Function Theorem are satisfied. Using it, we have

<sup>17</sup> $\mathfrak{R}_+$  stands for the set of the non-negative real numbers.

<sup>18</sup>Notice that  $\{j \in N \mid x_d < jx_{nd}\} \subset N$  and therefore  $\{j \in N \mid x_d < jx_{nd}\}$  has a minimum.

$$\frac{\partial \delta_m(j, x_d, x_{nd})}{\partial j} = - \frac{\frac{\partial h}{\partial j} \Big|_{(\delta, j) = (\delta_m(j, x_d, x_{nd}), j)}}{\frac{\partial h}{\partial \delta} \Big|_{(\delta, j) = (\delta_m(j, x_d, x_{nd}), j)}} = - \frac{x_{nd} [\delta_m(j, x_d, x_{nd})]^j \ln[\delta_m(j, x_d, x_{nd})]}{\frac{\partial h}{\partial \delta} \Big|_{(\delta, j) = (\delta_m(j, x_d, x_{nd}), j)}} < 0,$$

since  $1 > \delta_m(j, x_d, x_{nd}) > 0$ .

Therefore, the proof of the theorem is done.

There exists a number  $\delta_m(j, x_d, x_{nd}) \in (0, 1]$  such that,

**Lemma 2** •

• **Lemma 3** if  $\delta_m(j, x_d, x_{nd}) < 1$  and  $\delta < \delta_m(j, x_d, x_{nd})$ , then  $D \succ ND$ ;

**Lemma 4** if  $\delta_m(j, x_d, x_{nd}) = 1$  and  $\delta < \delta_m(j, x_d, x_{nd})$ , then  $D \succ ND$ ;

*L.1.3* if  $\delta_m(j, x_d, x_{nd}) < 1$  and  $\delta > \delta_m(j, x_d, x_{nd})$ , then  $ND \succ D$ ; and

*L.1.4* if  $\delta_m(j, x_d, x_{nd}) < 1$  and  $\delta = \delta_m(j, x_d, x_{nd})$ , then  $ND \sim D$ ;

Proof: Let be  $h(\delta, x_{nd}, x_d, j) = x_{nd} - x_d + \delta x_d - \delta^j x_{nd}$  and  $c(h) = |\{\delta \geq 0 \mid h(\delta, x_{nd}, x_d, j) = 0\}|$ . Observe that  $1 \leq c(h) \leq 2$ , since  $h$  is strictly concave on  $\delta \geq 0$  (given that  $\frac{\partial^2 h(\delta, x_{nd}, x_d, j)}{\partial \delta^2} = -j(j-1)\delta^{j-2} < 0$ , if  $\delta > 0$ ) and  $h(1, x_{nd}, x_d, j) = 0$ . We then define

$$\delta_m(j, x_d, x_{nd}) = \begin{cases} \delta^R & \text{if } c(h) = 2, h(\delta^R, x_{nd}, x_d, j) = 0 \text{ and } \delta^R < 1 \\ 1 & \text{if either } c(h) = 1, \text{ or } c(h) = 2, h(\delta^R, x_{nd}, x_d, j) = 0 \text{ and } \delta^R > 1 \end{cases} \quad (5)$$

Now we will show that this definition of  $\delta_m(j, x_d, x_{nd})$  satisfies *L.1.1-L.1.4* (clearly,  $0 < \delta_m(j, x_d, x_{nd})$ , because  $h(0, x_{nd}, x_d, j) = x_{nd} - x_d < 0$ ). Note that, if  $0 \leq \delta < 1$ , then,  $D \succ ND$  if and only if  $h(\delta, x_{nd}, x_d, j) < 0$ , since  $D \succ ND$  if and only if  $\frac{x_d}{1-\delta^j} > \frac{x_{nd}}{1-\delta}$ , if  $0 \leq \delta < 1$ . Suppose that  $\delta_m(j, x_d, x_{nd}) < 1$ . Thus, we have  $h(\delta_m(j, x_d, x_{nd}), x_{nd}, x_d, j) = 0$ . Therefore, we have  $h(\delta, x_{nd}, x_d, j) < 0$  if and only if  $\delta < \delta_m(j, x_d, x_{nd})$ , since, once again,  $h$  is strictly concave on  $\delta \geq 0$ . This concludes the proof of *L.1.1* and *L.1.3* (the proof of *L.1.4* is totally analogue to *L.1.1* and *L.1.3* and therefore omitted). Suppose now that  $\delta_m(j, x_d, x_{nd}) = 1$ : If  $c(h) = 1$ , we have  $h(\delta, x_{nd}, x_d, j) < 0$  for all  $0 < \delta < 1$ , because  $h$  is strictly concave on  $\delta \geq 0$  (1 is the maximum of  $h$  on  $\delta \geq 0$  and  $h(1, x_{nd}, x_d, j) = 0$ ). For the case when  $c(h) = 2$ ,  $h(\delta^R, x_{nd}, x_d, j) = 0$  and  $\delta^R > 1$  just note that  $h(\delta, x_{nd}, x_d, j) < 0$  for all  $0 \leq \delta < 1$ .

**Theorem 4** Take  $\delta_m(j, x_d, x_{nd})$ . Then,  $\delta_m(j, x_d, x_{nd})$  is a non-increasing function of  $j \in \mathfrak{R}_+$ .<sup>19</sup> More precisely, there exists a natural number  $j(x_d, x_{nd})$  such that

T.1.1 If  $j < j(x_d, x_{nd})$ , then  $\delta_m(j, x_d, x_{nd}) = 1$ ; and

T.1.2 If  $j \geq j(x_d, x_{nd})$ , then  $\delta_m(j, x_d, x_{nd}) < 1$  and  $\frac{\partial \delta_m(j, x_d, x_{nd})}{\partial j} < 0$ ; and

T.1.3  $\lim_{j \rightarrow \infty} \delta_m(j, x_d, x_{nd}) = 0$ .

Proof: Define

$$j(x_d, x_{nd}) = \min \{j \in N \mid x_d < jx_{nd}\}.$$
<sup>20</sup>

Take  $j(x_d, x_{nd})$  and  $\delta^c(j, x_d, x_{nd}) = \left[ \frac{x_d}{jx_{nd}} \right]^{\frac{1}{j-1}}$ . Then, if  $j < j(x_d, x_{nd})$  we have that  $\delta^c(j, x_d, x_{nd}) \geq 1$ . Now observe that  $\left. \frac{\partial h(\delta, x_{nd}, x_d, j)}{\partial \delta} \right|_{\delta=\delta^c(j, x_d, x_{nd})} = 0$  and, therefore (once again, because  $h(\delta, x_{nd}, x_d, j)$  is strictly concave with respect to  $\delta$ ),  $\delta_m(j, x_d, x_{nd}) = 1$ , since we have either  $c(h) = 1$  or  $c(h) = 2$ ,  $h(\delta^R, x_{nd}, x_d, j) = 0$  and  $\delta^R > 1$ , so the item T.1.1 is proven.

To prove that for  $j \geq j(x_d, x_{nd})$  the object  $\delta_m(j, x_d, x_{nd})$  is a differentiable function of  $j$  with  $\frac{\partial \delta_m(j, x_d, x_{nd})}{\partial j} < 0$  we will use the Implicit Function Theorem (see, for instance, bla, bla). Indeed, for  $j \geq j(x_d, x_{nd})$  the object  $\delta_m(j, x_d, x_{nd})$  is also defined by the equation  $h(\delta_m(j, x_d, x_{nd}), x_{nd}, x_d, j) = 0$  since, one more time,  $h(\delta, x_{nd}, x_d, j)$  is strictly concave with respect to  $\delta$ : This last claim is consequence of the fact that,  $\left. \frac{\partial h}{\partial \delta} \right|_{(\delta, j)=(\delta^c(j, x_d, x_{nd}), j)} = 0$  and that  $\delta^c(j, x_d, x_{nd}) < 1$  for  $j \geq j(x_d, x_{nd})$  and  $h(1, x_{nd}, x_d, j) = 0$ . We can also conclude, for the same argument of concavity of  $h$ , that  $\delta_m(j, x_d, x_{nd}) < \delta^c(j, x_d, x_{nd})$  and so  $\delta_m(j, x_d, x_{nd}) < 1$ . Now it leaves to check the assumptions of the Implicit Function Theorem. Clearly,  $h$  is a  $C^\infty$  function of  $(\delta, j)$  and,  $\left. \frac{\partial h}{\partial \delta} \right|_{(\delta, j)=(\delta_m(j, x_d, x_{nd}), j)} = x_d - jx_{nd} [\delta_m(j, x_d, x_{nd})_{nd}]^{j-1} > x_d - jx_{nd} [\delta^c(j, x_d, x_{nd})]^{j-1} = 0$ , since  $\delta_m(j, x_d, x_{nd}) < \delta^c(j, x_d, x_{nd})$ , as it was proven above. Therefore,  $\left. \frac{\partial h}{\partial \delta} \right|_{(\delta, j)=(\delta_m(j, x_d, x_{nd}), j)} < 0$  and thus the assumptions of the implicit Function Theorem are satisfied. Using it, we have

<sup>19</sup> $\mathfrak{R}_+$  stands for the set of the non-negative real numbers.

<sup>20</sup>Notice that  $\{j \in N \mid x_d < jx_{nd}\} \subset N$  and therefore  $\{j \in N \mid x_d < jx_{nd}\}$  has a minimum.

$$\frac{\partial \delta_m(j, x_d, x_{nd})}{\partial j} = - \frac{\frac{\partial h}{\partial j} \Big|_{(\delta, j) = (\delta_m(j, x_d, x_{nd}), j)}}{\frac{\partial h}{\partial \delta} \Big|_{(\delta, j) = (\delta_m(j, x_d, x_{nd}), j)}} = - \frac{x_{nd} [\delta_m(j, x_d, x_{nd})]^j \ln[\delta_m(j, x_d, x_{nd})]}{\frac{\partial h}{\partial \delta} \Big|_{(\delta, j) = (\delta_m(j, x_d, x_{nd}), j)}} < 0,$$

since  $1 > \delta_m(j, x_d, x_{nd}) > 0$ .

Therefore, the proof of the theorem is done.

These two results say that, if the parties are totally impatient (they do not care about the future at all) then, in any Nash equilibrium, the parties choose to divert resources, just because the voters do not have the opportunity to punish.

Some things to say before formal definitive proofs. First, in the case  $n = 2$ , it is clear that the parties know the parties' types, because it suffices only one shot play game for both to know if the other fails or not. For  $n > 2$ , we argue that, even it is not that clear, thinking the game may be coming from very long in the past, so the parties have had at least one shot play game with every other party.

So there is no need to think in a Bayesian game in each period of elections. In any case, there are two types for each party: good or bad, depending on the number of parties that are left to play the game, in case of it, that is, depending on the number of parties that have not been in office yet.

The strategies:

a)  $n = 2$ . The parties are good or bad *per se*, since there are no parties left to play. Then, if both parties are good, they do not divert resources, whenever they did not divert resources in the previous period; otherwise, they divert resources. The player  $V$  takes off both parties whenever at least one party diverted resources, otherwise,  $V$  confirms both parties.- So, it suffices the presence of one bath party to embed the society in a tragedy for ever and ever: both parties, in spite of having one good, will divert resources for ever and ever. (in this sense, to punish results in a perverse incentive: the good one party, imitates the bas one, and decides to divert resources)

b)  $n > 2$ . In this case there is a need of a lot structure. Some, probably for more than one, extreme simplifications, may help to say something.

Formal tentative:

$$s_j^F(h) = \begin{cases} x & \text{if } a_{t-3}^1 = j \text{ and } a_{t-2} = x \\ x_{nd}^F & \text{if } j \text{ is good and } a_{t-3}^1 \neq j \\ x_d^F & \text{if } j \text{ is bad and } a_{t-3}^1 \neq j \end{cases}$$

if  $p(h) = j$ . And, for the player  $V$

$$s_V(h) = \left\{ (i, j) \left| \begin{array}{l} i = a_{t-4} \text{ if } a_{t-3} = x_{nd}^F \\ i \neq a_{t-4} \text{ if } a_{t-3} \neq x_{nd}^F \\ \text{the same for } j \end{array} \right. \right.$$

intuition  $V$  Suppose that the player  $V$  at time  $t$  modifies its decision. If the history is such that  $V$  has decided to confirm both parties, that is, both parties have not diverted resources, then there are two possibilities: One, if the new two parties in office for the next period are again good type parties, and then  $V$  gets the same pay off as with the other decision; Second, if at least one of the new parties is of bad type, the alternative choices are to confirm again both parties or to confirm only one:

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