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DAVID JUAREZ-LUNA AND CHRISTIAN GHIGLINO Elite Capture of Democratic Politics: The Role of Social Identity



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# Abstract

In the present paper we uncover a novel mechanism through which a minority can gain a disproportionate power in a perfectly functioning democracy. In our model, a government elected by majority within a two party democracy, decides on a unique redistributive instrument, the tax rate. We show that a minority characterised by a high degree of social identification may, in the presence of ideological motives, influence the policy outcome. In particular, a rise in social identification among the rich minority may be able to reduce the tax rate. Importantly, this may happen even if the minority is more ideological than the majority. Finally, we attempt an explanation of the divide in the tax rate between the US and Europe.

## Keywords: Democracy, Influential elite, Social identity, Tax rate, Redistribution.

## Resumen

En el presente artículo dejamos al descubierto un novedoso mecanismo a través del cual una minoría puede ganar poder desproporcionado en una democracia que funciona perfectamente. En nuestro modelo un gobierno elegido por mayoría en una democracia bipartidista decide sobre un instrumento redistributivo único, la tasa de impuestos. Mostramos que una minoría que se caracteriza por un alto grado de identificación social puede, en presencia de incentivos ideológicos, influir en la política resultante. En particular, el aumento de la identificación social entre la minoría rica puede ser capaz de reducir la tasa de impuestos. Es importante destacar que esto puede ocurrir incluso si la minoría es más ideológica que la mayoría. Finalmente, intentamos brindar una explicación sobre la brecha existente en la tasa de impuestos entre los EE.UU. y Europa.

# Palabras clave: Democracia, Élite influyente, Identidad social, Tasa de impuestos, Redistribución.

# Introduction

he fundamental distinction between a democracy and a nondemocracy<sup>1</sup> is that in a democracy all agents possess equal political weight. As each citizen has a vote, common wisdom suggests that democracy will lead to the choice of policies that reflect the preferences of the poor, provided these are in greater number. This has important implications, as for example it would suggest high tax rates. However, occasional observation suggests that often minorities are disproportionately influential. Typical explanations are that the minority is able to control the party system, as the political agenda, to form an effective lobby against certain policies, or to gain monopoly on information (see Austen-Smith (1997) for a review).

In this literature, agents typically behave rationally in the pursuit of their self interest. However, social identity has been shown to affect a large portion of human social behavior, and in particular, the behavior of voters. As introduced by Akerlof and Kranton (2000) social identity is related to the "person's sense of self". The role of the status of the members of the group, seen as a pillar of this theory, often generates in-group altruism. This is true in the case considered here in which the issue is the level of taxation, or equivalently, the level of redistribution.

In view of what just said, one may wonder whether social identity may shake the democratic dictatorship of the median voter. This paper explores this question and shows that indeed social identification, modeled as in-group altruism, may provide an alternative way to allow a minority to be disproportionately influential. As we see below this indeed occurs, and through a novel and unexpected channel.

To gain intuition we first start to note that social identification alone is not enough to destroy the median voter theorem. However, the presence of preference uncertainty changes this conclusion. In probabilistic voting models with ideological concerns<sup>2</sup>, when the members of a social minority are more willing to change their party allegiance than the majority, the swing voters belong to this group and the political equilibrium is tilted toward the preferences of this group. Ideology alone therefore unambiguously reduces political power. However, group identification, modeled as in-group altruism, depreciates the relative weight given to ideology and therefore increases the political power of the social group. The present paper analyses the combined effects of this two forces. In particular we show that a minority with high social identification may gain a disproportionate power in the choice of public policies without the need of being less ideological.

As an application we try to solve the long standing puzzle of the wide differences

<sup>&</sup>lt;sup>1</sup>Following Acemoglu and Robinson (2006) we use the term nondemocracy to designate alternatives, such as dictatorship or authoritarian regime, since it has less specific connotations than any of the other words.

<sup>&</sup>lt;sup>2</sup>Probabilistic voting has some nice properties: equilibrium exists, cycles are avoided, and the equilibrium may correspond to the utilitarian social welfare optimum. Such conditions hold even in multidimensional policy space formulations. (see Coughlin (1982), Kirchgässner (2000). See Coughlin (1992) for a survey.

in the tax-rates in use in different democracies, in particular the divide US vs continental Europe. In fact, the model do propose a mechanism by which a country can vote an apparently low tax rate, opening the door to wide differences in tax rates across countries.

We now describe the model we use. As in Acemoglu and Robinson (2006) we consider the competition between two partisan parties in a representative democracy made up of two main social groups: Elite, that is a rich minority, and a poor majority. Voters derive utility both from their after tax income and from which party is in power, the ideological motive.

The model introduces identity through social preferences in a way similar to Akerlof and Kranton (2000). As noted before, improving the groups's status requires to increase the welfare of the other members of the group. However, this is not universal altruism as is commonly known, as it is directed at in-group members rather than being universal. We assume that this altruistic motive has two components: utility depends positively on the average income of the individuals in the group and negatively on the level of inequality in the group. The rationale for this formulation can be found in the maximization of the group's welfare (see Wittman (2005)).

The rest of the model is standard. Parties align their preferences to those of the poor and offer platforms on the only public policy we consider: a fully redistributive tax rate (we take gross income to be fixed and known). As each social group has different preferences on the tax rate, the choice of public policy is inherently conflictive. We allow ideology diversity within the social groups: individuals can support any political party regardless of the social group to which they belong.<sup>3</sup> We assume that ideology is only imperfectly observed by the parties; parties assign probability distributions to individuals' party preferences. As is standard in probabilistic voting models, both parties are assumed to make the same probability assignments.

Every individual votes for the party that best promotes his own utility, which has egotistic, altruistic and ideological terms. As the proceeds from the tax rate are redistributed via lump sum transfers to the whole population, a vote from an individual is a vote on tax rate. Each of the two political parties selects its redistribution policy so as to maximize its expected utility. Hence, the voters make use of the parties to obtain a government that promotes their utility and the parties make use of the voters to gain power - the political system thus being formed by the interaction between two categories of maximizing agents. This is the mechanism that allows the members of the minoritarian Elite to affect the equilibrium tax rate.

In the model, for each social group there are three competing factors that determine the equilibrium tax rate: ideology, the general level of in-group altruism and the relative weight given to inequality aversion relative to average income within the group. In general, we show that strong in-group altruism within members of a minority depreciates the relative weight given to ideology, giving to the minority a

<sup>&</sup>lt;sup>3</sup>This approach differs from Roemer (1999), who assumes that parties represent, imperfectly, different constituencies, or economic classes.

disproportionate weight in the choice of public policies. However, the relative weight given to inequality aversion also plays a role. These forces may lead to complex predictions as the following paragraphs illustrate.

We obtain three sets of results. First, we show that in-group inequality aversion has always a positive effect on the tax rate, that is, a rise in inequality aversion, keeping fixed all other parameters, increases the equilibrium tax rate. The role of the overall level of group identification, keeping fixed the relative weight of inequality aversion, depends on the identity of the group that is affected by the change and the level of inequality aversion. In particular, for low weights given to inequality aversion, when the rise in altruism concerns the rich minority, the rise will eventually lead to a decrease of the equilibrium tax rate. When the rise concerns the poor majority, the equilibrium tax rate always rises.

The second set of results concern the effect of simultaneous but opposite changes<sup>4</sup> in the level of group identification in the two groups. We find that if the rich and the poor place a low weight on their income inequality aversion, in a democracy in which the difference in in-group altruism across social groups is greater, the social group that exhibits greater in-group altruism pushes the tax rate in its "natural" direction.<sup>5</sup>

The third set of results concern simultaneous changes in ideology and in social identity. When changes in ideology affect similarly both social groups, the outcome is ambiguous and depends on the prevailing levels of altruism and the weight given to inequality aversion by the social groups. The results also depend on the details of the shifts in ideology.

A first result shows that a low level of ideology and a high level of altruism contribute to the political power of the group. In this case, increasing the general level of ideology of both groups will increase the political power of the group that has relatively low ideology. In particular, provided the rich place a low weight on their inequality aversion, the general rise in altruism rises the power of the minority that finally translates into a decrease in the tax rate, the "natural" direction for the rich.

The second result is surprising. Indeed, it shows that the condition that the rich are less ideological than the poor is not necessary to obtain a low tax rate. In fact, there exists an interval of ideologies for the rich, lying above the ideology of the poor, such that a rise in ideology reduces the equilibrium tax rate, provided the rich exhibit higher in-group altruism than the poor and their inequality aversion is not too high.

This result is important because it sheds light on the trade-off between ideology and social identity. At first sight, and in view of the literature, in a society where the rich are more ideological than the poor, as the rich individuals are more attached to the political parties, the rich receive low favour from the economic policy. However,

d ( $B_{pk}, B_{rk}$ ) designate the levels of in-group altruism of the poor and the rich in democracy k we focus on differences such that  $B_{rk} + B_{pk} = B_{rk'} + B_{pk'}$  for all k, k'.

<sup>&</sup>lt;sup>5</sup>This result holds irrespective of the prevailing levels of ideology and altruism in democracy 1.

we show that this fact could be reversed if, in addition, the rich exhibit greater social identification than the poor do.

The aim of the paper is related to the vast literature on how interest groups affect outcomes in elections and in legislatures. The literature has shown many ways the elite can manipulate the prevailing policy through contributions, lobbying and other informational activities (see Austen-Smith (1997) for a review). For example, the elites' ability to mobilize large groups of voters explains the turnout in elite-driven mobilization models (e.g., Uhlaner (1989), Morton (1991), Shachar and Nalebuff (1999), and Herrera and Martinelli (2006) cited in Evren (2010)). Often, the elite chooses entry barriers, regulations and inefficient contracting institutions that retard economic growth and create resource misallocations in order to protect their economic rents and redistribute resources to themselves (e.g., Mancur Olson (1982), Per Krusell and Jose-Victor Rios-Rull (1996), Stephen Parente and Edward J. Prescott (1999) cited in Acemoglu (2010)). A recent example of this line of thought is Bandiera and Levy (2010).<sup>6</sup> They propose a model with two main groups: the elite and the poor. Importantly, a segment of the poor, referred to as the ethnic group, have different preferences. The elite can gain weight in the choice of public goods by forming a stable coalition with the ethnic group. Then, despite the numerosity of the poor, the outcome is not the bliss point of the poor. The theoretical predictions are illustrated with evidence on the allocation of public goods in Indonesian villages, using differences in ethnicity as a measure of preference diversity.

Acemoglu and his coauthors - in particular Acemoglu and Robinson (2006 and 2008) - focus on the change of political regime and in particular the switch to democracy or the persistence of the elite control.<sup>7</sup> They adopt the framework of probabilistic voting and provide micro-political foundations to the fact that the elite may have disproportionate political power in a democracy. In fact, Acemoglu and Robinson (2006) analysis suggest that minorities can gain power beyond their vote share not only through lobbying, or capture the party system, but also indirectly as a consequence of ideology. To this analysis we add social identification, a motive that as we will see may dramatically affect the results.

The main special feature of the paper, is the role given to social identification. Akerlof and Kranton (2000) have developed and formalized the notion of social identity in economics. Recent work has mainly focused on experiments, see, for e.g., Kranton, Pease, Sanders and Huettel (2013). In political economy, Fowler and Kam (2007) have extended the participation model by adding a term related to general ingroup altruism, where the groups are defined by political party affiliation. They show that both altruism and social identification increase political participation.

Closer to our work, the role of social identity on the level of redistribution in a democracy has been analyzed in Shayo (2009). A first distinction is that in his model all

<sup>&</sup>lt;sup>6</sup>Another interesting is Bourguignon and Verdier (2009), who consider an elite trying to extract as much rent as possible from the economy while trying to minimize the probability of losing political power.

<sup>&</sup>lt;sup>7</sup>Within the institutional approach Hossain et al. **1999** establish that institutional change obey the behavior of the elite and their incentives to permit and invest in such change.

agents are assumed to vote directly and sincerely over the tax rate (that is, each agent votes for his most preferred tax rate), and the median tax rate is adopted. In fact, as noted by the author, this mechanism is very similar to a Downsian two-party electoral competition with pure majority rule. In his model, only the poor majority will determine the level of redistribution. The channel uncovered in the present paper is not present in a model without ideology, because of the median voter theorem. Second, in Shayo (2009) the available multiplicity of equilibria produces an apparent negative correlation between the level of class identification and the level of national identification. It is this mechanism that leads the US, which are characterized by a high national identification, to produce a low tax rate. However, there is no evidence of this pattern.

Finally note that the impact of universal altruism has been widely studied, including in political economy.<sup>8</sup> For example, Dixit and Londregan (1998) consider the care that citizens and politicians alike have for distributive equity, and relate this to altruism. They conclude that the voter with median income gains the most. Altruism has also been used to mend the nonexistence arising in the selfish Dawnsian model, and shown to lead to stable majorities and transitive preferences (see, e.g. Wittman (2005)). However, altruism has rarely been involved in the political process, even less in-group altruism. To the best of our knowledge, this is the first attempt to relate social identification, or in-group altruism, to the power of minorities.

The paper is organized as follows: In the next section we present our model. The equilibrium tax rate when there is no ideology is characterized in Section 2. In Section 3, we characterize the equilibrium tax rate in a general model. In Section 4 we analyze the role of altruism on equilibrium tax rate. In section 5 we attempt an explanation of the puzzle related to the cross-country data on tax rates. Section 6 concludes. Appendix contains some proofs.

## The model

To study political conflict over the tax rate, we develop a model of political competition with heterogeneous individuals, two political parties, and two social groups, based on Acemoglu and Robinson (2006). Individuals are ordered from poorest to richest. There is a proportional income tax with resulting lump sum transfers. Social identity plays a role in individual choices. We model this by assuming that individuals have altruistic preferences to members belonging to the same social group. All individuals vote according to their ideology and the tax rate they prefer most, following a probabilistic voting model. As the political parties are partisan they have preferences on policies as well as on whether they win the election. They compete to maximize their expected utility functions. Each political party aligns its preferences, including the altruistic component, to those of the poor to capture the

<sup>&</sup>lt;sup>8</sup>Fehr and Schmidt (2006) review the evidence gathered by experimental economists and psychologists. They also suggest how it can be best interpreted and how it should be modeled.

fact that the rich are not able to control the party system. We now present the model explaining its components in greater detail.

## The economy

We consider a society with n (odd) citizens. Individual i = 1, 2, 3, ..., n, has income  $y^i$ . Individuals are ordered from poorest to richest and let the median person be the individual with median income, denoted by  $y^M$ . The individual with median income is then individual M = (n+1)/2. Let  $\overline{y}$  denote the average income in this society,

$$\overline{y} = \left(\frac{1}{n}\right) \sum_{i=1}^{n} y^{i} \tag{1}$$

The political system determines a nonnegative income tax with rate  $0 \le \tau \le 1$ . Tax revenues are redistributed via lump sum transfers to all citizens. Let the resulting lump sum transfer to each individual be T.

We assume that it is costly to raise taxes. Let  $C : [0,1] \rightarrow \mathbb{R}_+$  be the deadweight cost of taxation associated to the tax rate. In the model, increases in the tax rate eventually reduces tax revenues due to the large loss associated with the distortions, captured by the assumption C(0)=0 and  $C'(\cdot)>0$ . Furthermore, we assume the cost to be strictly convex,  $C''(\cdot)>0$ , that C'(0)=0 and that C'(1)=1, ensuring the existence of an interior optimal tax rate. As total income in the economy is  $n\overline{y}$ , the cost induced by a tax rate  $\tau$  is given by  $C(\tau)n\overline{y}$ . These assumptions model the property that the disincentive effects of taxation become substantial as the tax rate increases. Finally, the lump sum transfer made by the government to each individual is

$$T = \frac{1}{n} \left( \sum_{i=1}^{n} \tau y^{i} - C(\tau) n \overline{y} \right) = (\tau - C(\tau)) \overline{y}$$
(2)

## Preferences of citizens and social groups

The society consists of two different socioeconomic groups of voters or social classes; the poor and the rich. The poor (p) have income below the average income in the society  $(y^{p} \leq \overline{y})$  and the rich (r) have income above the average income  $(y^{r} > \overline{y})$ . There are  $n_{p}$  poor individuals and  $n_{r}$  rich individuals, where  $n_{p} + n_{r} = n$  and  $n_{p} > n_{r}$ . as we assume individuals to be ordered from poorest to richest, the groups Poor and Rich are defined by:  $\Gamma^{p} = \{i \mid 1 \leq i \leq n_{p}\}, \ \Gamma^{r} = \{i \mid n_{p+1} \leq i \leq n\}$ . The sets of pre-tax

income of the poor and of the rich are, respectively,  $Y^p = \{y^k | k \in \Gamma^p\}$  and  $Y^r = \{y^k | k \in \Gamma^r\}$ . The average income of the poor is  $\overline{y}_p$  and of the rich is  $\overline{y}_r$ .

Individuals care about their own after-tax income and, departing from the Downsian model, the ideology of the party in power. The non-ideological component of their preferences is characterized by a direct utility function. Following a stripped down formulation of social identity theory (see Akerlof and Kranton (2000) and Shayo (2009), has altruistic motives towards his group. We assume that the altruistic motive has two components: it depends positively on the average income of the individuals in the group and negatively on the level of inequality in the group. The utility of an individual in group g is of the following form<sup>9</sup>

$$U^{g}(Y^{g}) = x^{g} + B_{g}\left[\overline{x}_{g} - E_{g}\sum_{k\in\Gamma^{g}}\frac{|x^{k} - \overline{x}_{g}|}{n_{g}}\right]$$

where  $x^g$  is the after tax income, that is  $x^g = (1-\tau)y^g + (\tau - C(\tau))\overline{y}$  and  $y^g$  is the pre-tax income of a voter in group g. The altruistic motive is modeled such that the utility of a voter in group g depends on the average income of the individuals in her group and by the level of income inequality in the group. The overall magnitude of the effect is captured by the parameter  $B_g$  while the relative weight associated to income inequality aversion by the parameter  $E_g$ .

The particular form of the utility function is borrowed from Wittman (2005) and has its justification in the maximization of the group's welfare. The realism of the two terms can be illustrated by the following. Suppose that the altruistic motive focuses only on the mean of the social group. An increase of the mean of the social group could be given by allocating all the income to one person. Clearly, there must be a better way to do this. Introducing aversion towards inequality can mend this weakness. In particular, we could then increase the median average income in a way that maximize the social welfare of the group. A rise in the average income of the peers would not be seen as beneficial if it is associated to a sharp rise in income inequality. There are, of course, many possible altruistic functions, but the linear formulation is a good compromise between tractability and realism.<sup>10</sup>

The utility function described above gives rise to an indirect (or ex-post) utility of voters in group g associated to the tax rate  $\tau$  that is noted  $V^{g}(\tau)$ . It is given by

<sup>&</sup>lt;sup>9</sup>With a slight abuse of notation, we use the superscript g to denote social classes as well as individuals in each social class, so for most of the discussion we have g = p or r.

<sup>&</sup>lt;sup>10</sup>See Wittman (2005) for the possible disadvantages of the utility function.

$$V^{g}(Y^{g} \mid \tau) = (1-\tau)y^{g} + B_{g}(1-\tau)\overline{y}_{g} - B_{g}E_{g}(1-\tau)\sum_{k\in\Gamma^{g}}\frac{|y^{k}-\overline{y}_{g}|}{n_{g}} + (1+B_{g})(\tau-C(\tau))\overline{y}$$
(3)

We use the notation  $V^{g}(Y^{g} | \tau)$  instead of  $V^{g}(\tau)$  because agents do not care directly about the tax rate, but taxes have an indirect effect because they affect the income of each agent, which is an argument of the utility function. Note that all individuals in a particular group have the same  $V^{g}(\tau)$  and that the egotistic utility a voter in group g obtains from income is linear in income as  $y^{g}$ . The weight the voter attributes to altruism is characterized by  $B_g > 0$ , the larger is  $B_g$ , the larger the voter weights the income in the group. The associated utility is  $B_g \overline{y}_g$ . On the other hand the disutility a voter obtains from income inequality in the group is characterized by a relative weight  $E_{\rm g}>0$ . Another way to see the same property is that the benefice of higher mean income of the peers  $B_{g} \overline{y}_{g}$ , is moderated by the multiplicative factor  $1 - \sum_{k \in \Gamma^g} (|y^k - \overline{y}_g|) / n_g \overline{y}_g$ , that is 1 minus the relative average absolute deviation in income. This view highlight an interesting feature of the indirect utility function  $V^{g}(Y^{g} | \tau)$ ; A rise in the average income of the peers would not be seen as beneficial if it is associated to a sharp rise in income inequality. Finally, it is worth noting that the assumptions about  $C(\tau)$  ensure that  $V^{s}(Y^{s} | \tau)$  is strictly concave and twice continuously differentiable, which are sufficient conditions for  $V^{s}$  to be single-peaked.

In the model agents are ideological. We assume that an individual j in group g has the following preferences over the tax rate and ideology

$$\tilde{V}^{gj}(\tau,m) = V^g(\tau) + \tilde{\sigma}_m^{gj}$$
(4)

when party m, m = L, R, comes to power. The term  $\mathcal{O}_m^{gj}$  can be interpreted as ideology (or non-policy) related benefits that the individual j receives from party m. Note that in the model individuals within the same economic group may have different ideological preferences. Let us define the difference in ideological benefits for individual j in group g by  $\mathcal{O}_R^{gj} = \mathcal{O}_R^{gj} - \mathcal{O}_L^{gj}$ . We assume that the  $\mathcal{O}_R^{gj}$  of an individual in group gis extracted from a given distribution characterized by a smooth cumulative distribution function  $F^g$  defined over  $(-\infty,\infty)$ , with the associated probability density function  $f^g$ .

Finally, note that the ideology position of group g is given by  $f^{s}(0)$ , that is the value at  $\sigma^{g} = 0$  of the probability density function associated  $\sigma^{g}$ . Indeed, with an argument similar to the one in Acemoglu and Robinson (2006), suppose that social

group p is more "ideological" than r, meaning that there are individuals in this group with strong preferences towards party L or R. This fact corresponds to the density function  $f^p$  having a relatively large share of its weight in the tails. In contrast, suppose group r is not very ideological, and the majority of the group vote for the party that gives them slightly better tax rate. This fact corresponds to having relatively little weight in the tails of  $f^r$ , and therefore a large value of  $f^r(0)$ . Then, the less ideological is a group g, the larger is the value of  $f^g(0)$ .

## **Voting behavior**

We adopt the framework of probabilistic voting. There is electoral competition between the two political parties, L and R. Let  $(\tau_L, \tau_R)$  be the policy platform proposed by the parties. Let  $\lambda^g$  be the proportion of voters in group g, with  $\lambda^p + \lambda^r = 1$ . Let  $\pi_m^g$  be the fraction, determined endogenously, of voters in group g voting for party m, where m = L, R. Then the expected share of voters of party m is

$$\pi_m = \lambda^r \pi_m^r + \lambda^p \pi_m^p. \tag{5}$$

The preferences on the tax rate by an individual j in group g is given by  $\tilde{V}^{gj}(\tau,m) = V^g(\tau) + \tilde{\sigma}_m^{gj}$ . As the difference in ideological benefits for individual j in group g is defined by  $\tilde{\sigma}^{gj} = \tilde{\sigma}_R^{gj} - \tilde{\sigma}_L^{gj}$ , the voting behavior of individual j can be represented by the following expression

$$p^{gj}(\tau_{L},\tau_{R}) = \begin{cases} 1 & \text{if } V^{g}(\tau_{L}) - V^{g}(\tau_{R}) > \tilde{\sigma}^{gj} \\ 1/2 & \text{if } V^{g}(\tau_{L}) - V^{g}(\tau_{R}) = \tilde{\sigma}^{gj} \\ 0 & \text{if } V^{g}(\tau_{L}) - V^{g}(\tau_{R}) < \tilde{\sigma}^{gj} \end{cases}$$
(6)

Equation (6) gives the conditions for individual j in group g to prefer  $\tau_L$  over  $\tau_R$ . Note that, as all that matters is the difference in ideological benefits, the condition involves only  $\sigma^{gj}$ . Expression (6) implies that the fraction of voters in group voting for party L (and its policy  $\tau_L$ ) is

$$\pi_L^g = F^g \left( V^g(\tau_L) - V^g(\tau_R) \right) \tag{7}$$

where  $F^{g}$  is the smooth cumulative distribution function, with the associated probability density function  $f^{g}$ , from which  $\tilde{\sigma}^{g}$  of individual j in group g is extracted.

## **Political** parties

Political parties L and R are partial in the sense that they have preferences over policies as well as over whether they win the election. Let  $w_L(\tau)$  and  $w_R(\tau)$  be the utility party L and R obtain from policy  $\tau$ . Let  $P(\tau_L, \tau_R)$  be the probability that party L wins the election when the parties offer the policy platform  $(\tau_L, \tau_R)$ . Party Rnaturally wins the election with probability  $1 - P(\tau_L, \tau_R)$ .

In probabilistic voting, the probability that a party wins the election is given by its vote share. Then, using equations (5) and (7), the probability that party L wins the election is given by

$$P(\tau_L, \tau_R) = \sum_{g=r, p} \lambda^g F^g \left( V^g(\tau_L) - V^g(\tau_R) \right)$$

Each political party chooses the tax rate that maximizes its expected utility for the given policy platform

Party 
$$L: Max[P(\tau_L, \tau_R)(K + w_L(\tau_L)) + (1 - P(\tau_L, \tau_R))w_L(\tau_R)]$$
 (8)  
Party  $R: Max_{\tau_R}[(1 - P(\tau_L, \tau_R))(K + w_R(\tau_R)) + P(\tau_L, \tau_R)w_R(\tau_L)]$ 

where K is the rent from being in office, which is assumed to be non-negative. As in the present model ideology and policy are completely independent dimensions of choice, there is no guide in choosing party preferences. We assume that both parties adopt the preference of the most numerous, the poor, that is  $w_L(\tau) = w_R(\tau) = V^p(\tau)$ . This is arguably the less favorable case for the elite, and therefore provides the strongest possible result. Furthermore it capture the fact that the rich are not able to control the party system.

Finally, we also assume that both parties choose their policies (policy platforms) simultaneously. So far we have defined all the basic blocs of the model. Now we proceed to obtain the equilibrium tax rate.

#### Political equilibrium: the model without ideology

First, we analyze the case without ideology motives. In this Downsian model all that matters to determine the equilibrium tax rate is the income of the voters. This simpler model allows us to obtain the tax rate preferred by an individual in each social group. Then, we obtain the tax rate preferred by the median voter.

## The tax rate preferred by each social group

We derive each individual g 's ideal tax rate from the indirect utility function. Indeed, this is the tax rate  $\tau^{g}$  that maximizes  $V^{g}(Y^{g} | \tau)$  given by expression (3). The first-order conditions, written in their Kuhn-Tucker form, allowing for the possibility that the preferred tax rate by an agent in group g is zero, can be written as

$$-y^{g} - B_{g}\overline{y}_{g} + B_{g}E_{g}\sum_{k\in\Gamma^{g}}\frac{|y^{k}-\overline{y}_{g}|}{n_{g}} + (1+B_{g})(1-C'(\tau^{g}))\overline{y} = 0 \text{ if } \tau^{g} > 0, \text{ and} -y^{g} - B_{g}\overline{y}_{g} + B_{g}E_{g}\sum_{k\in\Gamma^{g}}\frac{|y^{k}-\overline{y}_{g}|}{n_{g}} + (1+B_{g})(1-C'(\tau^{g}))\overline{y} \le 0 \text{ if } \tau^{g} = 0.$$
(9)

Note that a sufficient condition for a maximum is  $(1+B_g)(-C''(\tau^g))\overline{y} < 0$ , which always holds as  $C''(\tau^g) > 0$ . Furthermore, the preferred tax rate is positive  $\tau^g > 0$  whenever  $1 > C'(\tau^g) > 0$ . The following result is proved in the appendix.

**Lemma I** Let the preferences of a poor voter be given by expression (3) with g = p. There is a value  $\hat{E}_p > 0$  in the weight given by the poor to inequality aversion such that for any  $E_p \leq \hat{E}_p$  a poor voter prefers a positive income tax with rate  $0 < \tau^p < 1$ .

The previous analysis ensures that, in a model without ideology, a poor voter always prefers a strictly positive tax rate and that the poorer she is the greater is the tax rate she prefers. Furthermore, her preferred tax rate also positively depend on her altruism (size of  $B_p$ ), the income inequality among the whole population (size of  $\overline{y} - \overline{y}_p$ ), the weight on inequality aversion (size of  $E_p$ ), and finally the size of inequality among the poor (size of  $\sum_{k \in \Gamma^p} \frac{|y^k - \overline{y}_p|}{n_p}$ ).

A similar exercise can be performed for a rich individual. However in this case, when  $E_r > \hat{E}_r$ , it is not straightforward to see that a rich voter always prefers a positive tax rate. Consequently, we can only state the conditions for the rich preferring a zero tax rate as stated in the following Lemma:

**Lemma 2** Let the preferences of a rich voter be given by expression (3) with g = r. There is a value  $\hat{E}_r > 0$  in the weight given by the rich to inequality aversion such that for any  $E_r \leq \hat{E}_r$  a rich voter always prefer a zero tax rate,  $\tau^r = 0$ .

The previous analysis, derived from the Kuhn-Tucker conditions, imply that the poor always want a positive tax rate,  $0 < \tau^p < 1$ , whenever  $E_p \leq \hat{E}_p$ . On the other hand, the rich do not want any redistribution,  $\tau^r = 0$ , whenever  $E_r \leq \hat{E}_r$ . As the poor

favour high taxes and a large redistribution while the rich disfavor income redistribution, the choice of the tax rate is inherently conflictive. In this context the Median Voter Theorem implies that the equilibrium tax rate is the tax rate preferred by the median voter.

## The Median voter

As individuals are ordered from poorest to richest and there are more poor individuals than rich ones, the median voter is a poor voter  $(y^M < \overline{y})$ . From the discussion above, we conclude that the median voter prefers a strictly positive tax rate,  $0 < \tau^M < 1$ , that satisfies the condition in Lemma 1.

Form expression (9) we know that the tax rate preferred by the median voter is positively affected by several factors: 1) an increase in income inequality; 2) an increase in inner altruism,  $B_p$ ; 3) an increase in the the weight of inequality aversion,  $E_p$ ; and 4) an increase in inequality aversion. These factors in turn affect the equilibrium tax rate. We apply the implicit function theorem to expression (9) to analyze such effects. The effect of income inequality in the entire population on the tax rate is given by the following expression:

$$\frac{\partial \tau(y^p)}{\partial y^p} = \frac{-1}{\left(1 + B_p\right)C''\left(\tau(y^p)\right)y} < 0$$
(10)

Then, as inequality of income increases, keeping everything else constant and in particular  $\overline{y}$ , the median voter becomes poorer, and  $y^p$  decreases. Hence, from expression (10) we see that as the median voter corresponds to a poorer individual, then the median voter prefers a larger tax rate. This leads to the following Lemma:

**Lemma 3** There is a value  $\hat{E}_p > 0$  in the weight given by the poor to inequality aversion such that for any  $0 < E_p \leq \hat{E}_p$ , the grater is the inequality of income among the entire population, the greater is the tax rate preferred by the median voter.

Similarly, the implicit function theorem tells us that the impact of inner group altruism on tax rate is given by the following expression:

$$\frac{\partial \tau}{\partial B_p} = \frac{\left(1 - C'(\tau^p)\right)\overline{y} - \overline{y}_p + E_p \sum_{k \in \Gamma^p} \frac{|y^k - \overline{y}_p|}{n_p}}{\left(1 + B_p\right)C''(\tau^p)\overline{y}}$$
(11)

The sign of the derivative is determined by the sign of the numerator. We know that

$$(1 - C'(\tau^p))\overline{y} - \overline{y}_p$$
 is positive and  $E_p \sum_{k \in \Gamma^p} \frac{|y^k - \overline{y}_p|}{n_p}$  is positive as well.

From expression (28) in the appendix, we know that the poor prefer an interior tax rate whenever  $E_p \leq \hat{E}_p$  for  $\hat{E}_p = \frac{\overline{y}_p}{\sum_{k \in \Gamma^p} \frac{|y^k - \overline{y}_p|}{n_p}} > 0$ . Then  $\frac{\partial r}{\partial B_p} > 0$  if  $0 < E_p \leq \hat{E}_p$ .

Therefore, there always exists a bound below which an increase in altruism increases the tax level. The following proposition summarizes this result:

**Proposition 4** There is a value  $\hat{E}_p > 0$  in the weight given by the poor to inequality aversion such that for any  $0 < E_p \leq \hat{E}_p$  the greater is the inner group altruism exhibited by the median voter  $B_p$  the higher is the tax rate she prefers  $\tau^M$ .

Finally, we now focus on the direct impact of the weight the poor group places on inequality aversion. The fact that leads to the following result.

$$\frac{\partial \tau}{\partial E_p} = \frac{B_p \sum_{k \in \Gamma^p} \frac{|y^k - \overline{y}_p|}{p}}{(1 + B_p) C''(\tau^*) \overline{y}} > 0$$
(12)

**Lemma 5** There is a value  $\hat{E}_p > 0$  in the weight given by the poor to inequality aversion such that for any  $0 < E_p \leq \hat{E}_p$  the greater is the weight placed by the median voter on inequality aversion, the greater is the tax rate she prefers.

Lemma 3 and Lemma 5 along with Proposition 4 individuate the factors affecting the tax rate preferred by the median voter in the absence of ideology. By choosing the tax rate as the one preferred by the median voter and ignoring ideological concerns, the median voter model eliminates a powerful and surprising channel by which the minority of rich agents can influence the equilibrium tax rate. However, these results set the stage for our analysis with ideology.

#### Political equilibrium with ideology

The introduction of ideological concerns leads to a non-cooperative game between the parties. The resulting Nash equilibrium of the policy competition game between the two political parties is a pair of policies  $(\tau_L^*, \tau_R^*)$  that solve the maximization problems given by equation (8). The first-order condition of party L with respect to its own policy choice,  $\tau_L$ , taking the policy choice of the other party as given is:

$$\sum_{g=r,p} \lambda^{g} f^{g} \left( V^{g}(\tau_{L}^{*}) - V^{g}(\tau_{R}^{*}) \right) \frac{\partial V^{g}(\tau_{L}^{*})}{\partial \tau_{L}} \left( K + V^{p}(\tau_{L}^{*}) - V^{p}(\tau_{R}^{*}) \right) + \sum_{g=r,p} \lambda^{g} F^{g} \left( V^{g}(\tau_{L}^{*}) - V^{g}(\tau_{R}^{*}) \right) \frac{\partial V^{p}(\tau_{L}^{*})}{\partial \tau_{L}} = 0$$
(13)

In the appendix it is shown that the second-order condition associated to the above program is also satisfied, therefore the first order condition characterizes an interior maximum.

Since the problems of parties R and L are symmetrical, party R and party L promise the same policy. Hence, in equilibrium,  $\tau_R^* = \tau_L^* = \tau^*$ , satisfying:

$$\sum_{g=r,p} \lambda^{g} f^{g}(0) \frac{\partial V^{g}(\tau^{*})}{\partial \tau} K + \frac{\partial V^{p}(\tau^{*})}{\partial \tau} \sum_{g=r,p} \lambda^{g} F^{g}(0) = 0$$

Implying that

$$\sum_{g=r,p} \lambda^{g} f^{g}(0) \frac{\partial V^{g}(\tau^{*})}{\partial \tau} K + \frac{1}{2} \frac{\partial V^{p}(\tau^{*})}{\partial \tau} = 0$$
(14)

where the second line makes use of the fact that in equilibrium each party wins the election with probability  $\frac{1}{2}$ , thus  $\sum_{g=r,p} \lambda^g F^g(0) = \frac{1}{2}$ .

Equation (14), which gives equilibrium tax rates, also correspond to the solution to the maximization of the following weighted utilitarian social-welfare function:

$$\sum_{g=r,p} \chi^{g} \lambda^{g} V^{g}(\tau)$$
(15)

where  $\chi^r = f^r(0)K$  and  $\chi^p = f^p(0)K + \frac{1}{2\lambda^p}$  are the weights that different groups receive in the social-welfare function. We state this result as the following Lemma for future reference:

**Lemma 6** Consider a set of tax rates choices  $\tau \in [0,1]$  and let the preferences be given by (4) as a function of the tax rate and the ideological benefits from the party that is in power, with the distribution function of  $\tilde{\sigma}^{gi}$  given by  $F^{g}$ . Then, the equilibrium tax rate is given by  $\tau^*$  and it maximizes the weighted utilitarian social-welfare function, (15).

The equilibrium tax rate announced by both parties, obtained after substituting equation (3) into equation (14), and denoted by  $\tau^*$  satisfy

$$\lambda^{r} \left[ f^{r}(0) K \right] \left[ -y^{r} - B_{r} \overline{y}_{r} + B_{r} E_{r} \sum_{k \in \Gamma r} \frac{|y^{k} - \overline{y}_{r}|}{n_{r}} + (1 + B_{r}) (1 - C'(\tau^{*})) \overline{y} \right] + \lambda^{p} \left[ f^{p}(0) K + \frac{1}{2\lambda^{p}} \right] \left[ -y^{p} - B_{p} \overline{y}_{p} + B_{p} E_{p} \sum_{k \in \Gamma p} \frac{|y^{k} - \overline{y}_{p}|}{n_{p}} + (1 + B_{p}) (1 - C'(\tau^{*})) \overline{y} \right] = 0$$
(16)

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As seen from equation (16), the equilibrium tax rate depends on four central features of the society: i) the proportions of poor and rich ( $\lambda^p$ ,  $\lambda^r$ ); ii) The ideology position of each social group ( $f^p(0)$ ,  $f^r(0)$ ); iii) The degree of inner altruism of the social groups ( $B_p$ ,  $B_r$ ); and iv) the relative weight given to the aversion to income inequality in the social groups ( $E_p$ ,  $E_r$ ).

In the next sections we find conditions to guarantee the existence of an interior equilibrium tax rate and perform the comparative statics analysis.

## Equilibrium tax rate

We focus on situations in which the political equilibrium involves a positive tax rate, that is,  $\tau^* > 0$ . In this case, the outcome of the political competition among the parties leads to the equilibrium policy  $\tau_t^*$  which solves equation (14).

The first order conditions are necessary and sufficient for an interior equilibrium policy  $\tau^* > 0$  if and only if

$$\sum_{g=r,p} \lambda^{g} f^{g}(0) \frac{\partial V^{g}(0)}{\partial \tau} K + \frac{1}{2} \frac{\partial V^{p}(0)}{\partial \tau} > 0$$
(17)

Expression (17) is saying that the tax rate  $\tau^* = 0$  is not a critical point of the firstorder condition given by equation (14). We prove in the appendix that inequality (17) is satisfied for the set of parameter values we consider.

## The effect of ideology on the equilibrium tax rate

We may now apply the implicit function theorem to expression (16) to investigate the impact of ideology on the equilibrium tax rate. We have the following effects:

$$\frac{\partial \tau}{\partial f^{r}(0)} = \frac{\lambda^{r} \left[ \frac{dV^{r}(\tau^{*})}{d\tau} \right] K}{C''(\tau^{*}) \overline{y} \left( \lambda^{r} \chi^{r} (1+B_{r}) + \lambda^{p} \chi^{p} (1+B_{p}) \right)} < 0$$
(18)

$$\frac{\partial \tau}{\partial f^{p}(0)} = \frac{\lambda^{p} \left[ \frac{dV^{p}(\tau^{*})}{d\tau} \right] K}{C''(\tau^{*}) \overline{y} \left( \lambda^{r} \chi^{r} (1+B_{r}) + \lambda^{p} \chi^{p} (1+B_{p}) \right)} > 0$$
(19)

Expressions (18) and (19) show that, everything else constant, a social group pushes the tax rate in its "natural" direction by being less ideological. The less ideological are

the rich, the lower is the tax rate. The less ideological are the poor, the greater is the tax rate. The following proposition summarizes this result:

**Proposition 7** The less ideological are the rich (resp. the poor), the lower (resp. the higher) is the tax rate. In other words, a social group pushes the tax rate in its "natural" direction by being less ideological.

In a different model, Persson and Tabellini (2000) conclude that ideological neutral groups are more responsive to policy (and hence care less about ideology) in a neighborhood of the equilibrium policy; they are more likely to reward politicians with votes and get a policy closer to their bliss point.

## The effect of the size of the social groups on the equilibrium tax rate

We may also use (16) to investigate the effect of the size of the social groups on the equilibrium tax rate. In a democracy it is generally expected that economic policies reflect the preferences of the majority, via the principle one-person-one-vote. In the model, the effect of the size of the social groups on the tax rate is characterized by the following expressions:

$$\frac{\partial \tau}{\partial \lambda^{r}} = \frac{\chi^{r} \left[ \frac{dV^{r}(\tau^{*})}{d\tau} \right]}{C^{\prime\prime}(\tau^{*}) \mathcal{Y}(\lambda^{r} \chi^{r}(1+B_{r}) + \lambda^{p} \chi^{p}(1+B_{p}))} < 0$$
(20)

$$\frac{\partial \tau}{\partial \lambda^{p}} = \frac{\chi^{p} \left[ \frac{dV^{p}(\tau^{*})}{d\tau} \right]}{C''(\tau^{*}) \overline{y} \left( \lambda^{r} \chi^{r} (1 + B_{r}) + \lambda^{p} \chi^{p} (1 + B_{p}) \right)} > 0$$
(21)

Expression (20) shows, everything else constant, that the greater is the proportion of rich, the lower is the equilibrium tax rate. On the other hand, expression (21) shows that the greater is the proportion of the poor, the greater is the equilibrium tax rate. This leads to the following proposition.

**Proposition 8** The greater is the relative size of the rich minority, the lower is the equilibrium tax rate.

Note that an analogous result for the majority could also be obtained: The greater is the proportion of poor, the greater is the tax rate. The proposition then supports the idea that the more numerous are the rich (resp. the poor) more influential they are, and consequently the tax rate decreases (resp. increases).

#### The power of the Elite: the role of social identification

The main focus of the paper is on the role of social identification, modeled via in-group altruism (see Section 2.2), on the tax rate achieved under political equilibrium. In the model, in-group altruism is formalized as

$$U^{g}(Y^{g}) = x^{g} + B_{g}\left[\overline{x}_{g} - E_{g}\sum_{k \in \Gamma^{g}} \frac{|x^{k} - \overline{x}_{g}|}{n_{g}}\right]$$

where  $x^{g}$  is after-tax income and  $Y^{g}$  is set of pre-tax incomes of voters in group g. We assume that the altruism, generated by social identification, has two dimensions. First, agents care about the average income of their own group. Second, agents dislike income inequality in their group. The magnitude of these two effects are captured by the coefficients  $B_{g}$  and  $B_{g}E_{g}$ . In this sense, a rise in  $B_{g}$ , keeping  $E_{g}$  fixed, can be seen as a uniform rise in altruism. The coefficient  $E_{g}$  represents the relative weight agents attribute to inequality aversion. In the following sections we investigate the role of these factors on the equilibrium tax rate. The analysis is intended to capture both differences across two democracies as well as temporal shifts within a given democracy.

## The role of in-group inequality aversion

In this section we focus on the direct impact of the weight that a social group g places on income inequality on the tax rate, keeping all other factors unaffected. We obtain the following expression:

$$\frac{\partial \tau}{\partial E_g} = \frac{\lambda^g \chi^g B_g \sum_{k \in \Gamma^g} \frac{|y^k - \overline{y}_g|}{n_g}}{C''(\tau^*) \overline{y} (\lambda^r \chi^r + \lambda^p \chi^p)} > 0$$
(22)

As the derivative is always positive, expression (22) tells us that, everything else equal, the greater is the weight placed on inequality aversion, the greater is the equilibrium tax rate, this effect being true for both the poor majority and the rich minority.

**Proposition 9** Everything else equal, the greater is the weight placed by a social group on in-group inequality aversion, the greater is the equilibrium tax rate.

The results hold irrespective of the levels of ideology and altruism  $B_g$ . In particular, even for the elite a rise in their inequality aversion imply an increase of their preferred tax rate. Note that without ideology, only the properties of the median voter, which is a poor voter, would matter.

## The role of pure in-group altruism

Unlike the effect of in-group inequality aversion on the tax rate, which does not depend on the levels of ideology and altruism, the effect of the level of overall in-group altruism,  $B_g$ , depends on in-group inequality aversion. Formally, the marginal effect of  $B_g$  on the tax rate is given by the following expression

$$\frac{\partial \tau}{\partial B_{g}} = \frac{\lambda^{g} \chi^{g} \left[ \left( 1 - C'(\tau^{*}) \right) \overline{y} - \overline{y}_{g} + E_{g} \sum_{k \in \Gamma^{g}} \frac{|y^{k} - \overline{y}_{g}|}{n_{g}} \right]}{C''(\tau^{*}) \overline{y} \left( \lambda^{r} \chi^{r} (1 + B_{r}) + \lambda^{p} \chi^{p} (1 + B_{p}) \right)}$$
(23)

The sign of the derivative is determined by the sign of the numerator, the denominator being always positive. The effect of a rise in altruism  $B_g$  depends on the relative weight given to inequality aversion  $E_g$  as the following results shows. Importantly, in the Proposition the bounds  $\hat{E}_r$  and  $\hat{E}_p$  depends on all the parameters of the model via  $\tau^*$ , and in particular on ideology.

**Proposition 10** The comparative statics of the equilibrium concerning the level of altruism  $B_g$  is such that

- 1) There is a value  $\hat{E}_r > 0$  in the weight given by the rich minority to inequality aversion such that for any  $0 < E_r \leq \hat{E}_r$  the greater is the in-group altruism within the rich,  $B_r$ , the lower is the tax rate  $\tau^*$ .
- 2) There is a value  $\hat{E}_p > 0$  in the weight given by the poor majority to inequality aversion, such that for any  $0 < E_p \leq \hat{E}_p$  the greater is the in-group altruism within the poor,  $B_p$ , the higher is the tax rate  $\tau^*$ .

**Proof Part I.** As  $0 < C'(\tau^*) < 1$  the expression  $\overline{y}_r - (1 - C'(\tau^*))\overline{y}$  is always positive. Expression (30) in the appendix shows that according to their egoistic motives, the rich prefer a zero tax rate whenever  $0 < E_r \leq \hat{E}_r$  for  $\hat{E}_r = \frac{\overline{y}_r - \overline{y}}{\sum_{k \in \Gamma'} \frac{|y^k - \overline{y}_r|}{n_r}}$ . As

$$\left[ \left( 1 - C'(\tau^*) \right) \overline{y} - \overline{y}_r + E_r \sum_{k \in \Gamma'} \frac{|y^k - \overline{y}_r|}{n_r} \right] < 0 \text{ for } E_r \leq \hat{E}_r \text{ , we have that}$$

$$\frac{\partial \tau}{\partial B_r} < 0 \text{ if } 0 < E_r \le \hat{E}_r$$
(24)

Therefore, there always exists a bound in the in-group inequality aversion of the rich below which an increase in altruism decreases the tax level.

**Part 2**. On the other hand, focusing on the poor agents, the sign of  $\frac{\partial \tau}{\partial B_p}$  also depends on the sign of the numerator of expression (23). We can use the bound used to sign expression (12) in section 2 to obtain that

$$\frac{\partial \tau}{\partial B_p} > 0 \text{ if } 0 < E_p \le \hat{E}_p$$
(25)

Therefore, there always exists a bound in the in-group inequality aversion of the poor below which an increase in altruism increases the tax level. Note that the bound is affected by the level of ideology, as it depends on the tax rate  $\tau^*$  which is affected by ideology.

The intuition for the first result is as follows. Because the rich are a minority, their individual income is a decreasing function of the tax rate, and therefore rich agents tend to prefer a zero tax for any  $0 < E_r \leq \hat{E}_r$ , at least according to their egoistic motives. The bound exists because their inequality aversion pushes them to vote for a higher tax rate. Finally, by rising their in-group altruism,  $B_r$ , their political weight grows leading to a lower tax rate.

The intuition for the second result is as follows. Because the poor are the majority, their individual income is an increasing function of the tax rate, and therefore poor agents tend to prefer a high tax rate. Furthermore, their inequality aversion also pushes them to vote for a higher tax rate. Therefore, this holds as long the equilibrium exists, which is ensured by  $0 < E_p \leq \hat{E}_p$ .<sup>11</sup> Finally, by rising their altruism  $B_p$  their political weight grows leading to a higher tax rate.

The general mechanism operating here is that in-group altruism within members of a social group depreciates the relative weight given to ideology and therefore increases the political power of the social group. In particular, this means that an in-group altruistic Elite minority may gain a disproportionate power in the choice of public policies without the need of being less ideological.

## Opposite changes in in-group altruism across social groups

In the previous analysis we focus on the effect of variations in a single parameter on the equilibrium tax rate. However, often differences between countries affect several parameters. Similar shifts across time for a given country can also affect more than one parameter. We consider two main cases.

In the first scenario, suppose the degree of in-group altruism in both social

<sup>&</sup>quot;Existence is not a problem for the rich, as they never vote for a too high tax rate.

groups varies in exactly opposite directions but by the same magnitude. For example, consider two countries, democracy I and democracy 2. Let the degree of in-group altruism of the poor and the rich in country k be  $(B_{pk}, B_{rk})$  respectively. Suppose that the difference in altruism across the two social groups in democracy 2 is higher than in democracy I but assume that  $B_r + B_p$  is the same in the two democracies, that is,  $(B_{p2}, B_{r2}) = (B_{p1} - \varphi, B_{r1} + \varphi)$  with  $\varphi > 0$ .<sup>12</sup> The question we want to address here is to know in which democracy the tax rate is greatest. Although this way of capturing the effect of differences in inner altruism is obviously restrictive, it helps to gain an intuition on which democracy will face a higher tax rate. The result leads to the following proposition:

**Proposition 11** Assume that the change in  $B_r$  and in  $B_p$  are exactly opposite, that is  $B_r + B_p$ , is a constant. There exist values  $(\hat{E}_p, \hat{E}_r)$  as defined in Proposition 10 such that if  $0 < E_p \le \hat{E}_p$  and  $0 < E_r \le \hat{E}_r$ , then the following is true

- a) If the rich increase their inner altruism while the poor decrease theirs in the same amount in equilibrium the tax rate reduces.
- b) If the poor increase their inner altruism while the rich decrease theirs in the same amount in equilibrium the tax rate increases.

## Proof. See appendix.

Proposition 11 states that in a democracy in which the difference in social identification across social groups is greater, the social group that sees an increase in its identification pushes the tax rate in its "natural" direction. In particular, if, as given in part a) of the Proposition, in democracy 2 the rich exhibit greater group identification than the poor, then the equilibrium tax rate in this democracy is lower than the tax rate in democracy 1. On the other hand, if, as given in part b) of the Proposition, in democracy is greater group identification than the rich, then the equilibrium tax rate in this democracy 1. This result holds irrespective of the prevailing levels of ideology and altruism in democracy 1.

 $B_{r2} + B_{p2} = B_{r1} + B_{p1} \; .$ 

 $<sup>^{\</sup>rm 12} {\rm We}$  have the opposite case as well. Let  $B_{_{p2}}=B_{_{p1}}+\varphi$  ,  $B_{_{r2}}=B_{_{r1}}-\varphi$  with  $\varphi>0$  , We obtain

## Social identity and Ideology

In the previous section we looked at the case in which the level of ideology of the social groups is constant, or in the cross-country interpretation, the level of ideology of the rich (resp. the poor) in both countries is the same. Here we analyze the effect of changes in ideology on the tax rate. We will see that the effect on the equilibrium tax rate is ambiguous and depends on the prevailing levels of altruism and the weight given to inequality aversion by both social groups.

To simplify the analysis and obtain sharp result we consider changes in the level of ideology occurring in the direction of the vector representing the existing ideology. For example, consider two countries, democracy I and democracy 2. Let the levels of ideology of the poor and the rich in country k be  $(f_k^p(0), f_k^r(0))$  respectively. Supposing that  $(f_2^p(0), f_2^r(0)) = t(f_1^p(0), f_1^r(0))$  with t > 0, we would like to know in which democracy the tax rate is greatest. The following results hold

## **Proposition 12**

- 1) Suppose the rich are less ideological than the poor,  $f^r(0) > f^p(0)$ , and the rich exhibit higher in-group altruism than the poor,  $B_r > B_p$ , then, there exist values  $\tilde{E}_p > 0$  and  $\tilde{E}_r > 0$  such that if  $0 < E_p \le \tilde{E}_p$  and  $0 < E_r \le -\frac{\tilde{E}_r}{\tilde{E}_p}E_p + \tilde{E}_r$  an increase in the level of ideology in the direction  $f(0) = (f^p(0), f^r(0))$  reduces the equilibrium tax rate.
- 2) Suppose the rich exhibit higher in-group altruism than the poor,  $B_r > B_p > 0$ , then there exist  $\theta$ ,  $0 < \theta < 1$ , and values  $\tilde{E}_p > 0$  and  $\tilde{E}_r > 0$  such that if  $0 < E_p \leq \tilde{E}_p$  and  $0 < E_r \leq -\frac{\tilde{E}_r}{\tilde{E}_p}E_p + \tilde{E}_r$  and the rich are more ideological than the poor with  $\theta f^p(0) < f^r(0) < f^p(0)$ , an increase in the level of ideology in the direction  $f(0) = (f^p(0), f^r(0))$  reduces the equilibrium tax rate.

## Proof. See appendix.

The first part of the Proposition is intuitive in view of the previous sections. Indeed, both low level of ideology and high level of altruism generate political power to the group. Increasing ideology in this case, will increase the political power of the rich. Provided the rich place a low weight on their inequality aversion, the rise of minority power translates into a decrease in the tax rate, the "natural" direction for the rich.

The second part of the Proposition is more surprising. Indeed, it shows that the condition that the rich are less ideological than the poor is not necessary to obtain a

result similar to Part I. of the Proposition. In fact, there exists an interval of ideologies for the rich lying above the ideology of the poor such that a rise in ideology reduces the equilibrium tax rate, provided the rich exhibit higher in-group altruism than the poor and their inequality aversion is not too high.

This result is important because it sheds light on the trade-off between ideology and social identity. At first sight, and in view of the literature, in a society where the rich are more ideological than the poor, as the rich individuals are more attached to the political parties, the rich receive low favour from the economic policy. However, we show that this fact could be reversed if, in addition, the rich exhibit greater social identification than the poor do.

In this regard, Acemoglu and Robinson (2006) propose a mechanism by which a minority can influence the political outcome even in a purely democratic one man one vote democracy, with a 50% threshold. Indeed, in a probabilistic voting model ideology is a feature that unambiguously decreases political power so that a scarcely ideological minority can defeat a highly ideological majority in the political process. We show that in the presence of social identification this simply link may break. Indeed, even a highly ideological minority could defeat a less ideological majority, provided the minority exhibits a higher degree of social identification towards the members of the minority.

## Explaining the dispersion of tax rates

The model proposes a mechanism by which a country can vote a low tax rate as a result of the disproportionate power gained by the rich minority. The results are generated by the interplay between ideology, social identification and pre-tax income distribution.

First, suppose that ideology doesn't play any role, in which case the model is Downsian and what counts is the median voter, who is a poor voter. As a class is defined in the present model by its average income, the highest tax rate would be obtained for countries in which the poor class has the strongest class identification. Countries in which poor voters have a strong national identification but a low class identification would come next, but the difference would be small as the poor form the majority. Last would come those countries where voters show little social identification, at least along the dimension of wealth or income. In general, the stronger is the social identification the larger should be the tax rate and therefore the redistribution.

This simple mechanism outlined above is counterfactual as national identification is, according to the data, much stronger in the US than in Europe while the tax rate is much lower in the US. However, a twist may mend the problem. Assuming that the total amount of identification is constant would allow to link large national identification to low class identification. This mechanism would lead to a lower tax rate for countries with high national identity as the US (Figure 8, Shayo). However, it may be argued that as the majority is poor, it is unlikely that shifting the identification

of the poor majority from class identification to national identification would generate enough effect to explain the divide US/Northern Europe. In fact, only via a dynamic feedback, leading to multiple equilibria as in Shayo (2009), this channel may produce the expected large effect. In the present paper, we reveal a new channel by introducing ideology, in which case the vote intentions of swing voters matter and can affect the policy outcome.

With ideology, the swing voters are important and these can push the equilibrium in their direction. A first way swing voters can be highly redistributive is when these are poor. Another possibility is that the swing voters are rich but their national identity dominates their class identity or when they have a strong in-class inequality aversion. Putting together these pieces we can attempt the following explanation for the observed wide cross country variation in the tax rates. We now attempt a concrete picture.

The low tax rate in the US can be rationalized as follows. In the US the level of ideology is small and the swing voters play an important role. However, as the poor are more ideological than the rich, the swing voters are rich, and a low tax rate is produced provided the national identity of the rich is not too strong (that is still compatible with a strong average national identity) or the inequality aversion of the rich is not too large.

The northern European and Germanic countries have a high tax rate. This can be explained supposing that these are highly ideological countries in which the role of swing voters is reduced. Furthermore, due to the high education of the poor, the poor form a large share of the swing voters. Finally, if the swing voters are poor, or if they have a large in-group inequality aversion, the tax rate would be definitely high.

In the Latin countries, both the rich and the poor are highly ideological, reducing the role of swing voters. Still, we should expect the poor to be more ideological than the rich because of lower education. The swing voters are mostly, but not exclusively, rich individuals. Furthermore, inequality aversion is higher than in the US. As a result redistribution is still higher than in the US but lower than in Northern Europe. Finally, in Anglo-saxon countries, the poor are slightly less ideological than in other European countries, increasing the scope of the swing voters. Furthermore, because of the lack of strong class or national identity, the tax rate is lower than in many other European countries.

We would like to compare this scenario with the data. Unfortunately, data on the level of ideology of voters is difficult to obtain. Indeed, votes for a given party, or intention of votes, reflect both the ideology preferences as well as the pure, brand like, ideology. In other words, parties tend to be associated with policies, for example the Republican to low tax rates. On the other hand, in our model ideology is a brand name that is not correlated to the policies implemented. Because ideology is here the residual after taking into account the real policy, pure ideology it very hard to measure. Furthermore, the model asks for data on ideology by class, which is a further obstacle. Obtaining data on social identification is not much easier, particularly when specific to given classes.

# Conclusions

Why in democracies where power is apparently given to the most numerous class government decisions seem to be made which favour a minority, the elite? Our analysis shows that the answer may rely on the level of social identification of the groups. Among the many results delivered by the analysis, we show that if the rich place a low weight on their income inequality but care about the average income in their class, the higher their social identification they exhibit, the lower the equilibrium tax rate. Our analysis also compares democracies that differ along several dimensions, but in this case the effect of social identification depends generally on the weights given to inequality aversion. We find interesting the result that if in democracy A the rich are more ideological than in democracy B but they exhibit greater social identification than the poor, the equilibrium tax rate in democracy A may be lower than the tax rate in democracy B. Finally, we exploit this mechanism to attempt an explanation of the wide difference in the tax rate in place across otherwise similar countries, a long standing puzzle.

Altogether, these results highlight the role played by social identity in the political power of the minorities and in the role these have in determining equilibrium policies. There are several directions for future research. In the paper, social identity as well as the distribution of ideology are exogenously given. In a more comprehensive model, classes would realize the gains from losing their ideological component and strengthening their social identity and consequently would take actions to affect the distribution of these factors as a response to what they expect from the government.

## Annexes

Proof of Lemma I. Rearranging (9), we obtain the condition

$$1 > C'\left(\tau^{g}\right) = 1 - \frac{1}{\left(1 + B_{g}\right)\overline{y}}\left[y^{g} + B_{g}\overline{y}_{g} - B_{g}E_{g}\sum_{k\in\Gamma^{g}}\frac{|y^{k} - \overline{y}_{g}|}{n_{g}}\right] = 1 - \frac{U^{g}(Y^{g})}{\overline{y}\left(1 + B_{g}\right)}$$
(26)

However, from expression (26) it is not straightforward to conclude that the poor prefer a strictly positive tax rate. We know that for a poor individual there exists  $\varepsilon > 0$  such that  $y^p = \overline{y} - \varepsilon$ . Expression (g4) then becomes

$$C'(\tau^{p}) = \frac{1}{(1+B_{p})\overline{y}} \left[ \varepsilon + B_{p} \left( \overline{y} - \overline{y}_{p} + E_{p} \sum_{k \in \Gamma^{p}} \frac{|y^{k} - \overline{y}_{p}|}{n_{p}} \right) \right]$$
(27)

Note that expression (27) is positive because  $\mathcal{E} > 0$ ,  $B_p > 0$ ,  $\overline{y} - \overline{y}_p > 0$ ,  $E_p > 0$ , and  $\sum_{k \in \Gamma^p} \frac{|y^k - \overline{y}_p|}{n_p} > 0$ . Then, given the convexity of  $C(\cdot)$  and the fact that  $C'(\tau^p) > 0$ , a poor voter prefers a positive tax rate,  $\tau^p > 0$ .

In addition, we need to guarantee an interior optimal tax rate for the poor. This is,  $1 > \tau^p$ . Then, given the convexity of  $C(\cdot)$  such condition reduces to satisfy  $1 > C'(\tau^p)$ . For g = p in expression (9) we have

$$1 - C'(\tau^p) = \frac{1}{(1 + B_p)\overline{y}} \left[ y^p + B_p \overline{y}_p - B_p E_p \sum_{k \in \Gamma^p} \frac{|y^k - \overline{y}_p|}{n_p} \right]$$

Then,  $1 - C'(\tau^p) > 0$  implies

$$y^{p} + B_{p} \overline{y}_{p} - B_{p} E_{p} \sum_{k \in \Gamma^{p}} \frac{|y^{k} - \overline{y}_{p}|}{n_{p}} > 0$$

which reduces to

$$y^{p} + B_{p}\left[\overline{y}_{p} - E_{p}\sum_{k\in\Gamma^{p}}\frac{|y^{k}-\overline{y}_{p}|}{n_{p}}\right] > 0$$

As  $y^p > 0$ , for the previous expression to hold we need  $\overline{y}_p - E_p \sum_{k \in \Gamma^p} \frac{|y^k - \overline{y}_p|}{n_p} \ge 0$ . Then, whether  $1 - C'(\tau^p) > 0$  depends on the value of  $E_p$ . Let  $\hat{E}_p = \frac{\overline{y}_p}{\sum_{k \in \Gamma^p} \frac{|y^k - \overline{y}_p|}{n_p}} > 0$ such that  $\overline{y}_p - \hat{E}_p \sum_{k \in \Gamma^p} \frac{|y^k - \overline{y}_p|}{n_p} = 0$ . Therefore

In that  $y_p - E_p \sum_{k \in \Gamma^p} \frac{r}{n_p} = 0$  . Therefore

$$1 - C'(\tau^p) > 0 \text{ if } 0 < E_p \le \hat{E}_p$$
 (28)

As expression (28) is positive whenever  $0 < E_p \leq \hat{E}_p$  the poor has an interior solution,  $0 < \tau^p < 1$ .

#### Proof of Lemma 2.

In this case there exists  $\varepsilon_1 > 0$  such that  $y^r = \overline{y} + \varepsilon_1$ . For this rich individual expression (26) becomes

$$C'(\tau^{r}) = \frac{1}{(1+B_{r})\overline{y}} \left[ -\varepsilon_{1} + B_{r} \left( \overline{y} - \overline{y}_{r} + E_{r} \sum_{k \in \Gamma^{r}} \frac{|y^{k} - \overline{y}_{r}|}{n_{r}} \right) \right]$$
(29)

The tax rate preferred by the rich depends on the sign of expression (29). The rich individual would prefer a zero tax rate,  $\tau^r = 0$ , whenever  $C'(\tau^r) \le 0$  while she would prefer a positive tax rate,  $\tau^r > 0$ , whenever  $C'(\tau^r) \ge 0$ . As  $-\varepsilon_1 < 0$ ,  $\overline{y} - \overline{y}_r < 0$ ,  $B_r > 0$ ,  $E_r > 0$ , and  $\sum_{k \in \Gamma'} \frac{|y^k - \overline{y}_r|}{n_r} > 0$  the sign of expression (29) is determined by the sign of expression  $(\overline{y} - \overline{y}_r + E_r \sum_{k \in \Gamma'} \frac{|y^k - \overline{y}_r|}{n_r})$ . Then, whether expression (29) is negative depends on the value of  $E_r$  and on how big is  $\varepsilon_1$ . First, we focus on the value of  $E_r$ . Let  $\hat{E}_r = \frac{\overline{y}_r - \overline{y}}{\sum_{k \in \Gamma'} \frac{|y^k - \overline{y}_r|}{n_r}} > 0$  such that  $\overline{y} - \overline{y}_r + \hat{E}_r \sum_{k \in \Gamma'} \frac{|y^k - \overline{y}_r|}{n_r} = 0$ . Note that  $C'(\tau^r) = \frac{-\varepsilon_1}{(1+B_r)\overline{y}} < 0$  if  $E_r = \hat{E}_r$ . Therefore

$$C'(\tau^r) < 0 \text{ if } 0 < E_r \le \hat{E}_r$$
(30)

As expression (29) is negative whenever  $0 < E_r \leq \hat{E}_r$  the rich prefer a zero tax rate,  $\tau^r = 0$ . In this case, we have a corner solution and the first-order condition

given by expression (9) does not hold as an equality.<sup>13</sup> Note that when  $E_r > \hat{E}_r$  it is not straight forward to see that a rich voter always prefers a positive tax rate. In fact, in this case we would also need to specify the value of  $\mathcal{E}_1$  leading the rich voter to prefer a positive tax rate.

**Proof Sufficient condition for the partisan parties' maximization problem.** The first order conditions associated to the maximization of the expected vote share of party L is (according to expression (13))

$$\begin{split} \sum_{g=r,p} \lambda^{g} f^{g} \left( V^{g}(\tau_{L}) - V^{g}(\tau_{R}) \right) \frac{\partial V^{g}(\tau_{L})}{\partial \tau_{L}} \left( K + V^{p}(\tau_{L}) - V^{p}(\tau_{R}) \right) \\ &+ \sum_{g=r,p} \lambda^{g} F^{g} \left( V^{g}(\tau_{L}) - V^{g}(\tau_{R}) \right) \frac{\partial V^{p}(\tau_{L})}{\partial \tau_{L}} = 0 \end{split}$$

Taking the derivative of the previous expression with respect to  $\tau_L$  and evaluating it at the optimum  $(\tau_L^*, \tau_R^*)$  we obtain expression SC:

$$\begin{split} SC &= \sum_{g=r, p} \lambda^{g} \left[ f^{g} \left( V^{g} \left( \tau_{L}^{*} \right) - V^{g} \left( \tau_{R}^{*} \right) \right) \frac{\partial V^{g} \left( \tau_{L}^{*} \right)}{\partial \tau_{L}} \frac{\partial V^{p} \left( \tau_{L}^{*} \right)}{\partial \tau_{L}} + \left( K + V^{p} \left( \tau_{L}^{*} \right) - V^{p} \left( \tau_{R}^{*} \right) \right) \frac{\partial^{2} V^{g} \left( \tau_{L}^{*} \right)}{\partial \tau_{L}^{2}} + \left[ \frac{\partial V^{g} \left( \tau_{L}^{*} \right)}{\partial \tau_{L}} \right]^{2} f^{g'} \left( V^{g} \left( \tau_{L}^{*} \right) - V^{g} \left( \tau_{R}^{*} \right) \right) \frac{\partial^{2} V^{g} \left( \tau_{L}^{*} \right)}{\partial \tau_{L}^{2}} + \left[ \frac{\partial V^{g} \left( \tau_{L}^{*} \right)}{\partial \tau_{L}} \right]^{2} f^{g'} \left( V^{g} \left( \tau_{L}^{*} \right) - V^{g} \left( \tau_{R}^{*} \right) \right) \frac{\partial^{2} V^{p} \left( \tau_{L}^{*} \right)}{\partial \tau_{L}^{2}} + \frac{\partial V^{p} \left( \tau_{L}^{*} \right)}{\partial \tau_{L}} F^{g} \left( V^{g} \left( \tau_{L}^{*} \right) - V^{g} \left( \tau_{R}^{*} \right) \right) \frac{\partial^{2} V^{g} \left( \tau_{L}^{*} \right)}{\partial \tau_{L}^{2}} + \frac{\partial V^{p} \left( \tau_{L}^{*} \right)}{\partial \tau_{L}} F^{g} \left( V^{g} \left( \tau_{L}^{*} \right) - V^{g} \left( \tau_{R}^{*} \right) \right) \frac{\partial V^{g} \left( \tau_{L}^{*} \right)}{\partial \tau_{L}} \right] \end{split}$$

As we know that  $\tau^* = \tau^*_{\scriptscriptstyle R} = \tau^*_{\scriptscriptstyle L}$  we obtain

$$\begin{split} SC = & \sum_{g=r, p} \lambda^{g} \left[ f^{g} \left( 0 \right) \frac{\partial V^{g} \left( \tau^{*} \right)}{\partial \tau_{L}} \frac{\partial V^{P} \left( \tau^{*} \right)}{\partial \tau_{L}} + K \left\{ f^{g} \left( 0 \right) \frac{\partial^{2} V^{g} \left( \tau^{*} \right)}{\partial \tau_{L}^{2}} + \left[ \frac{\partial V^{g} \left( \tau^{*} \right)}{\partial \tau_{L}} \right]^{2} f^{g'} \left( 0 \right) \right\} \right] \\ & + \sum_{g=r, p} \lambda^{g} \left[ F^{g} \left( 0 \right) \frac{\partial^{2} V^{P} \left( \tau^{*} \right)}{\partial \tau_{L}^{2}} + \frac{\partial V^{P} \left( \tau^{*} \right)}{\partial \tau_{L}} f^{g} \left( 0 \right) \frac{\partial V^{g} \left( \tau^{*} \right)}{\partial \tau_{L}} \right] \end{split}$$

We need to show that SC < 0 in the equilibrium. As the probability density function  $f^{g}$  associated to  $\tilde{\sigma}^{gj}$  is smooth and symmetric, its mean equals its median and mode, so that at  $\tilde{\sigma}^{gj} = 0$  we have  $f^{g'}(0) = 0$ . Then

<sup>&</sup>lt;sup>13</sup>Note that it is not possible in general to compare  $\hat{E}_r$  and  $\hat{E}_p$  because  $\sum_{k \in \Gamma^g} \frac{|y^k - \bar{y}_g|}{n_g}$  cannot be compared.

$$\begin{split} SC &= \lambda^{r} [f^{r}(0) \frac{\partial V^{r}(\tau^{*})}{\partial \tau_{L}} \frac{\partial V^{p}(\tau^{*})}{\partial \tau_{L}} + K f^{r}(0) \frac{\partial^{2} V^{r}(\tau^{*})}{\partial \tau_{L}^{2}}] \\ &+ \lambda^{p} [f^{p}(0) \frac{\partial V^{p}(\tau^{*})}{\partial \tau_{L}} \frac{\partial V^{p}(\tau^{*})}{\partial \tau_{L}} + K f^{p}(0) \frac{\partial^{2} V^{p}(\tau^{*})}{\partial \tau_{L}^{2}}] \\ &+ \lambda^{r} \Big[ F^{r}(0) \frac{\partial^{2} V^{p}(\tau^{*})}{\partial \tau_{L}^{2}} + \frac{\partial V^{p}(\tau^{*})}{\partial \tau_{L}} f^{r}(0) \frac{\partial V^{r}(\tau^{*})}{\partial \tau_{L}} \Big] \\ &+ \lambda^{p} \Big[ F^{p}(0) \frac{\partial^{2} V^{p}(\tau^{*})}{\partial \tau_{L}^{2}} + \frac{\partial V^{p}(\tau^{*})}{\partial \tau_{L}} f^{p}(0) \frac{\partial V^{p}(\tau^{*})}{\partial \tau_{L}} \Big] \end{split}$$

Re-arranging we have:

$$\begin{split} SC &= \lambda^{r} f^{r} (0) \left[ \frac{\partial V^{r}(\tau^{*})}{\partial \tau_{L}} \frac{\partial V^{p}(\tau^{*})}{\partial \tau_{L}} + K \frac{\partial^{2} V^{r}(\tau^{*})}{\partial \tau_{L}^{2}} + \frac{\partial V^{p}(\tau^{*})}{\partial \tau_{L}} \frac{\partial V^{r}(\tau^{*})}{\partial \tau_{L}} \right] \\ &+ \lambda^{p} f^{p} (0) \left[ \frac{\partial V^{p}(\tau^{*})}{\partial \tau_{L}} \right]^{2} + K \frac{\partial^{2} V^{p}(\tau^{*})}{\partial \tau_{L}^{2}} + \left[ \frac{\partial V^{p}(\tau^{*})}{\partial \tau_{L}} \right]^{2} \right] \\ &+ \lambda^{r} F^{r} (0) \frac{\partial^{2} V^{p}(\tau^{*})}{\partial \tau_{L}^{2}} + \lambda^{p} F^{p} (0) \frac{\partial^{2} V^{p}(\tau^{*})}{\partial \tau_{L}^{2}} \end{split}$$

As  $\frac{\partial V^{P}(\tau^{*})}{\partial \tau_{L}} > 0$ ,  $\frac{\partial V^{r}(\tau^{*})}{\partial \tau_{L}} < 0$ ,  $\frac{\partial^{2} V^{P}(\tau^{*})}{\partial \tau_{L}^{2}} < 0$ , and  $\frac{\partial^{2} V^{r}(\tau^{*})}{\partial \tau_{L}^{2}} < 0$  the first and the third line of the previous expression are negative. There exists  $K_{0}$  such that  $2\left[\frac{\partial V^{P}(\tau^{*})}{\partial \tau_{L}}\right]^{2} + K_{0}\frac{\partial^{2} V^{P}(\tau^{*})}{\partial \tau_{L}^{2}} = 0$  so that for any  $K \ge K_{0}$  we have  $2\left[\frac{\partial V^{P}(\tau^{*})}{\partial \tau_{L}}\right]^{2} + K_{0}\frac{\partial^{2} V^{P}(\tau^{*})}{\partial \tau_{L}^{2}} \le 0$ . In this case the second line of the expression is also negative. As a consequence SC < 0 at  $\tau^{*} = \tau_{R} = \tau_{L}$  whenever  $K \ge K_{0}$ . Therefore  $\tau^{*}$  is a local maximum.

#### Proof of Proposition 11.

Assume that  $0 < E_p \leq \hat{E}_p$  and  $0 < E_r \leq \hat{E}_r$ , where the values  $(\hat{E}_p, \hat{E}_r)$  are defined as above. Then, as proved in proposition 10,  $\frac{\partial \tau}{\partial B_r} < 0$  for  $0 < E_r \leq \hat{E}_r$  and  $\frac{\partial \tau}{\partial B_p} > 0$  for  $0 < E_p \leq \hat{E}_p$ . Note the prevailing in-altruism exhibited by the social groups as  $B_1 = (B_{p1}, B_{r1})$ .

a) Assume the rich increase their inner altruism while the poor decrease theirs in the same amount,  $((B_{p2}, B_{r2}) = (B_{p1} - \varphi, B_{r1} + \varphi) = (B_{p1}, B_{r1}) + \varphi(-1, 1))$ , for  $\varphi > 0$ . This change implies that both social groups vary their in-altruism in direction  $B_2 = (-1, 1)$ . The impact on the equilibrium tax rate  $\tau^*$  when both groups modify their inner altruism on direction  $B_2$  is given by the following directional derivative

$$\frac{\partial \tau^*}{\partial B_2} = \left(\frac{\partial \tau^*}{\partial B_p}, \frac{\partial \tau^*}{\partial B_r}\right) \cdot \frac{B_2}{\|B_2\|} = \left(\frac{\partial \tau^*}{\partial B_p}, \frac{\partial \tau^*}{\partial B_r}\right) \cdot \frac{1}{\sqrt{2}} (-1, 1),$$

Then.

$$\frac{\partial \tau^*}{\partial B_2} = \frac{1}{\sqrt{2}} \left( -\frac{\partial \tau^*}{\partial B_p} + \frac{\partial \tau^*}{\partial B_r} \right) < 0.$$

Then, if the rich increase their inner altruism while the poor decrease theirs in the same amount in equilibrium the tax rate decreases.

b) Assume the poor increase their inner altruism while the rich decrease theirs in the same amount,  $((B_{p2}, B_{r2}) = (B_{p1} + \varphi, B_{r1} - \varphi) = (B_{p1}, B_{r1}) + \varphi(1, -1))$ , for  $\varphi > 0$ . This change implies that both social groups vary their in-altruism in direction  $B_3 = (1, -1)$ . The impact on the tax rate  $\tau^*$  when both groups modify their inner altruism in the direction  $B_3$  is given by the following directional derivative

$$\frac{\partial \tau^*}{\partial B_3} = \left(\frac{\partial \tau^*}{\partial B_p}, \frac{\partial \tau^*}{\partial B_r}\right) \cdot \frac{B_3}{\|B_3\|} = \left(\frac{\partial \tau^*}{\partial B_p}, \frac{\partial \tau^*}{\partial B_r}\right) \cdot \frac{1}{\sqrt{2}} (1, -1),$$

Then.

$$\frac{\partial \tau^*}{\partial B_3} = \frac{1}{\sqrt{2}} \left( \frac{\partial \tau^*}{\partial B_r} - \frac{\partial \tau^*}{\partial B_r} \right) > 0.$$

Then, if the poor increase their inner altruism while the rich decrease theirs in the same amount the equilibrium tax rate increases.

These results hold irrespective of the prevailing levels of in-group altruism  $B_{g1}$ .

**Proof of Proposition 12.** From Proposition 10 we know that  $\frac{\partial \tau}{\partial B_r} < 0$  for  $0 < E_r \leq \hat{E}_r$  and  $\frac{\partial \tau}{\partial B_p} > 0$  for  $0 < E_p \leq \hat{E}_p$ . Assume the prevailing levels of ideology of the social groups are given by the vector  $f_1(0) = (f_1^{\ p}(0), f_1^{\ r}(0))$ . We want to know how the equilibrium tax rate  $\tau^*$  is affected by a simultaneous change in ideology. We focus on changes such that both social groups vary their ideology in the direction  $f(0) = (f^{\ p}(0), f^{\ r}(0))$ . The new levels of ideology are given by the vector  $f_2(0) = (f_1^{\ p}(0) + tf^{\ p}(0), f_1^{\ r}(0) + tf^{\ r}(0)) = (f_1^{\ p}(0), f_1^{\ r}(0)) + t(f^{\ p}(0), f^{\ r}(0))$ , for  $t \in \mathbb{R}$ . To calculate the impact of this change of ideology we need to compute the directional derivative  $\frac{\partial \tau^*}{\partial f(0)}$ , which is given by

$$\frac{\partial \tau^*}{\partial f(0)} = \left(\frac{\partial \tau^*}{\partial f^p(0)}, \frac{\partial \tau^*}{\partial f^r(0)}\right) \frac{1}{\|f(0)\|} \left(f^p(0), f^r(0)\right)$$

Let us start finding the direction vector  $f_{\perp}(0) = (f_{\perp}^{\ p}(0), f_{\perp}^{\ r}(0))$  such that if both social groups change their ideology according to that direction, the impact on the equilibrium tax rate is null, that is,  $f_{\perp}(0) = (f_{\perp}^{\ p}(0), f_{\perp}^{\ r}(0))$  leads to  $\frac{\partial \tau}{\partial f_{\perp}(0)} = 0$ . Then

$$\frac{\partial \tau^*}{\partial f_{\perp}(0)} = \frac{1}{\|f_{\perp}(0)\|} \left( \frac{\partial \tau^*}{\partial f^{p}(0)} f_{\perp}^{p}(0) + \frac{\partial \tau^*}{\partial f^{r}(0)} f_{\perp}^{r}(0) \right) = 0$$

A simple manipulation gives

$$f_{\perp}(0) = \left(-\frac{\partial \tau^{*}}{\partial f^{r}(0)}, \frac{\partial \tau^{*}}{\partial f^{p}(0)}\right)$$

Then.

$$\frac{\partial \tau^*}{\partial f_{\perp}(0)} = 0 \text{ if } f_{\perp}(0) = \left(f_{\perp}^{p}(0), f_{\perp}^{r}(0)\right) = \left(-\frac{\partial \tau^*}{\partial f^{r}(0)}, \frac{\partial \tau^*}{\partial f^{p}(0)}\right) \quad (31)$$

Note that the vector  $f_{\perp}(0) = (f_{\perp}^{p}(0), f_{\perp}^{r}(0))$  is perpendicular to the gradient vector  $\nabla \tau (f_{0}^{p}(0), f_{0}^{r}(0)) = (\frac{\partial \tau}{\partial f^{p}(0)}, \frac{\partial \tau}{\partial f^{r}(0)})$ 

We start the analysis by a simplifying assumption, which will be relaxed in Part II. We assume that the vector  $f_{\perp}(0)$  is parallel to the vector (1,1), that is,  $f_{\perp}^{\ p}(0) = f_{\perp}^{\ r}(0)$ . Using this condition of colinearity with (1,1) we obtain

$$-\frac{\partial \tau^*}{\partial f^r(0)} = \frac{\partial \tau^*}{\partial f^p(0)}$$
(32)

From expressions (18) and (19) we have the following facts:

$$\frac{\partial \tau^{*}}{\partial f^{p}(0)} = \left(\frac{\lambda^{p}}{D}\right) \left[\frac{\partial V^{p}(\tau^{*})}{\partial \tau}\right] K > 0$$
$$\frac{\partial \tau^{*}}{\partial f^{r}(0)} = \left(\frac{\lambda^{r}}{D}\right) \left[\frac{\partial V^{r}(\tau^{*})}{\partial \tau}\right] K < 0$$

where  $D = C''(\tau^*)\overline{y}(\lambda^r \chi^r(1+B_r) + \lambda^p \chi^p(1+B_p))$ . Then, substituting in (32) we have

$$-\left(\frac{1}{D}\right)\lambda^{r}\left[\frac{\partial V^{r}(\tau^{*})}{\partial \tau}\right]K = \left(\frac{1}{D}\right)\lambda^{p}\left[\frac{\partial V^{p}(\tau^{*})}{\partial \tau}\right]K$$
$$-\frac{\lambda^{r}}{\lambda^{p}}\left[\frac{\partial V^{r}(\tau^{*})}{\partial \tau}\right] = \left[\frac{\partial V^{p}(\tau^{*})}{\partial \tau}\right]$$

(33)

or

To satisfy expression (32) we need to find conditions to satisfy expression (33). We know that

$$\frac{\partial V^{p}(\tau^{*})}{\partial \tau} = \left[ \left( 1 - C'(\tau^{*}) \right) \overline{y} - y^{p} \right] + B_{p} \left[ \left( 1 - C'(\tau^{*}) \right) \overline{y} - \overline{y}_{p} + E_{p} \sum_{k \in \Gamma^{p}} \frac{|y^{k} - \overline{y}_{p}|}{n_{p}} \right] > 0$$
  
and  
$$\frac{\partial V^{r}(\tau^{*})}{\partial \tau} = \left[ \left( 1 - C'(\tau^{*}) \right) \overline{y} - y^{r} \right] + B_{r} \left[ \left( 1 - C'(\tau^{*}) \right) \overline{y} - \overline{y}_{r} + E_{r} \sum_{k \in \Gamma^{r}} \frac{|y^{k} - \overline{y}_{r}|}{n_{r}} \right] < 0$$

First, supposing  $B_r = 0$  , we see that  $\exists \tilde{B}_p$  such that (33) holds. This is given by  $-\frac{\lambda^{r}}{\lambda^{p}}\left[\left(1-C'(\tau^{*})\right)\overline{y}-y^{r}\right]$  $= \left[ \left(1 - C'(\tau^*)\right) \overline{y} - y^p \right] + \widetilde{B}_p \left[ \left(1 - C'(\tau^*)\right) \overline{y} - \overline{y}_p + E_p \sum_{k \in \Gamma^p} \frac{|y^k - \overline{y}_p|}{n_p} \right]$ 

Then,

$$\widetilde{\boldsymbol{B}}_{p} = \frac{-\frac{\lambda^{r}}{\lambda^{p}} \left[ \left(1 - C'(\tau^{*})\right) \overline{y} - y^{r} \right] \cdot \left[ \left(1 - C'(\tau^{*})\right) \overline{y} - y^{p} \right]}{\left[ \left(1 - C'(\tau^{*})\right) \overline{y} - \overline{y}_{p} + E_{p} \sum_{k \in \Gamma^{p}} \frac{|y^{k} - \overline{y}_{p}|}{n_{p}} \right]}$$
(34)

Whether  $\widetilde{B}_p$  is positive or negative depends on the values of  $y^r$  and  $y^p$  as the denominator is positive for  $\, 0 \,{<}\, E_{_{p}} \,{\leq}\, \hat{E}_{_{p}}\,$  . The numerator is positive when

$$-\lambda^{r} \left[ \left( 1 - C'(\tau^{*}) \right) \overline{y} - y^{r} \right] - \lambda^{p} \left[ \left( 1 - C'(\tau^{*}) \right) \overline{y} - y^{p} \right] > 0$$
(35)

Consider two individuals, a poor and a rich. For a poor individual there exists  $\varepsilon > 0$  such that  $y^p = \overline{y} - \varepsilon$ . For a rich individual there exists  $\varepsilon_1 > 0$  such that  $y^r = \overline{y} + \varepsilon_1$ . Substituting into expression (35) we have

$$\lambda^{r}\left[\left(1-C'(\tau^{*})\right)\overline{y}-\overline{y}-\varepsilon_{1}\right]-\lambda^{p}\left[\left(1-C'(\tau^{*})\right)\overline{y}-\overline{y}+\varepsilon\right]>0$$

leading to

$$C'(\tau^*)\overline{y} + \lambda^r \varepsilon_1 - \lambda^p \varepsilon > 0$$

As  $C'(\tau^*)\overline{y} > 0$  then for  $\widetilde{B}_p$  to be positive we need that  $\lambda^r \varepsilon_1 - \lambda^p \varepsilon \ge 0$  which reduces to

$$\left[y^{r}-\overline{y}\right] \geq \frac{\lambda^{p}}{\lambda^{r}} \left[\overline{y}-\overline{y}^{p}\right]$$
(36)

We assume expression (36) holds, therefore  $\tilde{B}_p > 0$ .<sup>14</sup> Second, assume that  $B_p = 0$ , then  $\exists \tilde{B}_r$  such that (33) holds. This occurs if

$$-\frac{\lambda^{r}}{\lambda^{p}} \left[ \left( 1 - C'(\tau^{*}) \right) \overline{y} - y^{r} \right] - \widetilde{B}_{r} \frac{\lambda^{r}}{\lambda^{p}} \left[ \left( 1 - C'(\tau^{*}) \right) \overline{y} - \overline{y}_{r} + E_{r} \sum_{k \in \Gamma^{r}} \frac{|y^{k} - \overline{y}_{r}|}{n_{r}} \right]$$
$$= \left[ \left( 1 - C'(\tau^{*}) \right) \overline{y} - y^{p} \right]$$

Then,

$$\tilde{B}_{r} = \frac{\frac{-\frac{\lambda^{r}}{\lambda^{p}} \left[ \left( 1 - C'(\tau^{*}) \right) \overline{y} - \overline{y}^{r} \right] - \left[ \left( 1 - C'(\tau^{*}) \right) \overline{y} - \overline{y}^{p} \right]}{\frac{\lambda^{r}}{\lambda^{p}} \left[ \left( 1 - C'(\tau^{*}) \right) \overline{y} - \overline{y}_{r} + E_{r} \sum_{k \in \Gamma^{r}} \frac{|y^{k} - \overline{y}_{r}|}{n_{r}} \right]}$$
(37)

We already know that as  $\left| \left( 1 - C'(\tau^*) \right) \overline{y} - \overline{y}_r + E_r \sum_{k \in \Gamma'} \frac{|y^k - \overline{y}_r|}{n_r} \right| < 0$  for  $0 < E_r \le \hat{E}_r$ the denominator is negative. The numerator of expression (37) is exactly expression (35) which is positive. Then, in this case  $\tilde{B}_r < 0$ .

Now that we have computed the bounds  $(\tilde{B}_p, \tilde{B}_r)$  we can find  $(B_p, B_r)$  such that expression (33) holds. This occurs when

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<sup>&</sup>lt;sup>14</sup>For  $\widetilde{B}_p < 0$  we need to satisfy the condition  $C'(\tau^*)\overline{y} + \lambda^r [y^r - \overline{y}] < \lambda^p [\overline{y} - y^p]$ , which is stronger than expression (36).

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$$\begin{split} & \frac{\mathcal{L}'}{\lambda^{p}} \Big[ \Big( \mathbf{I} - C' \Big( \tau^{*} \Big) \Big) \overline{\mathbf{y}} - \mathbf{y}^{r} \Big] - B_{r} \frac{\mathcal{L}'}{\lambda^{p}} \Big[ \Big( \mathbf{I} - C' \Big( \tau^{*} \Big) \Big) \overline{\mathbf{y}} - \overline{\mathbf{y}}_{r} + E_{r} \sum_{k \in \Gamma'} \frac{|\mathbf{y}^{k} - \overline{\mathbf{y}}_{r}|}{n_{r}} \Big] \\ &= \Big[ \Big( \mathbf{I} - C' \Big( \tau^{*} \Big) \Big) \overline{\mathbf{y}} - \mathbf{y}^{p} \Big] + B_{p} \Big[ \Big( \Big( \mathbf{I} - C' \Big( \tau^{*} \Big) \Big) \overline{\mathbf{y}} - \overline{\mathbf{y}}_{p} + E_{p} \sum_{k \in \Gamma'} \frac{|\mathbf{y}^{k} - \overline{\mathbf{y}}_{p}|}{n_{p}} \Big] \end{split}$$

Some manipulations lead to

$$\boldsymbol{B}_{r} = \frac{\left[\left(1-C'(\tau^{*})\right)\overline{\boldsymbol{y}}-\overline{\boldsymbol{y}}_{p}+E_{p}\sum_{k\in\Gamma^{p}}\frac{|\boldsymbol{y}^{k}-\overline{\boldsymbol{y}}_{p}|}{n_{p}}\right]}{-\frac{\lambda^{r}}{\lambda^{p}}\left[\left(1-C'(\tau^{*})\right)\overline{\boldsymbol{y}}-\overline{\boldsymbol{y}}_{r}+E_{r}\sum_{k\in\Gamma^{r}}\frac{|\boldsymbol{y}^{k}-\overline{\boldsymbol{y}}_{r}|}{n_{r}}\right]}\boldsymbol{B}_{p} + \frac{\frac{\lambda^{r}}{\lambda^{p}}\left[\left(1-C'(\tau^{*})\right)\overline{\boldsymbol{y}}-\boldsymbol{y}^{r}\right]+\left[\left(1-C'(\tau^{*})\right)\overline{\boldsymbol{y}}-\boldsymbol{y}^{p}\right]}{-\frac{\lambda^{r}}{\lambda^{p}}\left[\left(1-C'(\tau^{*})\right)\overline{\boldsymbol{y}}-\overline{\boldsymbol{y}}_{r}+E_{r}\sum_{k\in\Gamma^{r}}\frac{|\boldsymbol{y}^{k}-\overline{\boldsymbol{y}}_{r}|}{n_{r}}\right]}$$
(38)

Substituting expressions (34) and (37) into expression (38) we have

$$B_r = -\frac{\widetilde{B}_r}{\widetilde{B}_p}B_p + \widetilde{B}_r$$

Then expression (33) holds whenever the social groups exhibit inner altruism given by

$$\left(B_{p},B_{r}\right) = \left(B_{p},-\frac{\widetilde{B}_{r}}{\widetilde{B}_{p}}B_{p}+\widetilde{B}_{r}\right)$$
(39)

for  $B_p > \tilde{B}_p$ .<sup>15</sup> As a consequence, expression (32) holds whenever the social groups exhibit inner altruism given by  $(B_p, B_r)$  such that  $B_p > \tilde{B}_p$  and  $B_r = -\frac{\tilde{B}_r}{\tilde{B}_p}B_p + \tilde{B}_r$ . This condition ensures that the derivatives are identical and have opposite sign. Condition (39) allows us to show that there are values  $(\tilde{B}_p, \tilde{B}_r)$  such that if the level of inner altruism exhibited by the poor and the rich  $(B_p, B_r)$  satisfy  $B_p > \tilde{B}_p$  and  $B_r = -\frac{\tilde{B}_r}{\tilde{B}_p}B_p + \tilde{B}_r$ , a change in the level of ideology of the groups in the direction  $f(0) = (f^p(0), f^r(0))$  generates the following comparative static effect:

- a)  $\frac{\partial \tau^*}{\partial f(0)} = 0$ , if  $f^p(0) = f'(0)$ ,
- b)  $\frac{\partial \tau^*}{\partial f(0)} > 0$ , if  $f^p(0) > f^r(0)$ , i.e., the tax rate increases when ideology decreases, provide the poor are less ideological.
- c)  $\frac{\partial t^*}{\partial f(0)} < 0$ , if  $f^p(0) < f^r(0)$ , i.e., the tax rate decreases when ideology decreases, provide the rich are less ideological.

<sup>&</sup>lt;sup>15</sup>We need to satisfy this condition to have  $B_r > 0$ . If  $B_p \le \widetilde{B}_p$  then  $B_r \le 0$ , which is meaningless in the model.

This first set of results are obtained for the knife-edge condition (32) and holds for a value of  $B_r$  that is uniquely determined by the choice of  $B_p$ . We now extend our result to an open set of values of  $(B_p, B_r)$ . We now extend this results by relaxing the condition (32) and instead analyze the case when  $-\frac{\partial \tau^*}{\partial f'(0)} > \frac{\partial \tau^*}{\partial f'(0)}$ . From the previous paragraphs we see that whenever  $(B_p, B_r)$  satisfy  $B_p > \tilde{B}_p$  and  $B_r > -\frac{\tilde{B}_r}{B_p}B_p + \tilde{B}_r$  we have that  $-\frac{\partial \tau^*}{\partial f'(0)} > \frac{\partial \tau^*}{\partial f'(0)}$ . We then have the second group of results:

1) There are values  $(\tilde{B}_p, \tilde{B}_r)$  such that if the level of inner altruism exhibited by the poor and the rich  $(B_p, B_r)$  satisfy  $B_p > \tilde{B}_p$  and  $B_r > -\frac{\tilde{B}_r}{\tilde{B}_p}B_p + \tilde{B}_r$ , a change in ideology in the direction  $f(0) = (f^p(0), f^r(0))$  generates the following effect:

$$\mathbf{a} ) \quad \frac{\partial \tau^{*}}{\partial f(0)} = 0 \quad \text{if} \quad \frac{f^{p}(0)}{f^{r}(0)} = \frac{-\frac{\partial \tau^{*}}{\partial f^{p}(0)}}{\frac{\partial \tau^{*}}{\partial f^{p}(0)}} > 1 \Longrightarrow f^{p}(0) > f^{r}(0)$$

$$\mathbf{b} ) \quad \frac{\partial \tau^{*}}{\partial f(0)} > 0 \quad \text{, if } \quad \frac{f^{p}(0)}{f^{r}(0)} > \frac{-\frac{\partial \tau^{*}}{\partial f^{p}(0)}}{\frac{\partial \tau^{*}}{\partial f^{p}(0)}} > 1 \implies f^{p}(0) >> f^{r}(0)$$

$$\mathbf{c} ) \quad \frac{\partial \tau^{*}}{\partial f(0)} < 0 \quad \text{, if } \quad \frac{f^{p}(0)}{f^{r}(0)} < \frac{-\frac{\partial \tau^{*}}{\partial f^{p}(0)}}{\frac{\partial \tau^{*}}{\partial f^{p}(0)}} \quad \text{which is satisfied for } f^{p}(0) < f^{r}(0) \text{ and for }$$

$$f^{p}(0) > f^{r}(0) > \frac{\frac{\partial \tau^{*}}{\partial f^{p}(0)}}{-\frac{\partial \tau^{*}}{\partial f^{p}(0)}} f^{p}(0) \quad .$$

Result 1c) is interesting as it allows for a reduction in the tax rate while the rich are more ideological than the poor. We now find conditions for this case to occur. From expression (39) we see that whether or not the poor are more altruistic than the rich,  $B_p$  vs  $B_r$ , depends on the values  $\tilde{B}_p$  and  $\tilde{B}_r$ . Given the functional form of  $B_r$  in expression (38), if  $-\tilde{B}_r \leq \tilde{B}_p$  then automatically  $B_r > B_p$  implies that the conditions for the second group of results are satisfied, as in this case  $B_p > \tilde{B}_p$  and  $B_r > -\frac{\tilde{B}_r}{B_p}B_p + \tilde{B}_r$ .<sup>16</sup> The case when  $-\tilde{B}_r > \tilde{B}_p$  is more complicated and do not offer so much additional information to the analysis. Therefore we just need to prove that  $-\tilde{B}_r \leq \tilde{B}_p$ .

<sup>&</sup>lt;sup>16</sup>There are some altruistic terms such that  $B_p < B_r$  that also satisfy the conditions for the second group of results. However, we do not consider them for the analysis.

Instead of working directly with the inequality  $-\tilde{B}_r \leq \tilde{B}_p$ , we start finding conditions that satisfy  $-\tilde{B}_r = \tilde{B}_p$ . We have:

$$\frac{\frac{\lambda^{r}}{\lambda^{p}}\left[\left(1-C'(\tau^{*})\right)\overline{y}-y^{r}\right]+\left[\left(1-C'(\tau^{*})\right)\overline{y}-y^{p}\right]}{\frac{\lambda^{r}}{\lambda^{p}}\left[\left(1-C'(\tau^{*})\right)\overline{y}-\overline{y}_{r}+E_{r}\sum_{k\in\Gamma^{r}}\frac{|y^{k}-\overline{y}_{r}|}{n_{r}}\right]}=\frac{\frac{\lambda^{r}}{\lambda^{p}}\left[\left(1-C'(\tau^{*})\right)\overline{y}-y^{r}\right]-\left[\left(1-C'(\tau^{*})\right)\overline{y}-y^{p}\right]}{\left[\left(1-C'(\tau^{*})\right)\overline{y}-\overline{y}_{p}+E_{p}\sum_{k\in\Gamma^{p}}\frac{|y^{k}-\overline{y}_{p}|}{n_{p}}\right]}$$

that implies

$$\left[ \left( 1 - C'(\tau^*) \right) \overline{y} - \overline{y}_p + E_p \sum_{k \in \Gamma^p} \frac{|y^k - \overline{y}_p|}{n_p} \right]$$

$$= -\frac{\lambda^r}{\lambda^p} \left[ \left( 1 - C'(\tau^*) \right) \overline{y} - \overline{y}_r + E_r \sum_{k \in \Gamma^r} \frac{|y^k - \overline{y}_r|}{n_r} \right]$$

$$(40)$$

We state conditions on  $E_p$  and  $E_r$  to satisfy expression (40).

First,  $\exists \tilde{E}_p$  such that if  $E_r = 0$  then expression (40) holds. This is

$$-\frac{\lambda^{r}}{\lambda^{p}}\left[\left(1-C'(\tau^{*})\right)\overline{y}-\overline{y}_{r}\right]=\left[\left(1-C'(\tau^{*})\right)\overline{y}-\overline{y}_{p}+\widetilde{E}_{p}\sum_{k\in\Gamma^{p}}\frac{|y^{k}-\overline{y}_{p}|}{n_{p}}\right]$$

Then,

$$\widetilde{E}_{p} = \frac{\frac{-\frac{\lambda'}{\lambda^{p}} \left[ \left( 1 - C'(\tau^{*}) \right) \overline{y} - \overline{y}_{p} \right] \left[ \left( 1 - C'(\tau^{*}) \right) \overline{y} - \overline{y}_{p} \right]}{\sum_{k \in \Gamma^{p}} \frac{|y^{k} - \overline{y}_{p}|}{n_{p}}} > 0$$
(41)

Therefore,  $\tilde{E}_p > 0$  exists and from the expression of  $\tilde{E}_p$  we see that  $\tilde{E}_p < \hat{E}_p$ . Second,  $\exists \tilde{E}_r$  such that if  $E_p = 0$  then expression (40) holds. This is

$$\left[\left(1-C'(\tau^*)\right)\overline{y}-\overline{y}_p\right]$$
$$=-\frac{\lambda^r}{\lambda^p}\left[\left(1-C'(\tau^*)\right)\overline{y}-\overline{y}_r+\widetilde{E}_r\sum_{k\in\Gamma^r}\frac{|y^k-\overline{y}_r|}{n_r}\right]$$

Then,

$$\widetilde{E}_{r} = \frac{-\left[\left(1 - C'(\tau^{*})\right)y - y^{p}\right] \frac{\lambda^{r}}{\lambda^{p}} \left[\left(1 - C'(\tau^{*})\right)y - y^{r}\right]}{\frac{\lambda^{r}}{\lambda^{p}} \sum_{k \in \Gamma'} \frac{|y^{k} - \overline{y}_{r}|}{n_{r}}} > 0$$
(42)

Therefore,  ${\tilde E}_r>0\,$  exists and from the expression of  ${\tilde E}_r\,$  we see that  ${\tilde E}_r<{\hat E}_r\,$  .

We can now use these values, to state that there exists  $(E_p, E_r)$  such that expression (40) holds. This condition can be written as

$$\begin{bmatrix} \left(1 - C'(\tau^*)\right)\overline{y} - \overline{y}_p + E_p \sum_{k \in \Gamma^p} \frac{|y^k - \overline{y}_p|}{n_p} \end{bmatrix}$$
$$= -\frac{\lambda^r}{\lambda^p} \begin{bmatrix} \left(1 - C'(\tau^*)\right)\overline{y} - \overline{y}_r + E_r \sum_{k \in \Gamma^r} \frac{|y^k - \overline{y}_r|}{n_r} \end{bmatrix}$$

After a few manipulations we end up with

$$E_{r} = \frac{\sum_{k \in \Gamma^{p}} \frac{|y^{k} - \overline{y}_{p}|}{n_{p}}}{\frac{\lambda^{r}}{\lambda^{p}} \sum_{k \in \Gamma^{r}} \frac{|y^{k} - \overline{y}_{r}|}{n_{r}}} E_{p} + \frac{-\frac{\lambda^{r}}{\lambda^{p}} \left[ \left(1 - C'(\tau^{*})\right) \overline{y} - y^{r} \right] - \left[ \left(1 - C'(\tau^{*})\right) \overline{y} - y^{p} \right]}{\frac{\lambda^{r}}{\lambda^{p}} \sum_{k \in \Gamma^{r}} \frac{|y^{k} - \overline{y}_{r}|}{n_{r}}}$$

$$(43)$$

Substituting (41) and (42) into (43) we have

$$E_r = -\frac{\widetilde{E}_r}{\widetilde{E}_p}E_p + \widetilde{E}_p$$

Then expression (40) holds, that is,  $-\tilde{B}_r = \tilde{B}_p$ , whenever the social groups place weights to inequality aversion given by  $(E_p, E_r)$  such that

$$\left(E_{p}, E_{r}\right) = \left(E_{p}, -\frac{\widetilde{E}_{r}}{\widetilde{E}_{p}}E_{p} + \widetilde{E}_{r}\right)$$
(44)

As a consequence, to satisfy the inequality  $-\tilde{B}_r \leq \tilde{B}_p$  we need weights on inequality aversion  $(E_p, E_r)$  such that  $0 < E_p \leq \tilde{E}_p$  and  $0 < E_r \leq -\frac{\tilde{E}_r}{\tilde{E}_p}E_p + \tilde{E}_p$ .

To conclude, Result 1c) implies the following, which Proposition 12, part 1. There are values  $(\tilde{E}_p, \tilde{E}_r)$ , with  $\tilde{E}_p \leq \hat{E}_p$  and  $\tilde{E}_r \leq \hat{E}_r$  for  $(\hat{E}_p, \hat{E}_r)$  as defined previously, such that if: 1) the weights given by the poor and the rich to inequality aversion  $(E_p, E_p)$  are such that  $E_p \leq \tilde{E}_p$  and  $E_r \leq -\frac{\tilde{E}_r}{E_p}E_p + \tilde{E}_p$ ; 2) the rich exhibit greater ingroup altruism than the poor  $(B_r > B_p)$ ; and 3) the rich are less ideological than the poor,  $f^r(0) > f^p(0)$ , then as the social groups change their level of ideology in the direction  $f(0) = (f^p(0), f^r(0))$  the equilibrium tax rate decreases. The second part of the Proposition rests again on Result 1c). However, the focus is now on the interval  $\frac{\frac{\partial f^r}{\partial f'(0)}}{\frac{\partial e^r}{\partial f'(0)}} f^p(0) < f^p(0)$  for which the rich are moderately more ideological than the poor.

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